
WALTON'S
ILLUSTRATIVE
PRACTICAL ARITHMETIC
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CONSTANCE WINSOR.

Constance Thurston

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Waltons' Normal Series.

THE

ILLUSTRATIVE PRACTICAL ARITHMETIC

BY A NATURAL METHOD,

WITH

DICTATION EXERCISES.

FOR COMMON SCHOOLS, HIGH SCHOOLS, NORMAL SCHOOLS, AND ACADEMIES.

BY

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ARITHMETICAL TABLE.

For Explanation, see "Manual and Key," pages 23, 25, etc.

	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	
A –	9	8	7	4	4	9	0	2	5	9	8	7	9	1	5	3	1	7	1	8	4	– A
B –	9	0	5	7	8	8	8	8	4	4	2	5	8	7	5	9	9	5	3	3	7	– B
C –	6	7	9	4	3	5	5	6	8	6	7	2	6	1	7	8	7	2	6	9	2	– C
D –	3	6	9	6	6	3	5	3	6	3	0	7	8	6	8	3	2	7	4	7	6	– D
E –	1	5	3	9	6	7	8	5	6	2	1	6	5	7	5	4	3	6	2	0	8	– E
F –	8	8	8	4	5	5	5	5	0	3	2	1	3	8	7	2	1	6	3	5	6	– F
G –	8	0	6	3	9	9	5	1	6	1	1	1	6	9	4	5	0	3	5	7	5	– G
H –	9	1	7	5	1	7	2	4	4	8	7	2	9	4	6	8	4	2	9	9	3	– H
I –	9	2	7	9	3	1	3	3	0	5	6	4	6	4	9	4	7	1	8	8	8	– I
J –	9	5	3	2	0	5	6	4	9	8	7	6	8	0	7	1	7	4	4	4	7	– J
K –	6	6	7	8	7	1	6	6	8	3	1	8	2	0	6	8	6	2	2	0	6	– K
L –	7	2	8	4	3	1	6	6	9	4	1	9	5	2	6	3	5	4	4	1	7	– L
M –	8	4	5	2	1	9	2	4	8	1	8	7	8	4	9	5	3	4	6	2	1	– M
N –	1	4	4	8	6	8	1	5	2	1	6	0	2	1	6	1	1	1	5	8	4	– N
O –	2	2	5	1	8	9	3	9	3	8	7	5	4	5	4	1	9	7	2	9	0	– O
P –	1	9	9	5	9	8	9	8	2	2	8	4	5	7	5	4	9	0	9	4	0	– P
Q –	9	4	1	5	2	9	5	5	5	2	9	4	1	8	0	8	7	6	7	0	0	– Q
R –	7	9	5	7	2	1	8	2	3	8	9	6	5	9	4	9	0	2	2	9	8	– R
S –	7	3	4	2	5	6	3	2	1	5	1	7	8	0	6	3	9	6	4	7	5	– S
T –	3	2	3	5	8	3	5	3	0	8	9	4	5	1	6	4	8	4	6	2	7	– T
U –	8	2	4	1	7	4	7	7	4	2	8	7	4	9	9	2	3	5	5	8	6	– U
V –	7	6	9	4	1	6	2	6	8	4	0	6	3	6	4	3	8	7	2	1	9	– V
W –	1	0	8	8	2	4	8	2	4	7	1	6	7	6	2	8	9	7	9	8	4	– W
X –	8	7	2	8	9	2	9	8	2	8	7	2	8	9	8	6	2	4	7	6	8	– X
Y –	4	4	4	7	6	4	4	2	5	5	7	4	4	5	7	4	7	6	5	1	1	– Y
	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	

PREFACE.

THE plan of the Illustrative Practical Arithmetic is indicated by its title; it embraces the following

GENERAL PRINCIPLES.

1. *The subjects taught are presented in their natural order.* Part I. contains an elementary course in the fundamental operations with applications to United States Money, Bills and Receipts. Part II. contains concise reviews of the fundamental operations, with rules; Properties of Numbers; Fractions, Common and Decimal; Compound Numbers and Metric System; Percentage, with its applications; Ratio and Proportion, with Partnership; Involution and Evolution; Mensuration. Contractions in Multiplication and Division, Annual Interest, etc., being incidental, are placed in an Appendix.

2. *Ideas are excited by familiar illustrations, in which reference is always had to the objects themselves.* See the treatment of the fundamental operations, Fractions, Percentage, etc.

3. *The unknown is taught through the known.* This principle is illustrated in every part of the book, which is so arranged that the occasion for knowledge which the pupil acquires is found in an illustration or in knowledge he previously possessed.

4. *Each synthetic statement follows from a previous analysis.* See the manner of deriving definitions and rules from the analysis of illustrative examples, with accompanying questions to be answered synthetically.

5. *The language is an exact expression of the ideas excited by the illustrations.* See definitions and explanations throughout the book.

It has been necessary in some instances to reject stereotyped forms of expression as meaningless, inappropriate, or contradictory, and to adopt language that describes the operations with greater accuracy, and which in many cases is much more simple. See explanation of Subtraction, Multiplication, Fractions, Mensuration, etc.

6. *Usually but one process is taught for a particular operation, and that the most practical.* See Subtraction; Division of Integral Numbers and of Decimals; Interest; Evolution, etc.

7. *Matter and methods which have become obsolete or useless to the general student are rejected;* such as English Notation; much of Com-

pound Numbers; Circulating Decimals; Duodecimals; Alligation Alternate; the Progressions; Annuities; also, many terms, as "local value," "borrowing and carrying," "improper fractions," etc. As "Analysis" and "Practice" are employed throughout the book, they have no separate treatment. Arithmetical puzzles are carefully avoided.

8. *Such new matter and such new methods as are demanded by the circumstances of the present time are introduced.* See the Metric System; the treatment of Percentage; a Table of legal rates of interest in the States and Territories at the present time, from official sources; Gold at a premium; U. S. Bonds; Average of Accounts; Partnership; Evolution, etc.

9. *Complete and thorough reviews both of principles and of processes are kept up throughout.* See Topical Reviews, with references, a feature not found in any other arithmetic; General Reviews and Dictation Exercises upon an original plan, by means of which additional examples with their answers are furnished to apply to every important subject in the book, without extra labor to the teacher.

By such a method and such a practice, it is believed that a knowledge of principles will be acquired, and the intellectual powers stimulated to healthful activity and to logical precision, and the pupil will be spared the mortifying experience, so common, of having to unlearn in the counting-room what he has learned in the school-room.

The authors' thanks are due to their fellow-teachers for many valuable suggestions, and for kindly assistance in the method of presenting the various topics and in the revision of the work in the manuscript and in the proofs; also to James A. Dupee, Esq., Banker of Boston, for criticisms and suggestions upon business forms, and to J. L. Woods, Esq., of Brimfield, for hints on the method of extracting the roots.

They avail themselves of this opportunity to express their gratitude for the hearty commendation and liberal patronage bestowed upon their former works by teachers and the public generally.

If the spirit with which they have studied anew the subject of numbers shall be communicated by these pages to their fellow-teachers, and especially if the book shall serve to awaken in the mind of the pupil greater love for study, greater activity and precision of thought, and if the true ends of education shall be more fully attained thereby, their labors will be richly compensated.

BOSTON, August, 1869.

SUGGESTIONS TO TEACHERS.

THOUGH every page of this arithmetic is an illustration of the authors' method of teaching the subject, yet a few additional hints may be of service to the inexperienced teacher.

The plan of the book contemplates the formation of classes in arithmetic, to each member of which is assigned the same daily lessons.

It is recommended that before a lesson is assigned it should be taught orally by the teacher, with the aid of appropriate objects and illustrations.

This teaching, if properly conducted, will so direct the attention of the pupil, that by observing for himself, the right ideas will be excited in his mind.

Having been assisted in this manner to make a full analysis of a subject or of an operation, he should then be required to give synthetically a statement of what he has observed.

The general method of teaching is indicated in the following

ILLUSTRATIVE LESSON.

[A method of teaching Arts. 1, 2, and 3 of the book.]

The teacher being provided with marbles, pebbles, shells, or other small objects, placed in collections on his table before the class, proceeds thus: —

Teacher. "John, show me a single pebble;" "a single shell." "Show me any single thing." "What have you now in your hand?" (*John.* "I have a single thing.") "A single thing is a **unit**. What is a unit?" (*John.* "A unit is a single thing.") "Each member of the class may show me a unit." "What is a unit?" (*Class.* "A unit," etc.)

Teacher. "James, show me a collection of units." "A collection of units is a **number**. What is a number?" (*James.* "A number is a collection of units.") "The class may answer." (*Class.* "A number," etc.)

Teacher [holding up five shells]. "What have I here?" (*Class.* "You have a number of shells.") "Name the number." (*Class.* "Five.") "Jane, you may write the word five on the blackboard." "You have now represented the number five by the word five; you may represent it by the letter V underneath the word five." "Do you know any other way of representing the number five?" [*Charles* passes to the board and writes under the letter V the character 5.] "How many ways have we found of representing the number five?" (*Class.* "Three ways; by using the word five, by using the letter V, and by using the character 5.") "Representing numbers by words, letters, or other characters is **expressing numbers**."

"What is expressing numbers?" (*Class.* "Expressing numbers is representing them by words, letters, or other characters.")

Teacher [writing upon the board the characters 1, 2, 3, 4, 5, 6, 7, 8, 9]. "For what are these characters used?" (*Class.* "They are used for expressing numbers.") "These characters, used for expressing numbers, are **figures**. What are figures?" (*Class.* "Figures are certain characters," etc.) "The expression of numbers by figures or by letters is **Notation**. What is notation?" (*Class.* "Notation is the expression," etc.)

For a class of quite young pupils, this lesson is sufficient. For older pupils, additional topics, as in Arts. 4, 5, etc., may be given.

The teacher should examine his class thoroughly as he proceeds, to see if they understand his teaching. After the above lesson is thoroughly taught, the following topics should be assigned to be illustrated and defined by the class at the next recitation:—*A unit; a number; expression of numbers; figures; notation* (more or fewer topics, as the teacher has previously illustrated), also an exercise in writing figures neatly upon the slate or paper, for inspection.

A part of each lesson assigned should consist of such exercises and examples to be performed upon the slate as are found in the book.

In preparing for recitation, the pupil will make the text-book his guide, where his teacher has already been his travelling companion.

At the recitations the pupil should be required to take the pebbles, shells, or other objects in his hand, and teach the lessons to his class in the manner and in the order observed by the teacher in assigning the topics.

The slate work of the pupils should be examined by the teacher. Neatness and order in arranging the work, in making figures, etc., should be required.

REVIEWS.—Each recitation should consist in part of a review of the topics of previous lessons.

THE TOPICAL REVIEWS at the close of each section of the book will be a guide to the pupil in preparing for his reviews. Ordinarily a few topics of one section should be reviewed with each advanced lesson of the succeeding sections.

The topics should be so prepared that they can be illustrated by the pupils of the class as they are called upon, without questions from the teacher. (See Illustrative Exercise above, also suggestions in the teachers' MANUAL AND KEY.

This form of review is a much more thorough test of the pupil's proficiency than the ordinary mode of questions and answers. It has the advantage also of occupying far less time, and, above all, of cultivating the *active powers* of the pupil's mind. The teacher may rest assured that it is

entirely practicable for pupils in all stages of development, less being required of younger than of older pupils.

A part of the time of each recitation should be devoted to performing examples upon the slate or board, in the presence of the teacher. In this exercise the teacher will be greatly aided by

THE DICTATION EXERCISES which are contained in the MANUAL AND KEY. These consist in part of miscellaneous examples; but mainly of the examples of the Arithmetic, so modified as to require an entirely different answer from that given in the Arithmetic, and to which the pupil has no clew. They are dictated thus: —

The teacher, turning to the article in the MANUAL AND KEY corresponding to any article in the Arithmetic which he wishes to review, assigns an example or examples, and announces the modification. A pupil having performed the example or examples assigned, announces his result to the teacher, who compares it with the result given in the Key.

It is only by practice of this kind that the pupil secures that self-reliance without which he may spend months in the counting-room before he is fitted to perform with accuracy and with facility the commonest arithmetical operations.

The pupils should have abundant practice in the fundamental operations, especially in addition. To facilitate this practice, Walton's ARITHMETICAL TABLE, or something similar, should be in the hands of every student of written arithmetic.

It is with a view to securing accuracy in these operations, that so large a number of examples without answers are given in the first part of the book. The pupil should be made as ready in performing these operations as he is in reading simple sentences. The teacher can excite a lively interest in his pupils by giving them occasionally examples drawn from the transactions of daily life with which they are familiar.

For further suggestions on the method of teaching the various topics, the teacher is referred to the MANUAL AND KEY. It is recommended that he should examine that before teaching any topic.

The above are but hints; they apply in the main to all teaching: they are merely suggestive, and will, of course, be adopted only so far as they can be incorporated with the methods of individual teachers: though the book is adapted to the method illustrated above, it does not require any peculiar method of teaching.

Though great care has been bestowed upon the book to secure accuracy, errors are almost unavoidable in the operations of so large a number of examples. By pointing out to the authors or publishers any errors which may be discovered, a benefit will be conferred upon them, also upon teachers and pupils using the book.

TABLE OF CONTENTS.

INTEGRAL NUMBERS.

ARTICLE **1.** When we see a collection of things, we think of the single things which compose the collection.

A single thing is a **Unit.**

What is a unit?

A collection of units is a **Number.** *What is a number?*

2. A number of units, as the fingers of the hand, may be represented by the word *five*, the letter V, or by another character, 5.

Representing numbers by words, letters, or other characters is **expressing numbers.**

What is expressing numbers?

NOTATION.

3. A	unit,		called		one,	is commonly expressed by the character		1.
one	unit	with	one unit,	called	two,	"		2.
one	"	"	two units,	"	three,	"		3.
one	"	"	three "	"	four,	"		4.
one	"	"	four "	"	five,	"		5.
one	"	"	five "	"	six,	"		6.
one	"	"	six "	"	seven,	"		7.
one	"	"	seven "	"	eight,	"		8.
one	"	"	eight "	"	nine,	"		9.

The characters 1, 2, 3, 4, 5, 6, 7, 8, 9, are **Figures.**

What are figures?

Ans. Figures are the characters commonly used to express numbers.

The expression of numbers by figures or by letters is **Notation.**

What is notation?

4. 1 unit with 9 units, called ten, cannot be expressed by any figure now introduced.

The figure 1 expresses a single thing; taken by itself, it can express but one.

One pebble is a single thing; one pile of ten pebbles is a single pile.

The one pebble, and the one pile of ten pebbles, are both units; but as one of the units is a single pebble, and the other is made up of ten single pebbles, to distinguish these units we will call the unit that is a single thing a **unit of the first order,** and the unit that is made up of ten units of the first order a **unit of the second order.**

What is a unit of the first order?

Ans. A unit of the first order is a unit that is a single thing.

What is a unit of the second order?

Ans. A unit of the second order is a unit that is made up of ten units of the first order.

To show that both units are units, the same figure, 1, may be employed: —

thus, 1 $\begin{cases}\text{pebble,}\\ \text{pile of ten pebbles;}\end{cases}$ or, $\begin{cases}\text{1 pebble,}\\ \text{10 pebbles.}\end{cases}$

The character 0 is called zero. By its use the figure 1, when it expresses one ten, stands second in order, beginning at the right, instead of standing first, as it would without the zero.

Then to express one ten, or a unit of the second order, place the figure 1 second in order from the right, writing a zero first in order; thus, 10.

A unit of the first order is generally called a unit simply, while a unit of the second order is called a ten.

5. Units and tens are expressed together as follows:—

1	ten	with	no	units,	called	ten,	is expressed thus,	10.
1	ten	"	1	unit,	"	eleven,	"	11.
1	ten	"	2	units,	"	twelve,	"	12.
1	ten	"	3	"	"	thirteen,	"	13.
1	ten	"	4	"	"	fourteen,	"	14.
1	ten	"	5	"	"	fifteen,	"	15.
1	ten	"	6	"	"	sixteen,	"	16.
1	ten	"	7	"	"	seventeen,	"	17.
1	ten	"	8	"	"	eighteen,	"	18.
1	ten	"	9	"	"	nineteen,	"	19.
2	tens	"	no	"	"	twenty,	"	20.
2	tens	"	1	unit,	"	twenty-one,	"	21.
2	tens	"	2	units,	"	twenty-two,	"	22.
2	tens	"	3	"	"	twenty-three,	"	23.
2	tens	"	9	"	"	twenty-nine,	"	29.
3	tens	"	no	"	"	thirty,	"	30.
3	tens	"	1	unit,	"	thirty-one,	"	31.
3	tens	"	2	units,	"	thirty-two,	"	32.
4	tens	"	no	"	"	forty,	"	40.
4	tens	"	1	unit,	"	forty-one,	"	41.
4	tens	"	5	units,	"	forty-five,	"	45.
5	tens	"	no	units,	"	fifty,	"	50.
6	tens	"	"	"	"	sixty,	"	60.
7	tens	"	"	"	"	seventy,	"	70.
8	tens	"	"	"	"	eighty,	"	80.
9	tens	"	"	"	"	ninety,	"	90.
9	tens	"	9	"	"	ninety-nine,	"	99.

EXERCISES.

6. The pupil may express the following in words and in figures : —

1. 1 ten. *Ans.* Ten; 10.
2. 1 ten with 1 unit. *Ans.* Eleven; 11.
3. 1 ten with 2 units; with 3 units, etc.
4. 2 tens.
5. 2 tens with 1 unit; with 2 units, etc.
6. 3 tens with 1 unit; with 2 units, etc.
7. 4 tens with 1 unit; with 4 units; with 5 units.
8. 6 tens with 4 units; with 8 units; with 2 units.
9. 8 tens with 2 units; with 7 units; with 9 units.

7. Ten tens can be expressed by no method yet explained.

A collection of ten tens is a single collection.

As the one pile of ten pebbles (Art. 4) is a unit, so the one collection of ten tens is a unit.

This new unit is called **one hundred** or **a unit of the third order.**

To express one hundred, or a unit of the third order, place the figure 1 the third in order from the right, and write zeros in the first and second places; thus, 100.

8. Units, tens, and hundreds are expressed together as follows : —

1	hundred	with	no units,		called	one hundred,	is expressed thus,	100.
1	"	"	1 unit,		"	one hundred one,	"	101.
1	"	"	2 units,		"	one hundred two,	"	102.
1	"	"	9 "		"	one hundred nine,	"	109.
1		"	1 ten,		"	one hundred ten,	"	110.
1	"	"	1 "	1 unit,	"	one hundred eleven,	"	111.
2	hundreds	"	no tens,	no units,	"	two hundred,	"	200.
2	"	"	1 ten,	" "	"	two hundred ten,	"	210.
5	"	"	3 tens,	7 "	"	five hundred thirty-seven,	"	537.
9	"	"	9 "	9 "	"	nine hundred ninety-nine,	"	999.

Exercises.

9. The pupil may express the following in words and in figures:—

10. 10 tens.
    Ans. One hundred; 100.
11. 1 hundred with 1 unit.
12. 1 hundred with 2 units.
13. 1 hundred with 3 units.
14. 1 hundred with 1 ten.
15. 1 hundred with 2 tens.
16. 2 hundreds with 9 units.
17. 3 hundreds with 7 tens.
18. 7 hundreds with 1 ten.
19. 8 hundreds with 5 tens and 1 unit.
20. 9 hundreds with 9 tens and 4 units.

10. The pupil may express the following in figures:—

21. Thirteen.
22. Eighteen.
23. Thirty-seven.
24. Fifty.
25. Seventy-nine.
26. One hundred.
27. One hundred five.
28. Two hundred seventy.
29. Seven hundred.
30. One hundred eleven.
31. One hundred ninety.
32. Three hundred sixty.
33. Nine hundred.
34. Five hundred eighteen.
35. Four hundred forty-four.
36. Four hundred four.

11. As a collection of ten tens, or one hundred, is a unit, so a collection of ten hundreds is a unit; this unit is called **one thousand,** or **a unit of the fourth order.**

To express one thousand, place the figure 1 the fourth in order from the right, and write zeros in the first, second, and third places; thus, 1000.

Exercises.

12. The pupil may express the following in words and in figures:—

37. 1 hundred with 3 tens and 5 units.
38. 1 thousand with 1 hundred, 1 ten, and 1 unit.
39. 1 thousand with 4 hundreds, no tens, and 5 units.

40. 2 thousands with 4 hundreds and 3 units.
41. 5 thousands with 7 tens and 2 units.
42. 8 thousands with 4 units.

13. A collection of ten thousands is a unit; it is called **one ten thousand**, or **a unit of the fifth order.**

To express one ten thousand, place the figure 1 the fifth in order from the right, and write zeros in the first, second, third, and fourth places; thus, 10000.

EXERCISES.

14. The pupil may express the following in words and in figures: —

43. 1 ten thousand.
44. 1 ten thousand with 1 thousand and 1 unit.
45. 1 ten thousand with 1 thousand and 5 hundreds.
46. 2 tens of thousands with 2 thousands, 7 hundreds, 5 tens, and 9 units.
47. 3 tens of thousands with 3 thousands and 4 units.
48. 5 tens of thousands with 5 hundreds and 4 units.
49. 8 tens of thousands with 7 tens and 1 unit.

15. From what has been taught, it appears that there are different orders of units, and that ten units of any lower order may be united and form one unit of a higher order; thus, —

Ten units	are united	and	form	one ten.
Ten tens	"	"	"	"	one hundred.
Ten hundreds	"	"	"	"	one thousand.
Ten thousands	"	"	"	"	one ten thousand.

It also appears that the order of each unit is expressed by the place in which its sign, the figure 1, is written.

The units of the successive orders to the tenth are expressed in the following

TABLE OF UNITS.

Billion.	Hundred million.	Ten million.	Million.	Hundred thousand.	Ten thousand.	Thousand.	Hundred.	Ten.	Unit.	
1	1	1	1	1	1	1	1	1	1	
10th.	9th.	8th.	7th.	6th.	5th.	4th.	3d.	2d.	1st.	Orders of Units.

EXERCISES UPON THE TABLE.

16. 50. The pupil may give the names of the units of the several orders from 1 unit to 1 million; from 1 million to 1 unit.

In which place, beginning at the right, is expressed

51. 1 hundred?
52. 1 ten?
53. 1 thousand?
54. 1 ten thousand?
55. 1 hundred thousand?
56. 1 million?

17. 57. Give the names of the units of the several orders from 1 unit to 1 billion; from 1 billion to 1 unit.

58. What is the name of the unit expressed in the 6th place? *Ans.* 1 hundred thousand.

59. 1st place?
60. 2d place?
61. 3d place?
62. 7th place?
63. 5th place?
64. 9th place?

EXERCISES.

18. The pupil may express the following in figures: —

65. 10 hundreds, or 1 thousand. *Ans.* 1000.
66. 1 thousand with 1 hundred. *Ans.* 1100.
67. 2 thousands with 5 hundreds. *Ans.* 2500.

68. 7 thousands with 8 hundreds and 2 tens. *Ans.* 7820.
69. 3 thousands with 6 hundreds, 8 tens, and 5 units.
70. 7 thousands with 5 tens and 2 units. *Ans.* 7052.
71. 9 thousands with 7 hundreds and 6 units. *Ans.* 9706.
72. 5 thousands with 4 hundreds and 5 units.
73. 7 thousands with 3 tens and 4 units.
74. 8 thousands with 5 tens.
75. 8 thousands with 5 units.
76. 10 thousands, or 1 ten thousand. *Ans.* 10000.
77. 1 ten thousand with 3 thousands, 2 hundreds, and 5 units. *Ans.* 13205.
78. 7 tens of thousands with 5 hundreds, 3 tens, and 7 units.
79. 7 tens of thousands with 4 units.
80. 3 tens of thousands with 2 thousands, 6 tens, and 4 units.
81. 9 millions with 9 hundreds of thousands.
82. 9 tens of thousands with 9 thousands, 9 hundreds, and 9 units.

NUMERATION.

19. If we have a collection of figures, as 111, let us see what number is expressed.

We have learned (Arts. 4, 7) that a figure 1 written in the first place expresses a unit of the first order, in the second place a unit of the second order, in the third place a unit of the third order.

We have also learned that a unit of the second order equals ten units of the first order, that a unit of the third order equals ten units of the second order.

The expression 111 must then mean 1 unit, 1 ten of units, and 1 hundred (10 tens) of units; or, without re-

peating the word unit, 1 hundred, 1 ten, and 1 unit, which, expressed together, are one hundred eleven units.

20. Let us now see what number is expressed by the collection of figures 111111.

We have already learned that the first three figures at the right of the collection express one hundred eleven units.

The figures in the fourth, fifth, and sixth places express 1 thousand, 1 ten thousand, and 1 hundred thousand, which, expressed together, are one hundred eleven thousand.

Because the first three figures in the collection express units, and the next three express thousands, the collection may be separated into two divisions containing three figures each. The first division, then, will express units, the second thousands.

A collection of figures expressing units, tens, and hundreds of the same kind is called a **group** of figures.

What is a group of figures?

NOTE.—A corresponding collection of the orders of units is called a group of units.

Below is the collection of figures 111111 separated into groups.

Hundred.	Ten.	Unit.		Hundred.	Ten.	Unit.
1	1	1	,	1	1	1

$$\underbrace{1\ 1\ 1}_{\text{Thousands.}}\ ,\ \underbrace{1\ 1\ 1}_{\text{Units.}}$$

21. Each group of figures receives its name from the units which it expresses; the first group is named the units' group, the second the thousands' group.

Of the units' group, the orders of units are units, tens, hundreds; of the thousands' group, the orders of units are the same, units, tens, hundreds.

The first figure of the first group expresses a unit of that group, *units;* the first figure of the second group expresses a unit of that group, *thousands.*

The above collection of figures, then, expresses 1 hundred, 1 ten, and 1 unit of thousands, 1 hundred, 1 ten, and 1 unit of units, or one hundred eleven thousand, one hundred eleven (units being understood).

We have now named the orders of units, and the groups expressed by this collection of figures; we have also named the entire number expressed.

Naming the orders of units and the groups expressed by any collection of figures is **Numeration.**

What is numeration?

Naming the number expressed by a collection of figures is called **reading the number.**

NOTATION AND NUMERATION.

22. The manner of expressing numbers generally in groups is illustrated in the following

NUMERATION TABLE.

Names of Groups.	Orders of Units.	Expression	The order of the Groups.
Trillions.	hundreds of, tens of, units of	4 6 2,	5th.
Billions.	hundreds of, tens of, units of	3 5 7,	4th.
Millions.	hundreds of, tens of, units of	1 8 2,	3d.
Thousands.	hundreds of, tens of, units of	5 7 6,	2d.
Units.	hundreds of, tens of, units of	9 0 8.	1st.

EXERCISES UPON THE TABLE.

23. 1. Name the groups in order from units to trillions; from trillions to units.

What is the name of

2. The second group?	4. The first group?
3. The third group?	5. The fourth group?

6. The fifth group?

24. 7. Which is the units' group?

8. The millions' group?	10. The billions' group?
9. The thousands' group?	11. The trillions' group?

25. 12. Name the orders of units expressed in each group of figures from the lowest to the highest; from the highest to the lowest.

13. In which place of each group are expressed units? tens? hundreds?

26. In which place and which group are expressed

14. Units of thousands?	20. Tens of millions?
15. Tens of thousands?	21. Hundreds of millions?
16. Hundreds of thousands?	22. Units of billions?
17. Hundreds of units?	23. Hundreds of billions?
18. Units of units?	24. Units of trillions?
19. Units of millions?	25. Tens of trillions?

27. What are expressed in the

26. 1st place of 2d group?	30. 2d place of 4th group?
27. 1st place of 3d group?	31. 3d place of 1st group?
28. 1st place of 4th group?	32. 3d place of 3d group?
29. 2d place of 1st group?	33. 2d place of 5th group?

28. If it is required to read the number expressed by any collection of figures, as

23150467,

Beginning at the right we separate the collection into groups of three figures each, by commas, thus,

23,150,467.

We then name the number expressed in the left-hand group, together with the group, and thus proceed with each succeeding group from left to right, omitting the name of the units' group.

The above collection will then be read, "twenty-three million, one hundred fifty thousand, four hundred sixty-seven."

EXERCISES.

29. In a similar manner read the following:—

34.	1111.	41.	60892.
35.	5110.	42.	168790.
36.	7208.	43.	465369.
37.	23426.	44.	990083.
38.	31500.	45.	1187607.
39.	70862.	46.	2000332.
40.	75060.	47.	87065000.

30. If it is required to express a number in figures, as four hundred twenty-six thousand, seven hundred eighty-four,

Beginning with the highest group, we express the number of that group, 426; then, at the right, the number of the next group, 784; and the entire number is expressed as follows:—

426,784.

EXERCISES.

31. In a similar manner express the following: —

48. One hundred ninety-eight.
49. Two thousand two hundred twenty.
50. Seventeen thousand five hundred twenty-one.
51. Twenty thousand eight hundred seventy.
52. Seventy thousand sixty-five.
53. Ten thousand four hundred eleven.
54. Six hundred ten thousand eighty.
55. One million one hundred thirty thousand nine hundred ninety-nine.
56. Thirty million.
57. Sixteen million fifty thousand four hundred eight.

32. How many tens with how many units are expressed by each of the following?

58. 25.	61. 99.	64. 119.
59. 64.	62. 89.	65. 125.
60. 80.	63. 109. *Ans.* {10 tens with 9 units.}	66. 260.

33. How many hundreds, with how many tens and how many units, are expressed by each of the following?

67. 125.	70. 990.	73. 1118.
68. 781.	71. 999.	74. 1305.
69. 979.	72. 1075.	75. 2760.

34. 76. In 150 how many tens? How many hundreds, and how many tens remain?

77. In 24 tens how many units?

How many hundreds and how many tens remain

78. In 38 tens?	80. In 59 tens?	82. In 100 tens?
79. In 30 tens?	81. In 99 tens?	83. In 112 tens?

35. 84. In 325 how many tens and how many units remain? How many hundreds and how many units remain? *Ans.* 3 hundreds and 25 units.

36. 85. In 1500 how many tens? How many hundreds? How many thousands and how many units remain? How many thousands and how many hundreds remain?

86. In 25 hundreds how many thousands and how many hundreds remain?

How many thousands and how many hundreds remain

87. In 99 hundreds?	89. In 110 hundreds?
88. In 106 hundreds?	90. In 207 hundreds?

37. In which place shall I write 9 so that it shall express

91. 9 units?	93. 9 thousands?	95. 9 tens of thousands?
92. 9 tens?	94. 9 hundreds?	96. 9 millions?

38. 97. In 1 ten how many units?

98. In 1 hundred how many tens? how many units?

99. In 1 thousand how many hundreds? tens? units?

100. In 9 tens how many units?

101. In 2 hundreds how many tens? units?

102. In 2 tens of thousands how many thousands? hundreds? tens? units?

103. In 12 thousands how many hundreds? tens? units?

104. In 25 hundreds of thousands how many tens of thousands? thousands? hundreds? tens? units?

COUNTING.

39. One marble with one marble is two marbles; one marble with two marbles is three marbles.

One of three marbles taken leaves two marbles.

Taking a unit or a number of units with another to find how many are made, or out of another to find how many are left, is called **counting**.

What is counting?

EXERCISES.

40. Count by ones

1. From one to twenty; from twenty to one.

   Count by twos

2. From two to twenty; from twenty to two.
3. From one to twenty-one; from twenty-one to one.

   Count by threes

4. From three to thirty; from thirty to three.
5. From one to thirty-one; from thirty-one to one.
6. From two to thirty-two; from thirty-two to two.

   Count by fours

7. From four to forty; from forty to four.
8. From one to forty-one; from forty-one to one.
9. From two to forty-two; from forty-two to two.
10. From three to forty-three; from forty-three to three.
11. Count by fives, sixes, sevens, eights, and nines in a manner similar to the above.

41. In counting as above, from two to twenty, all the twos of twenty are counted together.

In counting from twenty to two, first one two, then two twos, then three twos, are counted apart from the rest of twenty.

At each step in counting we have a new combination.

Counting numbers of units together, or apart, is called **combining numbers.**

What is combining numbers?

Numbers are combined by processes illustrated in the following lessons.

ADDITION.

42. If I have one pebble in my hand, and put with it one pebble, how many pebbles have I?

If I put two pebbles with one pebble, how many pebbles have I?

To find how many I have, you count the pebbles together, thus: "one, two"; "one, three."

43. If I put one apple with one pebble, have I two pebbles? Have I two apples?

Objects counted together must be of the same kind.*

44. The process of counting together numbers of the same kind is **Addition.**

What is addition?

45. The number found by counting numbers together is the **sum** or **amount.**

What is the sum or amount?

* I may count one apple as one *thing* with one pebble as another *thing*, and then have *two things*, though they will be neither two apples nor two pebbles; but in this case, I only consider them as *things*, in which particular they are alike.

If 2 pebbles are added to 5 pebbles, what is the sum or amount?

46. Examples.

Commencing at the bottom of each of the following columns, add the numbers as expressed from the bottom upward.

	(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)	(9.)	(10.)
	Tops.	Pens.	Men.	Boys.	Hats.	Caps.	Pins.	Fans.	Days.	Figs.
	3	4	3	5	1	7	3	5	2	4
	2	3	2	2	2	3	4	5	4	2
	1	1	4	1	4	0	3	2	2	3
	1	5	2	4	3	3	2	1	4	5
Sum.	7	13	11	12	10	13	12	13	12	14

Note to the Pupil. — In adding orally the numbers in any example, name the results simply; thus, in Example 1, say "1, 2, 4, 7," and not "1 and 1 are 2, and 2 are 4, and 3 are 7."

Proof. — To prove the work, add the numbers in the reverse order; that is, if you commenced at the bottom, commence at the top; if the work is right, the same sum will be obtained as before.

Note. — The pupil should prove every example till he can add accurately, and continue to practice adding until he can add rapidly as well as accurately.

(11.)	(12.)	(13.)	(14.)	(15.)	(16.)	(17.)	(18.)	(19.)	(20.)
3	1	2	1	4	3	1	3	6	1
4	5	1	4	3	3	2	5	1	6
2	3	3	2	2	4	5	1	2	4
1	2	5	5	1	1	6	2	6	3
2	3	4	2	6	3	2	4	1	5
12	14	15	14	16	14	16	15	16	19

(21.)	(22.)	(23.)	(24.)	(25.)	(26.)	(27.)	(28.)	(29.)	(30.)
1	1	4	2	1	3	3	1	1	5
1	2	1	1	0	1	1	2	4	1
2	5	3	4	1	8	2	5	1	1
3	1	2	1	4	2	2	1	3	2
1	2	3	5	4	5	1	1	3	2
4	2	1	3	5	1	2	6	1	4
3	5	2	1	3	2	4	0	3	1
4	1	2	1	1	2	1	3	3	1
19									

(31.)	(32.)	(33.)	(34.)	(35.)	(36.)	(37.)	(38.)	(39.)	(40.)
4	2	4	5	6	3	3	1	6	5
3	4	1	2	4	6	4	2	3	4
3	3	3	2	3	2	5	1	6	4
2	2	4	2	8	8	4	4	5	6
1	5	3	6	3	7	4	3	4	6
4	2	4	3	2	2	4	6	6	2
17									

(41.)	(42.)	(43.)	(44.)	(45.)	(46.)	(47.)	(48.)	(49.)	(50.)
4	5	4	6	5	5	6	0	2	7
1	8	6	0	4	7	2	5	0	4
3	4	1	4	5	0	8	9	3	7
4	3	6	4	2	3	7	5	1	9
1	7	6	6	7	8	6	4	9	8
5	1	0	7	5	3	4	4	6	9
4	5	4	5	4	6	8	4	4	2
8	0	5	4	1	4	5	1	9	8
6	7	6	5	9	9	2	7	8	4
1	4	3	8	3	2	1	2	0	9
38									

47. When I have one pebble in my hand and put with it another pebble, I hold in my hand *one more* pebble. The word **plus** means more.

Explain the meaning of "five plus seven."

Ans. "Five with seven more" or "seven added to five."

Explain the meaning of "three plus eight"; "nine plus six."

48. The sign of addition is an upright cross made thus, +, and read "plus"; this sign is placed between the figures which express the numbers to be added.

Read and explain the expressions

3 chairs + 4 chairs; 7 books + 6 books.

49. 7 books + 6 books equal how many books?

To express the equality of two numbers, as 7 + 6 equals 13, a sign is used; it consists of two short horizontal marks made thus, =, and is read "equals" or "equal to"; it is placed between the written expressions of two numbers that are equal, as

$$7 + 6 = 13.$$

Read and explain the expression

2 slates + 5 slates = 7 slates.

50. EXAMPLES.

51. 19 + 5 + 8 + 6 + 9 + 4 + 3 + 7 = how many? *Ans.* 61.

52. 36 + 2 + 8 + 3 + 6 + 7 + 9 + 4 = how many? *Ans.* 75.

53. 48 + 1 + 2 + 3 + 4 + 5 + 6 + 7 = how many?

54. 55 + 9 + 8 + 7 + 6 + 5 + 4 + 3 = how many?

51. ILLUSTRATIVE EXAMPLE I. John gathered 42 apples under one tree, 35 under another, and 12 under another; how many apples did he gather?

OPERATION EXPRESSED AND EXPLAINED.

Tens.	Units.	
4	2	apples.
3	5	"
1	2	"
8	9	apples.

To find how many he gathered, we add the numbers 42, 35, and 12.

We express these numbers so that units of the same order shall be expressed in the same column, and draw a line underneath.

42 equals 4 tens with 2 units; 35 equals 3 tens with 5 units; 12 equals 1 ten with 2 units.

We first add the units, thus, 2, 7, 9, and have 9 units; we write a figure 9 under the line in the units' place.

We then add the tens, thus, 1, 4, 8, and have 8 tens; we write a figure 8 under the line in the tens' place, and have 8 tens with 9 units, or 89, for the sum, which is the number of apples he gathered.

EXAMPLES.

55. Henry earned 14 cents on Monday, 22 cents on Tuesday, and 31 cents on Wednesday; how many cents did he earn? *Ans.* 67 cents.

56. Mr. Ray had 15 acres of woodland, 12 of pasture-land, 21 of meadow-land, and a wheat-field of 40 acres; how many acres had he? *Ans.* 88 acres.

57. Mrs. Chase owed 13 dollars to one person, 2 to another, 22 to another, and 21 to another; how many dollars did she owe? *Ans.* 58 dollars.

58. How many sheep are 23 sheep, 11 sheep, and 34 sheep added together?

59. Add together 211 books, 4 books, 322 books, 10 books, and 11 books.

60. Add together 123 days, 31 days, 201 days, 314 days, and 20 days.

61. What is the sum of 401, 312, 53, 222, and 101?

62. What is the sum of 140, 200, 32, 316, 231, and 110?

52. ILLUSTRATIVE EXAMPLE II. A fisherman caught 33 fishes on Monday, 67 on Tuesday, 25 on Wednesday, and 38 on Thursday; how many fishes did he catch?

OPERATION EXPRESSED AND EXPLAINED.

	Tens.	Units.	
	3	3	fishes.
	6	7	"
	2	5	"
	3	8	"
1	6	3	fishes.

Here, adding the units (8, 13, 20, 23), we have 23 units, which equals 2 tens with 3 units; we write a figure 3 under the line in the units' place, and reserve the 2 tens to add with the tens.

Adding the tens, we have, with the 2 reserved tens, (2, 5, 7, 13, 16,) 16 tens, which equals 1 hundred with 6 tens; we write a figure 6 in the tens' place and a figure 1 in the hundreds' place, and have 163 for the sum, which is the number of fishes he caught.

NOTE.—In practice, the pupils need only say "8, 13, 20, 23; express 3, reserve 2. 2, 5, 7, 13, 16. Answer, 163."

EXAMPLES.

☞ For test examples in addition, which may profitably be given at each recitation, see "Waltons' Manual and Key," pages 25, 26.

(63.)	(64.)	(65.)	(66.)	(67.)
Men.	Women.	Boys.	Girls.	Shoes.
62	68	65	218	382
35	24	38	364	769
41	81	42	207	27
68	75	85	595	824
206	248	230	1,384	2,002

(68.)	(69.)	(70.)	(71.)	(72.)
Horses.	Sheep.	Dollars.	Trees.	Books.
381	981	1486	5678	3579
576	654	7890	9876	1357
287	782	8901	5432	9135
781	407	2345	1987	7913
809	554	6789	6543	2468
763	273	1234	1298	4862
3,597	3,651	28,645	30,814	29,314

	(73.)		(74.)		(75.)		(76.)		(77.)
78.	868	+	517	+	9162	+	2201	+	9752
79.	123	+	649	+	3703	+	3043	+	8413
80.	456	+	233	+	7942	+	2424	+	9913
81.	789	+	821	+	1262	+	7216	+	8355
82.	328	+	833	+	9951	+	6156	+	1386
	2,564		3,053		32,020		21,040		37,819

83. Add together 542, 868, 325, and 672.

84. Add together 79, 620, 58, 585, 978, 798, and 380.

85. Add together 1369, 2468, 7654, 5432, 3218, and 9287.

86. How many are 36847 + 59218 + 100762 + 48076 + 89007?

87. Find the sum of all the numbers from 1 to 25 inclusive. *Ans.* 325.

88. Find the sum of all the numbers from 132 to 144 inclusive.

89. 32 + 81 + 545 + 2368 + 9 + 78 + 1684 + 10000 + 89 equals what number?

90. 450 thousand plus 26 thousand plus 5 million 8 hundred, equals what number?

91. In the first class of a school there are 62 pupils; in the second, 98; in the third, 102; and in the fourth, 130: how many pupils are there in all the classes?

92. How many days are there in the year, there being in January, 31 days; in February, 28; in March, 31; in April, 30; in May, 31; in June, 30; in July, 31; in August, 31; in September, 30; in October, 31; in November, 30; and in December, 31?

93. In taking an account of stock, a grocer found he had grain worth 284 dollars, sugar worth 193 dollars, cheese worth 219 dollars, butter worth 85 dollars, and miscellaneous articles worth 528 dollars; what was his entire stock worth?

94. Mrs. Clarke spent 38 dollars for dress silk, 17 dollars for lace and other trimmings, and 12 dollars for making the dress. She also spent 55 dollars for a shawl, 12 for a bonnet, 2 for gloves, and 5 for a pair of boots; what did she spend for all?

95. A man travelled 196 miles on Monday, 37 on Tuesday, 218 on Wednesday, 229 on Thursday, 173 on Friday, and rested on Saturday; how many miles did he travel during the week?

96. If 189200 bricks are required to build a house, 50000 to build a stable, and 1360 to lay a walk, how many bricks in all are required?

97. In 1860 there were in the New England States, 19514 manufacturing establishments; in the Middle States, 52364; in the Western States, 34301; in the Southern States, 18026; and in the Pacific States, 4095: how many manufacturing establishments were there in all?

SUBTRACTION.

53. If you have five marbles in your hand and take away two of them, how many marbles have you left?

Taking two marbles, a part of five marbles, away, we find there are three marbles left.

The process of taking part of a number away to find how many are left is **Subtraction.**

What is subtraction?

54. The number, a part of which is taken away, is the **minuend.**

What is the minuend?

Which is the minuend in the given illustration?

55. The part of the minuend taken away is the **subtrahend.**

What is the subtrahend?

Which is the subtrahend in the given illustration?

56. That part of the minuend left, after a part is taken away, is the **remainder.**

What is the remainder?

What is the remainder in the given illustration?

57. ORAL EXERCISES. — 1. If you have 11 apples, and give 8 of them away, how many remain? How many remain if you give away 2? 4? 9? 5? 3? 6? 7?

2. If there are 12 pears and you take away 5 of them, how many pears remain? How many remain if you take away 9? 7? 6? 8? 4? 3?

3. Mary has 13 peaches, and Henry has 4 less than Mary; how many has Henry? How many are 13 less 4? 13 less 5? 13 less 8? 13 less 7? 13 less 6? 13 less 9?

4. How many are 14 less 8? 14 less 5? 14 less 9? 14 less 7? 14 less 6?

5. The sum of two numbers is 15; if one of the numbers is 7, what is the other? If one is 6, what is the other? If one is 9, what is the other? If one is 8, what is the other?

6. 8 oranges and how many are 16 oranges? 7 and how many are 16? 9 and how many are 16?

7. 17 are how many more than 8? than 9? 18 are how many more than 9?

58. ILLUSTRATIVE EXAMPLE I. If Henry has 5 apples and Mary has 3, how many more apples has Henry than Mary?

We may find how many more Henry has by taking away a number of his apples equal to Mary's. 3 of Henry's apples taken leaves 2.

We see, then, that Henry has 2 apples more than Mary has, or that the difference of the numbers 5 and 3 is 2.

Hence to find the difference of two numbers, we take away a part of the larger number equal to the smaller.

This process is the same as that illustrated in Art. 53, and is therefore **Subtraction.**

How many more are 17 eggs than 9 eggs? Tell how you know this?

59. The process of subtraction may be thus expressed:

6 marbles, whole number, called minuend.
2 marbles, one part given, called subtrahend.
4 marbles, other part found, called remainder.

REMARK. — On comparing subtraction with addition, it will be seen that one is the reverse of the other, for while in addition we increase a number by the number added, in subtraction we diminish a number by the number taken away.

60. The sign of subtraction is a short horizontal mark made thus, —, and read "minus" or "less." The expression

5 cents — 2 cents = 3 cents,

shows that if 2 of 5 cents are taken away, 3 cents will remain.

Read and explain the expressions

15 apples — 6 apples = 9 apples.

18 — 12 = 8 — 2 = 6.

61. ILLUSTRATIVE EXAMPLE II. Jane had 27 apples, and gave 13 of them to Mary; how many apples had she left?

OPERATION.

Tens	Units	
2	7	whole number.
1	3	part given.
1	4	part remaining.

Explanation. — To find how many she had left, we take 13 of the number 27 away. We express the number 27 and 13 under it so that units of the same order shall be expressed in the same column, and draw a line underneath.

27 equals 2 tens with 7 units; 13 equals 1 ten with 3 units.

If 3 of the 7 units are taken away, 4 units remain;* we write a figure 4 under the line in the units' place.

If 1 of the 2 tens is taken away, 1 ten remains; we write a figure

* TO THE TEACHER. — This expression is preferable to the expression, "If 3 units are taken from 7 units," etc., since "from" commonly denotes "leaving behind" or "departure" rather than "out of."

If 3 men should walk away from 7 men, 7 men would be left, and not 4 men; but if 3 *of* 7 *men* should walk away *from the rest*, 4 would be left.

"If three units are taken out of 7 units" is a correct and convenient form of expression.

1 in the tens' place, and have 1 ten with 4 units, or 14, for the entire remainder, which is the number of apples Jane had left.

NOTE. — The above illustrates the manner of explaining an example in subtraction; but in practice the pupil needs only say "3 of 7 taken leaves 4; 1 of 2 taken leaves 1;" or "3 out of 7 leaves 4; 1 out of 2 leaves 1."

62. PROOF OF ILLUSTRATIVE EXAMPLE II.

13	part given.
14	part remaining.
27	whole number.

The number of apples Jane has left, or 14, added to the number she gave away, or 13, must equal the number she had at first; adding them, we have 27 for the result; we may therefore conclude that the work was correct; and, generally,

To prove the work of any example in subtraction, add the remainder to the subtrahend; if the work is correct, their sum must equal the minuend.

63. EXAMPLES.

8. John had 28 cents, and spent 17 of them for a slate; how many cents had he left? *Ans.* 11 cents.

9. Albert had 56 cents, and spent 32 of them for a hat, and the rest for a book; how many cents did he spend for the book? *Ans.* 24 cents.

10. Olive earned 98 cents in a day, and Sarah earned 72 cents; how many more cents did Olive earn than Sarah?

NOTE. — She earned as many more than Sarah as 98 cents are more than 72 cents, which may be found by taking 72 of the number 98 away.

Ans. 26 cents.

11. Mr. Goss paid 275 dollars for his horse, and 124 dollars for his oxen; how many more dollars did he pay for his horse than for his oxen? *Ans.* 151 dollars.

12. How many more sheep are there in a flock of 389 sheep than in a flock of 278 sheep? *Ans.* 111 sheep.

13. If of 278 bricks 235 bricks are taken away, how many bricks remain? *Ans.* 43 bricks.

14. If of a regiment of 785 men 572 were taken prisoners, how many remained? *Ans.* 213 men.

15. If of a drove of 1648 cattle 524 are sold, how many will remain?

NOTE. — When the answer is not given, the pupil should prove his work. (Art. 62.)

16. In the schools of a certain town, in one year there were 3697 cases of tardiness; the next year there were 2253 less; how many were there in the second year?

64. ILLUSTRATIVE EXAMPLE III. Annie had 243 plums, and gave Arthur 125 of them: how many plums had she left?

OPERATION.

Hundreds.	Tens.	Units.
	(3)	(13)
2	4	3
1	2	5
1	1	8

Explanation. — As there are but 3 units in the minuend, we cannot take 5 units away; we will therefore change one of the 4 tens (leaving 3 tens) to units. 1 ten equals 10 units; ten units with 3 units are 13 units. If 5 of these 13 units are taken away, 8 units remain.

If 2 of the 3 tens are taken away, 1 ten remains.

If 1 of the 2 hundreds is taken away, 1 hundred remains. We have, therefore, 118 for the entire remainder; which is the number of plums Annie had left.

NOTE. — In practice, the pupil needs only say "5 of 13 taken leaves 8; 2 of 3 taken leaves 1," etc, or "5 out of 13 leaves 8, 2 out of 3 leaves 1," etc.

65. EXAMPLES.

Find the remainders in the following: —

	(17.)	(18.)	(19.)
Minuend	362 sheep.	475 lambs.	827 horses.
Subtrahend	257 sheep taken.	259 "	235 "
Remainder	105 sheep left.	216 lambs.	592 horses.

(20.)	(21.)	(22.)	(23.)	(24.)
Boats.	Soldiers.	Flags.	Balls.	Bags.
564	3781	5318	3629	6234
272	2968	2882	1848	785
292	813	2,436	1,781	5,449

	(25.)		(26.)
	Miles.		Miles.
27.	45809	—	28365
28.	16776	—	7584

29. Find the sum of the last four answers.
Ans. 76,450 miles.

	(30.)		(31.)
	Men.		Men.
32.	10708	—	8542
33.	6291	—	1938

34. Find the sum of the last four answers.
Ans. 17,540 men.

	(35.)		(36.)
	Nails.		Nails.
37.	23456	—	17932
38.	12478	—	6189

39. Find the sum of the last four answers.
Ans. 34,534 nails.

	(40.)		(41.)
	Seconds.		Seconds.
42.	90270	—	28145
43.	19187	—	10728

44. Find the sum of the last four answers.
Ans. 159,084 seconds.

66. ILLUSTRATIVE EXAMPLE IV. A man who had 300 dollars gave 257 dollars for a horse; how many dollars had he left?

OPERATION.

```
(2)(9)(10)
 3 0 0
 2 5 7
 -----
   4 3
```

Explanation.—In this example, as there are no units and no tens in the minuend, we change one of the 3 hundreds (leaving 2 hundreds) to tens; 1 hundred equals 10 tens.

We change one of the 10 tens (leaving 9 tens) to units; 1 ten equals 10 units.

We have now changed 300 to 2 hundreds with 9 tens and 10 units, 2 hundreds 5 tens and 7 units of which are to be taken away.

☞ For test examples in subtraction, which may profitably be given at each recitation, see "Waltons' Manual and Key," pages 29, 30.

67. EXAMPLES.

45. How many are 200 less 77? *Ans.* 123.

46. How many are 23000 less 22?

47. How many are 10000 less 123?

48. How many are 10027 less 9231?

49. What is the sum of the last four answers? *Ans.* 33,774.

50. What is the difference between 23456 and 9876?

51. What is the difference between 34267 and 9876?

52. What is the difference between 45628 and 6439?

53. How many more are 982654 than 98365?

54. How many more are 876523 than 98765?

55. How many more are 765432 than 27483?

56. What is the sum of the last six answers? *Ans.* 2,477,156.

57. How many more in 250 thousand than in 1 thousand 7 hundred 9? *Ans.* 248,291.

58. How much larger is the number 1 million 32 thousand than 35 thousand 17? *Ans.* 996,983.

59. How much larger is 365 million than 365 thousand 365? *Ans.* 364,634,635.

60. 390000 — 8700 — 390 are how many?

NOTE.—In this example 8700 may first be taken away, and then 390 of the remainder taken, or the sum of 8700 and 390 may be taken at once. *Ans.* 380,910.

61. 560009 — 3270 — 999 are how many? *Ans.* 555,740.

62. A man in journeying 375 miles travelled 239 miles the first day; how many miles then remained to be travelled? *Ans.* 136 miles

63. A woman paid for 304 yards of cloth, but found on measuring it that she had received but 286 yards; how many more yards should she have received?

Ans. 18 yards.

64. Charles and Andrew counted their steps in going to school; Charles took 4320 steps, and Andrew 3862 steps; how many more steps did Charles take than Andrew?

65. How many more sheep are there in a flock of 422 sheep than in one of 318 sheep?

66. How many more are 9000 silkworms than 7280 silkworms?

67. When a balloon is 1300 feet high, how many feet higher is it than the top of a steeple that is 130 feet high?

68. How many feet higher is the Great Salt Lake, which is 4200 feet above the level of the sea, than Lake Superior, which is 630 feet above the level of the sea?

69. Lake Superior has 32000 square miles of surface; how many more square miles of surface has it than the State of Maine, which has 31760 square miles of surface?

70. How much smaller is the State of Rhode Island, which contains 1306 square miles, than Delaware, which contains 2120 square miles?

71. How many more square miles has Texas, which contains 237321 square miles, than California, which is the next State in size, and contains 188982 square miles?

72. In an army of 8624 men, 232 men became ill and 38 others were detailed as nurses; how many men remained for active service?

68. GENERAL REVIEW, No. 1.

1. In 287 how many tens with how many units? How many hundreds, and what number remains?

2. In 125 hundreds how many tens? How many thousands and how many hundreds remain?

3. Express and add the following: 9 millions, 46 thousands, 2 hundreds, 15 units; 490 millions, 902 thousands, 7 hundreds, 6 tens, 4 units; and 25 thousands, 2 tens, 1 unit.

4. If a subtrahend is 74 thousand 9 hundred 85, and a minuend is 372 thousand 2 hundred 92, what is the remainder?

5. If the difference of two numbers is 87, and the smaller number is 278, what is the larger number?

6. If there are 172 bushels of grain in a bin which can hold 1000 bushels, how many more bushels of grain may be put into it?

7. In an orchard there are 800 trees, 72 of which are peach trees, 35 are pear trees, and the rest are apple trees; how many are apple trees?

8. If a clerk collects 2834 dollars of one person, 478 dollars of another, and pays out 17 dollars to one and 459 dollars to another, how many dollars will he have left?

9. A man who has 2810 dollars owes 125 dollars to one person, 1642 dollars to another, and 842 dollars to another; how much money will he have left after paying what he owes?

10. How much more money must a person get who has 4398 dollars, that he may pay for a house worth 3500 dollars, furniture worth 1200 dollars, and land worth 575 dollars?

☞ **For Dictation Exercises upon this Review, see "Manual and Key," page 31.**

MULTIPLICATION.

69. If each of 3 boys has two marbles, how many marbles have all?

To find how many they have, we unite three twos of marbles; that is, we unite a number of equal numbers.

The process of uniting a number of equal numbers at once is **Multiplication.**

What is multiplication?

70. Any one of the equal numbers to be united is a **multiplicand.**

What is a multiplicand?

Which is the multiplicand in the given illustration?

71. The number which shows how many equal numbers are to be united is the **multiplier.**

What is the multiplier?

Which is the multiplier in the given illustration?

72. The result of the multiplication is the **product.**

What is the product?

What is the product in the given illustration?

73. The product resulting from multiplication may be considered as the whole, of which each of the numbers united is a part.

The process of multiplication may be thus expressed: —

2 marbles, one of the equal parts, called multiplicand.
3 parts, number of equal parts, called multiplier.
6 marbles, result, called product.

NOTE. — Because by using the multiplicand and multiplier, the product is obtained, the multiplicand and multiplier are called the *factors* (makers) of the product.

REMARK. — On comparing multiplication with addition, it will be seen that they differ mainly in this, that while in addition one number is united to another, and another to their sum, in multiplication the numbers are united at once. It will be seen, also, that in multiplication only equal numbers are united, while in addition unequal numbers may be taken.

74. An oblique cross made thus, ×, is the sign of multiplication. The expression,

$$5 \text{ lemons} \times 3 = 15 \text{ lemons},$$

shows that if three collections of 5 lemons each are united, the result will be 15 lemons.

The expression is read, "5 lemons multiplied by 3 equal 15 lemons"; "three 5's of lemons equal 15 lemons," or "3 times 5 lemons equal 15 lemons."

"Multiplied by" may be considered equivalent to "multiplied by using the given number as a multiplier."

NOTE TO THE TEACHER. — There is an objection to the use of the word "times" in illustrating the subject of multiplication, for in multiplication one number is not taken a number of times, but a number of equal numbers are taken at once.

We can take a number of things, as 3 apples, once, then we can take the same 3 apples a second time, and yet a third time, and have only 3 apples in the end; in this case 3 times 3 apples are 3 apples. But if we take 3 apples at one time, then another 3 apples, and then another 3 apples, we first have 3 apples, then 6 apples, and finally 9 apples; in this case 3 times 3 apples are 9 apples; but here the union was made by addition, not by multiplication.

75. ILLUSTRATIVE EXAMPLE I. Charles brought home from market 4 pecks of russet apples, which cost him 32 cents a peck; how many cents did the 4 pecks cost?

OPERATION BY ADDITION.

```
    3 2
    3 2
    3 2
    3 2
Sum 1 2 8
```

Explanation. — If one peck cost 32 cents, 4 pecks cost four 32's of cents.*

We can find four 32's by addition, as in the margin at the left.

The following is the method by multiplication: —

OPERATION BY MULTIPLICATION.

	Tens.	Units.
Multiplicand	3	2
Multiplier		4
Product	1 2	8

Express in figures 32 and 4 underneath; then proceed as follows: —

32 equals 3 tens with 2 units.

Four 2's of units are 8 units; we write a figure 8 under the line in the units' place.

Four 3's of tens are 12 tens, which equal 1 hundred with 2 tens; we write a figure 2 in the tens' place, and 1 in the hundreds' place, and have 128 for the product, which is the number of cents 4 pecks will cost.

76. ILLUSTRATIVE EXAMPLE II. At 37 cents a yard for cloth, how many cents will 8 yards of cloth cost?

OPERATION.

Tens.	Units.
3	7
	8
2 9	6

Explanation. — At 37 cents a yard, 8 yards of cloth will cost eight 37's of cents.

37 equals 3 tens with 7 units.

Eight 7's of units are 56 units, which equal 5 tens with 6 units; we write a figure 6 under the line in the units' place, and reserve the 5 tens to add to the product of the tens.

Eight 3's of tens are 24 tens, which with the 5 reserved tens are 29 tens, equal to 2 hundreds with 9 tens; we write a

* If the word "times" is used in explanations, the following solution may be adopted: —

If 1 peck costs 32 cents, 4 pecks will cost 4 times 32 cents. 32 equals 3 tens with 2 units. 4 times 2 units are 8 units; we write a figure 8 under the line in the units' place.

4 times 3 tens are 12 tens, which equal, etc

figure 9 in the tens' place, and 2 in the hundreds' place, and have 296 for the product, which is the number of cents 8 yards of cloth will cost.

NOTE.—In practice the pupil needs only say: "Eight 7's are 56; express the 6 and reserve the 5. Eight 3's are 24, and 5 added are 29, etc. "*Ans.* 296 cents."

77. EXAMPLES.

1. In 1 day there are 24 hours; how many hours are there in 7 days? *Ans.* 168 hours.

2. What will 4 cows cost at 52 dollars apiece? *Ans.* 208 dollars.

3. What will 5 horses cost at 248 dollars apiece? *Ans.* 1,240 dollars.

4. If a boy earns 62 cents every day, how many cents will he earn in 6 days? *Ans.* 372 cents.

5. Ellen studied 25 minutes every hour for 9 hours; how many minutes did she study? *Ans.* 225 minutes.

6. How many days are there in 2 years of 365 days each? *Ans.* 730 days.

7. If a steam car goes 233 miles every day, how many miles does it go in 9 days? *Ans.* 2,097 miles.

8. How many pounds of poultry are there in 3 lots of 928 pounds each? *Ans.* 2,784 pounds.

9. How many pounds of butter in 6 lots of 3268 pounds each? *Ans.* 19,608 pounds.

10. The multiplicand being 7264 and the multiplier 7, what is the product? *Ans.* 50,848.

11. What is the sum of 2468 $\times$ 2 and 3579 $\times$ 3? *Ans.* 15,673.

12. What is the sum of 2468 × 3 and 3579 × 2?
Ans. 14,562.

13. What is the sum of 2468 × 4 and 3579 × 5?
Ans. 27,767.

14. What is the sum of 2468 × 5 and 3579 × 4?
Ans. 26,656.

15. What is the sum of 2468 × 6 and 3579 × 7?
Ans. 39,861.

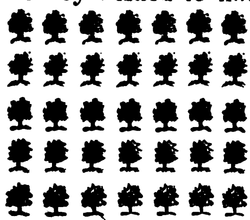
16. What is the sum of 2468 × 7 and 3579 × 6?
Ans. 38,750.

17. What is the sum of 2468 × 8 and 3579 × 9?
Ans. 51,955.

18. What is the sum of 2468 × 9 and 3579 × 8?
Ans. 50,844.

☞ **For test examples in multiplication, which may profitably be given at each recitation, see Walton's "Manual and Key," pages 33, 34.**

78. ILLUSTRATIVE EXAMPLE III. A boy wished to know how many trees there were in his father's orchard; counted in one way, there were 7 rows of 5 trees each, and in another way, there were 5 rows of 7 trees each; how many trees were there in the orchard?



Ans. 35 trees.

From the above illustration, we see that the number of trees in 7 rows of 5 trees each is equal to the number in 5 rows of 7 trees each, or that seven 5's are equal to five 7's.

19. How many cents must be paid for 52 pounds of beef at 8 cents a pound?

Explanation. — 52 pounds, at 8 cents a pound, will cost fifty-two 8's of cents; but fifty-two 8's of cents equal eight 52's of cents (Art. 78), which equal 416 cents; or we may reason thus: 52 pounds at 1 cent a pound will cost 52 cents; 52 pounds at 8 cents a pound will cost eight 52's of cents, or 416 cents.

Ans. 416 cents.

20. How many cents must be paid for 725 pounds of pork at 9 cents a pound? *Ans.* 6,525 cents.

21. What will 298 quarts of milk cost at 7 cents a quart? *Ans.* 2,086 cents.

22. What will 927 gallons of vinegar cost at 11 cents a gallon? *Ans.* 10,197 cents.

23. What will 4372 barrels of flour cost at 12 dollars a barrel? *Ans.* 52,464 dollars.

79. ILLUSTRATIVE EXAMPLE IV. How many men are there in 10 military companies of 87 men each? In 100 companies? In 1000 companies?

In	10	companies	of 87	men	each,	there	are	ten	87's of men.
"	100	"	" 87	"	"	"	"	1 hundred	87's " "
"	1000	"	" 87	"	"	"	"	1 thousand	87's " "

Ten	87's	equal	87 tens (Art. 78),	which equal	870 (Art. 38)
One hundred	87's	"	87 hundreds,	" "	8700
One thousand	87's	"	87 thousands,	" "	87000

From the above we see that *when the multiplier is* 10, *the product may be expressed by annexing to the expression for the multiplicand one zero; when the multiplier is* 100, *by annexing two zeros; and so on.*

80. EXAMPLES.

24. What will 100 bushels of wheat cost at 2 dollars a bushel? *Ans.* 200 dollars.

25. What will 10 saddles cost at 42 dollars each? *Ans.* 420 dollars.

26. What will 100 acres of land cost at 25 dollars an acre? *Ans.* 2,500 dollars.

27. What is the sum of ten 275's, one hundred 275's, and one thousand 275's? *Ans.* 305,250.

28. What is the sum of one hundred 689's, ten 4296's, and ten thousand 88's? *Ans.* 941,860.

29. Multiply 285 by 10, 100, 1000, and add the products. *Ans.* 316,350.

30. Multiply 4692 by 100, 10, 1000, 10000 and add the products. *Ans.* 52,128,120.

31. Multiply 2469 by 10, 100, 10000, 1000, and add the products. *Ans.* 27,430,590.

81. ILLUSTRATIVE EXAMPLE V. What will 20 acres of land cost at 37 dollars an acre?

OPERATION.

$$\begin{array}{r} 37 \\ 20 \\ \hline 740 \end{array}$$

Here the multiplier is 20; 20 equals 2 tens. Ten 37's equal 37 tens, and 2 tens of 37's are 74 tens.

Writing 74 so that it shall represent tens, by placing a zero in the units' place, we have 740, for the product.

From this and similar examples, we learn that *when zeros occur at the right of the expression for the multiplier, we may*

multiply as if there were none, and annex as many zeros to the expression for the product as there were zeros disregarded.

What is the sum of

32. 3685 × 20 and 4682 × 40? *Ans.* 260,980.

33. 762 × 500 and 1923 × 6000? *Ans.* 11,919,000.

34. 98 × 8000 and 24076 × 9000? *Ans.* 217,468,000.

35. Multiply 364 by 2 and by 20, and add the products. *Ans.* 8,008.

36. Multiply 682 by 8 and by 30, and add the products. *Ans.* 25,916.

37. Multiply 328 by 4 and by 60, and add the products. *Ans.* 20,992.

82. ILLUSTRATIVE EXAMPLE VI. What is the product when 78 is the multiplicand and 64 the multiplier?

OPERATION EXPRESSED AND EXPLAINED.

	7 8		
	6 4		
78 × 4 =	3 1 2		first partial product.
78 × 60 =	4 6 8	(tens)	second partial product.
78 × 64 =	4,9 9 2		entire product.

Here the multiplier is 6 tens with 4 units, or 60 with 4. Four 78's equal 312, the first partial product, which we express. 60 equals 6 tens; ten 78's equal 78 tens; 6 tens of 78's equal 468 tens. We write 468 so that it shall represent tens, placing the right-hand figure of the expression under the tens' figure in the first product.

Adding these partial products, we have for the entire product, 4992.

PROOF. Since 78 sixty-fours are equal to 64 seventy-eights (Art. 78), to prove the work of the above example, take for a multiplicand a number equal to the multiplier 64, and for a multiplier a number equal to the multiplicand 78; as the product thus obtained is equal to the product first obtained, the work may be presumed to be correct.

```
   6 4
   7 8
 -----
  5 1 2
 4 4 8
 -----
 4,9 9 2
```

In a similar manner, all examples in multiplication may be proved.

83. EXAMPLES.

38, 39. How many are 87×82? 62×47?
Ans. 7,134; 2,914.

40, 41. How many are 29×65? 45×72?
Ans. 1,885; 3,240.

42, 43. How many are 82×97? 72×76?
Sum of *Ans.* 13,426.

44, 45. How many are 98×52? 59×23?
Sum of *Ans.* 6,453.

84. ILLUSTRATIVE EXAMPLE VII. What is the product when 538 is the multiplicand and 345 the multiplier?

OPERATION EXPRESSED AND EXPLAINED.

	5 3 8		
	3 4 5		
$538 \times 5 =$	2 6 9 0		first partial product.
$538 \times 40 =$	2 1 5 2	(tens)	second partial product.
$538 \times 300 =$	1 6 1 4	(hundreds)	third partial product.
$538 \times 345 =$	1 8 5,6 1 0		entire product.

In this example, the multiplier consists of 3 hundreds with 4 tens and 5 units. Taking the 5 units as a multiplier, we have 2690 as the first partial product; taking the 4 tens as a multiplier, we have

2152 tens as the second partial product; taking the 3 hundreds as a multiplier, we have 1614 hundreds as the third partial product.

Adding these partial products, as before, we have 185610 for the entire product.

NOTE. — Care should be taken to put the first figure of the expression of each partial product under the figure which expresses that part of the multiplier used.

85. EXAMPLES.

46, 47. How many are 246 × 347? 329 × 519?
Ans. 85,362; 170,751.

48, 49. How many are 842 × 468? 912 × 395?
Ans. 394,056; 360,240.

50, 51. How many are 8902 × 9006? 4581 × 8724?
Sum of *Ans.* 120,136,056.

52, 53. How many are 2345 × 9028? 1526 × 5678?
Sum of *Ans.* 29,835,288.

54, 55. How many are 4120 × 7800? 57000 × 9200?
Ans. 32,136,000; 524,400,000.

NOTE. — In performing the last two examples, it is better to disregard the zeros at the right at first, and then annex to the expression for the product as many zeros as were disregarded.

OPERATION OF 55TH EXAMPLE EXPRESSED.

```
    57000
     9200
    -----
   114
  513
  -----------
  524,400,000
```

56, 57. How many are 5470 × 2600? 18007 × 520?
Sum of *Ans.* 23,585,640.

58, 59. How many are 2000 × 5670? 48000 × 720?
Sum of *Ans.* 45,900,000.

60, 61. How many are 3800 × 6000? 21000 × 940?
Sum of *Ans.* 42,540,000.

62. How many are 125 × 37 × 18? *Ans.* 83,250.

63. How many are 340 × 62 × 55? *Ans.* 1,159,400.

64. How many are 268 × 10 × 107? *Ans.* 286,760.

65. How many are 829 × 98 × 206? *Ans.* 16,735,852.

66. How many are 327 × 79 × 870? *Ans.* 22,474,710.

67. I have 6 bins, each of which contains 32 bushels of corn; how many bushels do all contain?

68. How many tomato plants are there in 12 rows of 38 tomato plants each? *Ans.* 456 tomato plants.

69. How many trees are there in an orchard which contains 18 rows of 27 trees each? *Ans.* 486 trees.

70. What will 284 barrels of flour cost at 13 dollars a barrel? *Ans.* 3,692 dollars.

71. What will 427 bushels of corn cost at 56 cents a bushel? *Ans.* 23,912 cents.

72. What will 595 hats cost at 62 cents for each?

73. What will 1276 bushels of wheat cost at 87 cents a bushel? *Ans.* 111,012 cents.

74. At 1 dollar a gallon for oil, what must I pay for 82 casks of oil, each containing 184 gallons?
Ans. 15,088 dollars.

75. If there are 8 rows of desks in your schoolroom, and each row contains 8 desks, and each desk seats 2 pupils, how many pupils can be seated in all?
Ans. 128 pupils.

76. At 18 cents a yard, what is the cost of cloth for 12 shirts, if it takes 3 yards of cloth to make each shirt?

77. There is a kind of snail that has upon its tongue 160 rows of teeth, and 180 teeth in each row; how many teeth have 2 such snails?

78. If a machine can make 46 paper bags in a minute, how many paper bags can the same machine make in 2 hours of 60 minutes each? *Ans.* 5,520 bags.

79. If 2760 bags can be made in one hour, how many can be made in 6 days of 11 hours each?

80. How much money does a person make in 12 weeks who gives 4 lectures a week, receiving 75 dollars more than his expenses for each lecture? *Ans.* 3,600 dollars.

81. In one year there are 8766 hours; how many hours has a boy lived who is 12 years old?

82. In one year there are 525960 minutes; how many minutes has a boy lived who is 15 years old?

83. If 15000 types can be set by one machine in one hour, how many can be set by 2 machines in 3 days of 9 hours each?

84. How much will a photographer receive for 42 dozen photographs at 3 dollars a dozen, and 5 photographs at 8 dollars apiece? *Ans.* 166 dollars.

85. How many days are there in 35 years of 365 days each, and 12 years of 366 days each?

86. Arthur sold 23 boxes of strawberries at 35 cents a box, and spent 500 cents of the money he received, for flour; how many cents had he left?

87. Mr. Lee owned 45 shares of railroad stock worth 100 dollars a share, 38 shares of the stock of a coal company worth 10 dollars a share, and enough real estate to make his entire property worth 14500 dollars; what was the worth of his real estate?

DIVISION.

86. William has six tops, which he gives away to some boys; if he gives two tops to each boy, to how many boys does he give them?

That we may know to how many boys he gives them, we must find how many of the equal numbers, twos of tops, there are in another number of tops, six.

The process of finding how many equal numbers, one of which is given, there are in another number, is **Division.**

What is division?

87. In the above illustration, six tops is the number divided; the number divided is the **dividend.**

What is the dividend?

88. In the illustration, two tops is one of the equal numbers which there are in another number, six tops. Any one of the equal numbers which there are in another number is a **divisor.**

What is a divisor?

89. The result of division, or the number of equal parts obtained, is the **quotient.**

What is the quotient?

90. If in the above illustration the boy had had seven tops instead of six, and had given away twos of tops, as before, would he have had any tops left? How many?

The part of the dividend left after equal numbers are taken away, is the **remainder.**

What is the remainder?

91. The dividend may be regarded as a whole, of which it is required to find the number of equal parts, one of the equal parts being given.

The process of division may be thus expressed: —

One of the equal parts, called *divisor*. } 2 tops,) 6 tops, { The whole number, called *dividend*.

3 parts, { Number of equal parts there are in the whole, called *quotient*.

REMARK. — Division may be regarded as the reverse of multiplication; for, while in division the whole and one of the equal parts are given, and it is required to find the number of parts, in multiplication one of the equal parts and the number of parts are given, and it is required to find the whole.

92. The sign of division is a short horizontal line between two dots; thus, $\div$. The expression

$$6 \div 2 = 3$$

shows that there are three 2's in 6. It may be read, "2's in 6, 3," or "6 divided by 2 equals 3."

NOTE. — Sometimes the dividend and divisor are expressed in place of the dots; thus,

$$\frac{6}{2} = 3,$$

read "1 half of 6 equals 3," or "6 divided by 2 equals 3."

TO THE TEACHER. — As this form of expression is also used for fractions, it has been urged that its use in division should be discontinued; but as it is a convenient form, and one extensively used, it is here retained. If the following could be universally adopted, it would, on some accounts, be preferable: —

$$2 \mid 6 = 3.$$

93. ILLUSTRATIVE EXAMPLE I. Among how many pupils must 48 pencils be divided that each pupil may receive 2 pencils?

Explanation. — If each pupil receives 2 pencils, 48 pencils must be divided among as many pupils as there are 2's in 48.

We may ascertain how many 2's there are in 48 by taking away first one 2, and then another, and then another, till no 2's remain, and then counting the 2's taken; this is the method by subtraction. The following is the method by division: —

OPERATION.

Units of pencils. Tens, Units } of pencils.

$$\text{Divisor. } 2\,)\,\underline{4\ 8}\ \text{ Dividend.}$$
$$\text{Quotient. } 2\ 4\ \{\text{Twos of pencils.}$$

We express in figures the 48 with 2 at the left, drawing a curved line between the expressions and a straight line under that for the dividend.

48 equals 4 tens or 40, with 8.

In 40 there are twenty 2's, or 2 tens of 2's; we write a figure 2 under the line in the tens' place.

In 8 there are four 2's; we write a figure 4 under the line in the units' place and have 2 tens with 4 units, or 24 for the result; therefore, that each pupil shall receive 2 pencils, 48 pencils must be divided among 24 pupils.

NOTE. — In practice, the pupils need only say "2's in 4, 2; 2's in 8, 4: answer, 24 pupils."

94. EXAMPLES.

(1.)	(2.)	(3.)	(4.)
Divisor. 3) 3 9 6 Dividend.	4) 8 0 4	2) 8 2 4 6	5) 5 0 5 5
1 3 2 Quotient.	2 0 1		

95. ILLUSTRATIVE EXAMPLE II. How many 3's are there in 246?

OPERATION.

$$3\,)\,\underline{2\ 4\ 6}$$
$$8\ 2$$

Explanation. — In the number 2 hundred there are no hundreds of 3's; we therefore first divide 24 tens.

In 24 tens, or 240, there are 80 threes, or 8 tens of 3's; we write a figure 8 under the line in the tens' place.

In 6 there are two 3's; we write a figure 2 under the line in the units' place, and have 82 for the quotient, which is the number of 3's in 246.

96. EXAMPLES.

5. A man has 306 quarts of milk which he wishes to put into cans that hold 6 quarts each; how many cans must he use? *Ans.* 51 cans.

6. In 420 days how many weeks? *Ans.* 60 weeks.

7. In 1 peck there are 8 quarts; how many pecks are there in 488 quarts? *Ans.* 61 pecks.

8. A boy has 639 hours' work to do; how many days will it take him to do it, if he works 9 hours a day? *Ans.* 71 days.

97. ILLUSTRATIVE EXAMPLE III. At 8 cents apiece, how many oranges can be bought for 1945 cents?

OPERATION.

Thousands. Hundreds. Tens. Units.

(16)(32)

8) 1 9 2 5 — 5

2 4 0

Explanation. — As in 19 there are two 8's and 3 over, in 19 hundred there are 2 hundred 8's and 3 hundred over; we write a figure 2 below the line in the hundreds' place, and unite the 3 hundred which remain, with the tens, making 32 tens.

In 32 tens, or 320, there are 40 eights, or 4 tens of 8's; we write a figure 4 below the line in the tens' place.

In 5 there are no 8's; we write a zero in the units' place, and a figure 5 at the right to indicate the remainder.

NOTE. — In practice, the pupil may say "8's in 19, 2 and 3 remain; 8's in 32, 4; 8's in 5 none, and 5 remain. *Ans.* 240 oranges; 5 cents remain."

98. PROOF. — Since the quotient shows how many parts equal to the divisor there are in the dividend, take the divisor for a multiplicand and the quotient for a multiplier, and to the product, when found, add the remainder if there is one. If the result is like the dividend, the work may be presumed to be correct.

99. EXAMPLES.

9. At 9 dollars a barrel for flour, how many barrels of flour can be bought for 200 dollars, and how many dollars will remain? *Ans.* 22 barrels; 2 dollars remain.

10. At 8 cents apiece, how many oranges can be bought for 500 cents, and how many cents will remain?

Ans. 62 oranges; 4 cents remain.

11. At 11 dollars for a coat, how many coats can be bought for 127 dollars, and how many dollars will remain?

Ans. 11 coats; 6 dollars remain.

12. How many weeks are there in 1367 days?

Ans. 195 weeks; 2 days remain.

13. At 3 dollars apiece, how many books can be bought for 725 dollars, and how many dollars will remain?

Ans. 241 books; 2 dollars remain.

14. What is the largest number of 5-dollar bills which may be used to pay a debt of 752 dollars, and how many 1-dollar bills may be used besides? What number of 10-dollar bills may be used? Of 2-dollar bills?

Ans. { 150 five-dollar bills, 2 one-dollar bills.
75 ten-dollar bills, 2 one-dollar bills.
376 two-dollar bills.

15. Divide 12342 by 2; by 3, and add the quotients.

Ans. 10,285.

16. Divide 41472 by 3; by 4, and add the quotients.

Ans. 24,192.

17. Divide 98760 by 4; by 5, and add the quotients.

Ans. 44,442.

18. Divide 27690 by 5; by 6, and add the quotients.

Ans. 10,153.

19. Divide 42336 by 6; by 7, and add the quotients.

Ans. 13,104.

20. Divide 69104 by 7; by 8, and add the quotients.

Ans. 18,510.

21. Divide 74304 by 8; by 9, and add the quotients.
Ans. 17,544.

22. Divide 42264 by 9; by 12, and add the quotients.

23. Divide 340692 by 11; by 12, and add the quotients.

☞ For test examples in division, which may profitably be given at each recitation, see "Waltons' Manual and Key," pages 37, 38.

NOTE. — In the preceding examples in division, each divisor has been 12 or less than 12. When the divisor is greater than 12, it is generally necessary to express more of the work. Because the former process takes less time and space, it is called *short division*, while the process illustrated below is called *long division*.

100. ILLUSTRATIVE EXAMPLE IV. How many shad at 32 cents apiece can be bought for 175 cents?

OPERATION.

```
Tens. Units.  Hundred. Tens. Units.  Units.
 3     2   )   1        7     5   (   5
               1        6     0
               ----------------
                        1     5
```

Explanation. — As many shad at 32 cents apiece can be bought for 175 cents as there are 32's in 175.

Sometimes, as in this case, the quotient figures are more easily found by using a trial divisor; thus, 32 equals 3 tens with 2 units; 175 equals 17 tens with 5 units. There are about as many 32's in 175 as there are 3's of tens in 17 tens, or as there are 3's in 17. There are five 3's in 17; we therefore assume that there are five 32's in 175, and write 5 as the quotient figure at the right of the expression for the dividend. Five 32's equal 160. Taking 160 of the 175 away, we have a remainder of 15. Therefore 5 shad can be bought for 175 cents, and 15 cents will remain.

101. EXAMPLES.

24. 359 ÷ 43?	*Ans.* 8; 15 rem.	29. 233 ÷ 63?	44 rem.
25. 546 ÷ 72?	*Ans.* 7; 42 rem.	30. 427 ÷ 82?	17 rem.
26. 760 ÷ 91?	*Ans.* 8; 32 rem.	31. 461 ÷ 65?	6 rem.
27. 522 ÷ 84?	*Ans.* 6; 18 rem.	32. 309 ÷ 53?	
28. 527 ÷ 63?	23 rem.	33. 823 ÷ 74?	

102. ILLUSTRATIVE EXAMPLE V. How many barrels can be filled with 40337 pounds of flour, if 196 pounds are put into each barrel?

OPERATION.

```
h. t. u.      h. t. u.   h. t. u.
1 9 6 ) 4 0 3 3 7 ( 2 0 5
        3 9 2
        -------
          1 1 3 7
            9 8 0
        ---------
            1 5 7
```

Explanation. — As many barrels can be filled as there are 196's in 40337.

As 196 is nearly 200, there are about as many 196's in 403 as there are 200's in 400, or as there are 2's in 4; we therefore make 2 the trial divisor. In 4 there are two 2's, we therefore assume that there are two 196's in 403 (or 2 *hundred* 196's in 403 hundreds), and make 2 the first quotient figure which will occupy the hundreds' place. 2 (hundred) 196's are 392 (hundred).

Taking 392 (hundred) of the 403 (hundred) away, we have left 11 (hundred). Uniting with this the 3 (tens) of the dividend, we have 113 (tens).

As 113 is smaller than 196, there are no (tens of) 196's in 113 (tens); we therefore write a zero as the next quotient figure.

Uniting the 7 units of the dividend with the 113 tens, we have 1137. Applying the trial divisor as before, we find the last quotient figure to be 5 and the remainder to be 157. Therefore 205 barrels can be filled, and 157 pounds will remain.

NOTE. — In practice the pupils need only say "196's in 403, two; two 196's are 392; 11 remain; annex 3; 196's in 113, none; annex 7; 196's in 1137, five; five 196's are 980; 157 remain.

"*Ans.* 205 barrels; 157 pounds remain."

103. EXAMPLES.

34. How many chests which contain 46 pounds each will be required to contain 5704 pounds of tea?

Ans. 124 chests.

35. How many acres of land at 55 dollars an acre can be bought for 10795 dollars? *Ans.* 196 acres; 15 dollars rem.

36. How many cows at 34 dollars apiece can be bought for 19207 dollars? *Ans.* 564 cows; 31 dollars rem.

37. How many numbers of 468 units each are there in 32842 units? *Ans.* 70; 82 units remain.

38, 39. How many 238's are there in 69549? in 79200?
Ans. 292; 53 remain. 332; 184 remain.

40, 41. How many 763's are there in 59003? in 38761?
252 remain. 611 remain.

42, 43. How many 597's are there in 30954? in 43007?

44, 45. Divide 678090 by 4008; by 3907.

46, 47. Divide 69542 by 1239; 567890 by 89721.

104. ILLUSTRATIVE EXAMPLE VI. In 87000 how many 10's? 100's? 1000's?

Ans. In 87000 there are 8700 tens (see Art. 36).
" " " " 870 hundreds.
" " " " 87 thousands.

ILLUSTRATIVE EXAMPLE VII. At 100 cents a pound how many pounds of tea may be bought for 235 cents, and how many cents will remain?

OPERATION.

1|0 0) 2|3 5
2 Quotient.
3 5 Remainder.

Explanation.—The 2 of the dividend by expressing the 100's of cents contained in 235 cents also expresses the number of pounds of sugar that can be bought, and the 35 expresses the cents that remain.

From the above and similar examples, we see that *when the divisor is* 10 *the quotient and remainder are indicated by cutting off one figure at the right of the expression for the dividend; when the divisor is* 100, *by cutting off two figures at the right of the expression for the dividend, and so on.*

The figures at the left of the separating line express the quotient, and those at the right express the remainder.

105. EXAMPLES.

48. 100 cents equal 1 dollar; how many dollars are there in 200 cents? in 1000 cents? in 1500 cents?

Ans. 2 dollars; 10 dollars; 15 dollars.

49. In 2550 cents how many dollars, and how many cents remain?

50. How many centuries of 100 years each in 1475 years, and how many years remain?

51. Divide 36000 by 10; by 1000; and add the quotients.

Ans. 3,636.

106. ILLUSTRATIVE EXAMPLE VIII. How many 300's are there in 1327?

OPERATION.

```
Divisor.  Dividend.  Quotient.
3|0 0 ) 1 3|2 7 ( 4
        1 2
        -----
          1 2 7  Remainder.
```

Explanation. — In 1327 there are 13 hundreds with 27 units remaining; in 13 hundreds there are four 3 hundreds, with 1 hundred remaining. Uniting the 1 hundred remaining with the 27 units, we have for the entire remainder 127.

From the above and similar examples we learn that, *when zeros occur at the right of the expression for the divisor, we may cut them off, and also cut off an equal number of figures at the right of the expression for the dividend, and then divide as if the remaining figures expressed the true divisor and dividend.*

If there be no remainder after this division, the undivided part of the dividend will be the true remainder.

If there be a remainder after this division, at the right of the expression for the remainder, express the undivided part of the dividend; the true remainder will be thus expressed.

107. EXAMPLES.

52. How many are 3678 ÷ 20? *Ans.* 183; 18 remain.

53. How many are 5849 ÷ 700? *Ans.* 8; 249 remain.

54. How many are 19876 ÷ 290? 156 remain.

55. How many are 10000 ÷ 3700? 2,600 remain.

56. How many are 68301 ÷ 1020? 981 remain.

57. Divide 49756000 by 80; by 500; by 4000, and add the quotients. *Ans.* 733,901.

58. Divide 665280 by 56, 63, 70, 616, and 5544, and add the quotients. *Ans.* 33,144.

59. Divide 1310400 by 26, 14, 130, 36, 1040, and 112, and add the quotients. *Ans.* 203,440.

108. ILLUSTRATIVE EXAMPLE IX. If 86 marbles are given to 2 boys, so that they share them equally, how many marbles will each boy receive?

Explanation. — To find how many marbles each boy will receive, we may give one marble to each of the boys and then another and then another, till all the marbles are given away, and then count each boy's marbles; but the following is a shorter method.

OPERATION.

$$\begin{array}{r|l} 2 & 86 \\ \hline & 43 \end{array}$$

When 1 marble is given to each of 2 boys, 2 marbles are given away; if 2 marbles are given away in order that each boy may receive 1 marble, when 86 marbles are shared between them, each boy will receive as many marbles as there are 2's of marbles in 86 marbles, or as there are 2's in 86. There are 43 twos in 86, therefore each boy will receive 43 marbles.

From the above illustration, we see that *in one of the* 2 *equal parts of a number there are as many units as there are* 2'*s in the given number.*

By a similar process we should find *that in one of the* 3 *equal parts of a number there are as many units as there are* 3'*s in the given number; in one of the* 4 *equal parts of a number, as many units as there are* 4'*s in the given number, and so on.*

109. When any number is divided into 2 equal parts, each of the parts is called 1 *half* of the number; when into 3 equal parts, each of the parts is called 1 *third* of the number; when into 4 equal parts, 1 *fourth* of the number; when into 5 equal parts, 1 *fifth*, and so on;* hence,

To find 1 half of a number, make 2 a divisor; to find 1 third of a number, make 3 a divisor, and so on.

NOTE TO THE TEACHER.—In the Illustrative Example above we were required to find one of the equal parts of a number, the number of parts being given, while in previous examples we have been required to find the number of equal parts contained in a number, one of the parts being given.

Comparing both these forms of division with multiplication, we see that the dividend in division corresponds to the product in multiplication; but while the first form of division (Arts. 86 – 107) consists in finding the multiplier (or number of parts), when the multiplicand (one part) is given, the second form of division (Arts. 108 – 112) consists in finding the multiplicand (one of the equal parts), the multiplier (or number of equal parts) being given. Though the written operation is the same in both forms of division, the explanations of practical examples should differ. Compare explanations of examples in Arts. 93 to 102 with explanations in Arts. 108 and 110.

110. ILLUSTRATIVE EXAMPLE X. James had 63 nuts, and divided them equally among his 3 brothers; how many nuts did he give to each?

OPERATION.

$$\begin{array}{r} 3\,)\,\underline{6\;3} \\ 2\;1 \end{array}$$

Explanation.—He gave 1 third of 63 nuts to each, which are as many nuts as there are 3's in 63. There are twenty-one 3's in 63; therefore he gave 21 nuts to each.

ILLUSTRATIVE EXAMPLE XI. Horace had 172 apples, and sold 1 fourth of them; how many did he sell?

OPERATION.

$$\begin{array}{r} 4\,)\,\underline{1\;7\;2} \\ 4\;3 \end{array}$$

Explanation.—He sold 1 fourth of 172 apples, which is as many apples as there are 4's in 172, etc.

In 17 tens or 170, there are 40 or 4 tens of 4's; we write a figure 4 under the line in the ten's place, etc.

* See Intellectual Arithmetic, Section XVI.

NOTE I. — After the pupil can readily give equal parts of the smaller numbers, explanations like the following of Illustrative Example XI. may be preferred.

OPERATION.

$$4\,)\underline{1\;7\;2}$$
$$4\;3$$

Explanation. — 17 tens equal 16 tens plus 1 ten; 1 fourth of 16 tens is 4 tens; 1 ten with 2 units equals 12 units; 1 fourth of 12 units is 3 units, and we have for the quotient 4 tens with 3 units, or 43, the number of apples sold.

NOTE II. — In practice the pupils need only say, "1 fourth of 17, 4; 1 fourth of 12, 3. *Ans.* 43."

111. EXAMPLES.

60. What is the price of one chair, when 6 chairs cost 132 dollars? *Ans.* 22 dollars.

61. Charles bought 8 watch-guards for 600 cents, what was the price of one watch-guard? *Ans.* 75 cents.

62. When 12 towels can be bought for 444 cents, what is the price of 1 towel? *Ans.* 37 cents.

63. A man whose income was 3564 dollars gave 1 ninth of it in charity; how many dollars did he give in charity?

64. I have a square field with 592 rods of fencing around it; what is the length of one side of the field? *Ans.* 148 rods.

112. ILLUSTRATIVE EXAMPLE XII. What is 1 fourth of 5847 dollars?

OPERATION.

$$4\,)\underline{5\;8\;4\;7}$$
$$1\;4\;6\;1\tfrac{3}{4}$$

Explanation. — Here, after dividing what we can of the dividend and have whole numbers in the quotient, we have 3 units of the dividend yet undivided. These we can also divide by taking 1 fourth of each of them; if 1 fourth of each of the 3 is taken, we shall have 3 fourths (see Int. Arith.). This is expressed by a figure 3 written over a figure 4 with a short line between them, thus $\frac{3}{4}$. The whole answer is read 1461 and 3 fourths dollars.

65. What is 1 sixth of 4879? *Ans.* $813\frac{1}{6}$.

66. What is 1 third of 36701? *Ans.* $12{,}233\frac{2}{3}$.

67. What is 1 ninth of 8972? *Ans.* $996\frac{8}{9}$.

68. What is 1 seventh of 98764? *Ans.* $14{,}109\frac{1}{7}$.

69, 70. What is 1 fifth of 4567? 1 eighth of 64321?

71. If there are 1461 days in 4 years, what is the average number of days in a year? *Ans.* $365\frac{1}{4}$ days.

113. Examples in Both Forms of Division.

72. At 8 cents apiece, how many pears can be bought for 464 cents?

73. How many slates, at 17 cents apiece, may be bought for 1360 cents?

74. In an orchard there are 495 trees, 45 trees in a row; how many rows are there?

75. How many barrels of flour, at 14 dollars a barrel, can be bought for 175 dollars, and how many dollars will remain? 7 dollars remain.

76. How many cows, at 35 dollars apiece, can be bought for 250 dollars, and how many dollars will remain?
5 dollars remain.

77. How many yoke of oxen, at 165 dollars a yoke, can be bought for 1000 dollars? 10 dollars remain.

78. How many acres of land, at 140 dollars an acre, can be bought for 1756 dollars? 76 dollars remain.

79. What is the price of 1 yard of cloth, when 40 yards can be bought for 525 cents? *Ans.* $13\frac{5}{40}$ cents.

80. Mr. Prescott had 93 hens, which laid 465 eggs in one week; what was the average number of eggs laid by each hen? *Ans.* 5 eggs.

81. 5000000 pounds of cotton were raised in Illinois in one year, the land under cultivation yielding 1000 pounds to the acre; how many acres were under cultivation?

114. Miscellaneous Examples in Review.

1. Water freezes at a temperature of 32 degrees Fahrenheit, which is 180 degrees lower than the temperature at which water boils; at what degree of temperature does water boil? *Ans.* 212 degrees.

2. 133416 emigrants arrived in New York in 1867, which was 9731 more than arrived in 1866; how many arrived in 1866? *Ans.* 123,685 emigrants.

3. At 2 dollars for pasturing each sheep during the summer, and 3 dollars for feed during the rest of the year, what is the cost of feeding a flock of 35 sheep for 2 years? *Ans.* 350 dollars.

4. What will be the cost of fencing 49 miles of railroad at 728 dollars a mile?

5. A man who has 1847 dollars wishes to purchase a house worth 3072 dollars; in how many years can he pay for it from his savings, if he saves 245 dollars a year? *Ans.* 5 years.

6. A man sold a farm of 42 acres at 90 dollars an acre, and with the money received for it purchased 63 acres of woodland; what did he pay an acre for his woodland? *Ans.* 60 dollars.

7. How many yards of cloth can be made of 5000 pounds of wool, if 62 pounds are allowed for waste and 3 pounds are put into every yard? 1,646 yards.

8. How many yards of cloth at 27 cents a yard will pay for 9 pounds of butter at 42 cents a pound, and 1 basket worth 81 cents?

UNITED STATES MONEY.

Nickel Cent.

Gold Dollar.

Silver Dime.

Gold Eagle.

115. The cent, dime, dollar, and eagle, here represented, are pieces of United States money; they are called **coins.**

116. Each of these coins represents a unit of value. Besides these four units, there is another unit called a mill.

Ten of any lower order of these units are equal to one of the next higher order.

The relative values of these units are as follows:—

10 mills (m.)	=	1 cent,	marked	ct.
10 cents	=	1 dime,	"	d.
10 dimes	=	1 dollar,	"	$
10 dollars	=	1 eagle,	"	E.

117. One unit of each order may be expressed thus:—

Eagle.	Dollar.	Dime.	Cent.	Mill.
1	1	1	1	1

118. Eagles being tens of dollars, and dimes tens of cents, eagles are generally expressed as tens of dollars and dimes as tens of cents.

One unit of each order, then, is expressed thus: —

$ 11.111

and the expression is read 11 dollars, 11 cents, 1 mill. The dot (.) used in the expression is called a *decimal point.*

Dollars are expressed at the left of the decimal point, cents in the first two places at its right, and mills in the third place.

119. Read the following expressions: —

1.	$ 8.19	7.	$ 30.181
2.	$ 1.324	8.	$ 824.70
3.	$ .728	9.	$ 103.082
4.	$ .024	10.	$ 908.007
5.	$ 32.20	11.	$ 3000.25
6.	$ 75.26	12.	$ 1708.025

120. Express the following in figures: —

13. Fifty-four dollars, twenty-nine cents. $ 54.29.
14. Ninety-seven dollars, sixty-two cents, five mills. $ 97.625.
15. Sixty-nine cents, two mills. $ .692.
16. Five cents, five mills. $ .055.
17. Eight dollars, three cents, two mills.
18. Three hundred dollars, twenty-seven cents.
19. Nine hundred and eight dollars, nine cents.
20. Ten thousand and thirty dollars, two cents.

121. To change numbers from units of higher to units of lower orders.

1 cent is equal to how many mills?

1 dime is equal to how many cents? how many mills?

1 dollar is equal to how many dimes? cents? mills?

ILLUSTRATIVE EXAMPLE I. In 5 cents how many mills?

Explanation. — Since 1 cent equals 10 mills,
5 cents equal 5 tens of mills, or 50 mills.

ILLUSTRATIVE EXAMPLE II. In 8 dimes how many cents? how many mills?

Explanation. — Since 1 dime equals 10 cents or 100 mills,

$$8 \text{ dimes equal} \begin{cases} 8 \text{ tens of cents, or } 80 \text{ cents.} \\ 8 \text{ hundreds of mills, or } 800 \text{ mills.} \end{cases}$$

ILLUSTRATIVE EXAMPLE III. In 5 dollars how many dimes? how many cents? how many mills?

Explanation. — Since $ 1 equals 10 dimes, 100 cents, or 1000 mills,

$$\$\,5 \text{ equal} \begin{cases} 5 \text{ tens of dimes, or } 50 \text{ dimes.} \\ 5 \text{ hundreds of cents, or } 500 \text{ cents.} \\ 5 \text{ thousands of mills, or } 5000 \text{ mills.} \end{cases}$$

From the above illustrations, it will be seen that,

To change numbers from units of higher to units of lower orders, *we multiply the given number by the number of units it takes of the lower order to make one unit of the higher order.*

NOTE. — To express the product of a number multiplied by 10, 100, or 1000, see Art. 79. When the decimal point is given in any example, it is only necessary to move the point one, two, or three places to the right as occasion may require.

122. EXAMPLES.

Change

21. $ 2 to dimes. *Ans.* 20 dimes.
22. $ 15 to cents. *Ans.* 1,500 cents.
23. $ 68 to mills. *Ans.* 68,000 mills.
24. $ .55 to mills. *Ans.* 550 mills.
25. $ 7.50 to cents. *Ans.* 750 cents.
26. $ .02 to mills.
27. $ 1 to cents; to mills. *Ans.* 100 ct.; 1,000 m.
28. $ 10 to dimes; to cents. *Ans.* 100 d.; 1,000 ct.
29. $ 8.20 to cents.
30. $ 15.45 to cents.
31. $ 50.07 to mills.
32. $ 1000 to cents.
33. $ 29.185 to mills.
34. 34 dollars 7 cents to mills.
35. 158 dollars 20 cents to mills.

123. To change numbers from units of lower to units of higher orders.

ILLUSTRATIVE EXAMPLE IV. In 70 dimes how many dollars?

Explanation. — Since 10 dimes equal 1 dollar, 70 dimes will equal as many dollars as there are 10's in 70, or 7 dollars.

ILLUSTRATIVE EXAMPLE V. In 400 cents how many dimes? how many dollars?

Explanation. — Since 10 cents equal 1 dime,
and 100 cents equal 1 dollar,
400 cents will equal as many dimes as there are 10's in 400; and as many dollars as there are 100's in 400.

$$\text{Hence, 400 cents equal}\begin{cases}\text{40 dimes,}\\ \text{4 dollars.}\end{cases}$$

ILLUSTRATIVE EXAMPLE VI. In 5000 mills how many cents? how many dimes? how many dollars?

Explanation. — Since 10 mills equal 1 cent,
100 mills equal 1 dime,
and 1000 mills equal 1 dollar,
5000 mills will equal as many cents as there are 10's in 5000; as many dimes as there are 100's in 5000; and as many dollars as there are 1000's in 5000.

$$\text{Hence, 5000 mills equal}\begin{cases}\text{500 cents,}\\ \text{50 dimes,}\\ \text{5 dollars.}\end{cases}$$

From the above illustrations, it will be seen that

To change numbers from units of lower to units of higher orders, *we divide the given number by the number of units it takes of the lower order to make one unit of the higher order.*

NOTE. — To divide by 10, 100, or 1000. See Art. 104.

124. EXAMPLES.

Change the following to units of higher orders: —

36. 3000 mills to cents. *Ans.* 300 cts.

37. 15000 mills to dollars. *Ans.* $15.

38. 15000 cents to dollars. *Ans.* $150.

39. 1524 cents to dollars with cents. *Ans.* $15.24.

40. 11111 mills to dollars with cents and mills. *Ans.* $11.111.

41. 375 mills to cents with mills. *Ans.* $.375.

42. 146 cents to dollars with cents.

43. 75000 mills to cents.

44. 120000 mills to dollars.

45. 8500 cents to dollars.

46. 11424 cents to dollars with cents.

47. 50341 mills to dollars with cents and mills.

48. 462 mills to cents with mills.

ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF UNITED STATES MONEY.

125. ILLUSTRATIVE EXAMPLE VII. What is the sum of $2.07, $80, $50.306, and $78.245?

OPERATION.

```
$  2.07
  80.
  50.306
  78.245
--------
$210.621
```

Explanation. — We express these numbers in figures so that units of the same order shall be expressed in the same column, and draw a line underneath.

Commencing with the mills, we find their sum to be 11 mills, which equal 1 cent with 1 mill: we write a figure 1 under the line, etc., etc.

126. EXAMPLES.

What is the sum of

49. $37.007 + $2.05 + $.375? *Ans.* $39.432.

50. $257.08 + $19 + $5.32? *Ans.* $281.40.

51. $1.50 + $.875 + $2.25 + $.165 + $1.25?

52. $1000.50 + $250 + $4.875 + $900.25?

53. Mr. Day earned at one time $13.50, at another $25.08, at another $50.125, and at another $6.25; how much did he earn in all?

127. ILLUSTRATIVE EXAMPLE VIII. If I have $767.18, and pay $88.20 for a suit of clothes, how much shall I have left?

OPERATION.

```
$767.18
   88.20
 -------
$678.98
```

Explanation. — We express $767.18, and beneath we express $88.20, so that units of the same order shall be expressed in the same column, and draw a line underneath. If none of the 8 cents are taken, 8 cents remain; we write a figure 8 under the line, etc., etc.

128. EXAMPLES.

54. How much more is $99.20 than $78.60?
Ans. $20.60.

55. What is the difference between $27 and $6.38?

NOTE. — $27 = $27.00. (Art. 121.) 27.00 — 6.38 = 20.62.
Ans. $20.62.

56. How much more must a man get who has $25.60 to purchase a coat worth $30? *Ans.* $4.40.

57. How much more is $1.80 than $.985?
Ans. $.815.

58. If a man who has $50, spends $17.625, how much will he have left? *Ans.* $32.375.

129. ILLUSTRATIVE EXAMPLE IX. If I earn $2.25 each day for 6 days, how much money do I earn in all?

OPERATION.

$$\begin{array}{r} \$2.25 \\ 6 \\ \hline \$13.50 \end{array}$$

Explanation. — We express the multiplicand $2.25 and the multiplier 6 under its right-hand figure, and draw a line beneath; six 5's of cents are 30 cents, which equal 3 dimes with no cents; we write a zero, etc.

130. EXAMPLES.

59. If each of 9 persons has $58.28, how much money have all? *Ans.* $524.52.

60. If a person earns $1000 a year, what will he earn in 16 years? *Ans.* $16,000.

61. Multiply $200.25 by 105. *Ans.* $21,026.25.

62. Multiply $18.64 by 24. *Ans.* $447.36.

63. What cost 6 tons of coal at $10.36 a ton?

64. If a man earns $42 a month, how many dollars will he earn in a year, or 12 months?

65. At $1.25 apiece, what will 2 dozen or 24 hats cost?

131. ILLUSTRATIVE EXAMPLE X. How many boxes at 7 cents a box may be bought for $2.75?

OPERATION.

$$\begin{array}{r} 7\,)\,\underline{2\;7\;5}—2 \\ 3\;9\quad \end{array}$$

Explanation. — As many boxes at 7 cents a box may be bought for $2.75 as there are 7's of cents in $2.75. Before performing the division it is necessary that the divisor and dividend should be expressed in units of the same order. As the divisor is cents, we change the dividend to cents. $2.75 equal 275 cents, in which there are thirty-nine 7's of cents and 2 cents remain. Therefore 39 boxes can be bought, and 2 cents remain.

132. EXAMPLES.

66. How many candles at 8 cents apiece can be bought for $ 5, and how many cents will remain?

Ans. 62 candles, 4 cts. rem.

67. Alvin spends $ 9 for cloth at 60 cents a yard; how many yards did he buy? *Ans.* 15 yards.

68. How many toys at 17 cents each can be bought for $ 10, and how many cents will remain?

69. A woman who had $ 3.92 bought as many yards of cloth as she could at 18 cents a yard, and spent the rest of her money for thread; how many yards of cloth did she buy, and how many cents did she spend for thread?

133. 70. Mr. Greene divided $ 13.52 equally among his 8 children; what did he give to each? *Ans.* $ 1.69.

71. If 15 pounds of sugar cost $ 1.65, what will 1 pound cost? *Ans.* $ .11.

72. If 6 chairs cost $ 16.50, what will 1 chair cost?

73. What is the cost of 1 concert ticket, if 5 can be bought for $ 1.75?

74. If 8 hens cost $ 6, what is the cost of 1 hen?

NOTE.—$ 6 equals 600 cents; 1 eighth of 600 cents is 75 cts. *Ans.* $ .75.

75. What is 1 sixth of $ 29? (Ans. in dollars with cents and mills.) *Ans.* $ 4.833$\frac{2}{6}$.

76. What is 1 seventh of $ 32? *Ans.* $ 4.571$\frac{3}{7}$.

77. What is 1 fifth of $ 424? *Ans.* $ 84.80.

134. MISCELLANEOUS EXAMPLES.

78. Mary bought flour for $1.35, oil for $.75, rice for $.63, and butter for $1.125; what was the entire cost?
Ans. $3.855.

79. John earned $1.75 by picking berries, $1.84 by piling wood, and $.38 by running upon errands; after paying $2 of his money for boots, how much money had he left? *Ans.* $1.97.

80. Charles sold 7 dozen eggs at 24 cents a dozen, and 3 chickens at 42 cents apiece; what did he receive for both eggs and chickens? *Ans.* $2.94.

81. A boy received $5.40 for 6 days' work; how much did he receive for one day's work?

82. At 40 cents a box for strawberries, how many boxes may be bought for $8.40?

83. Among how many girls may $5 be distributed that each girl may receive 25 cents?

84. What cost 34 yards of cloth at 13 cents a yard?

85. How many yards of cloth at 10 cents a yard will pay for 2 pecks of peas at 50 cents a peck? *Ans.* 10 yards.

86. How many pairs of shoes at 88 cents a pair may be bought for 16 yards of cloth at 33 cents a yard?

87. Mr. Smith owed Mr. Brown $12.37, and gave in payment 1 sheep worth $8.75, and the rest in money; how much did he give in money?

88. In one year there are 52 weeks of 7 days each, and 1 day besides; if a man spends 10 cents a day for cigars, how much does he spend in 1 year? *Ans.* $36.50.

COINS AND PAPER MONEY.

135. The legal coins of the United States are: —

Gold.			Silver.		
Double-Eagle	=	$ 20.00	Dollar	=	$ 1.00
Eagle	=	10.00	Half-dollar	=	.50
Half-Eagle	=	5.00	Quarter-dollar	=	.25
Quarter-Eagle	=	2.50	Dime	=	.10
Three-dollar piece	=	3.00	Half-dime	=	.05
One-dollar piece	=	1.00	Three-cent piece	=	.03

Nickel and copper 1-cent, 3-cent, and 5-cent pieces.
Copper 1-cent and 2-cent pieces.

NOTE I. — The gold coin is hardened by an alloy of $\frac{1}{10}$ copper and silver (the silver not to exceed the copper). The silver coin is hardened by $\frac{1}{10}$ copper. The cent and the two-cent piece, coined since 1866, have 95 parts of copper to 5 parts of tin and zinc. The three-cent and five-cent pieces have 75 parts of copper to 25 parts of nickel.

NOTE II. — At present (1869) gold and silver coins are rarely used in this country, and their place is supplied by paper money, intended to represent the same value as the coins themselves. This paper money, government promises to exchange for coin at a future time.

136. EXAMPLES.

NOTE. — As many as possible of the following questions should be performed without the use of the slate.

89. How many cents are there in one 50-cent piece with two 25-cent, four 10-cent and three 3-cent pieces?

90. How many more cents must a person get to have 2 dollars, who has three 25-cent pieces and four 10-cent pieces?

91. How many dollars with how many cents in all has a man who has two 3-dollar bills, five 1-dollar bills, one 10-dollar bill, seven 2-dollar bills, four 50-cent pieces, two 25-cent, three 10-cent, and three 5-cent pieces?

Ans. $ 37.95.

92. Jane bought 2 yards of cloth at 37 cents a yard, 1 paper of pins for 20 cents, and 2 dozen buttons at 30 cents a dozen; she gave in payment a 2-dollar bill; how many cents should she receive in return? *Ans.* 46 cts.

93. Park spent 18 cents for a fish-line, 8 cents for fish-hooks, and 50 cents for a fishing-rod; he gave in payment three 25-cent pieces and one 10-cent piece; how many cents should he receive in return? *Ans.* 9 cts.

94. Henry had four 10-cent pieces, one 5-cent, and four 3-cent pieces; with his money he bought two quires of paper at 20 cents a quire, and as many pencils at 6 cents apiece as he could pay for, and gave the rest of his money for rubber; what did he give for rubber? *Ans.* 5 cts.

☞ Exercises in counting money itself, and in "making change," are earnestly recommended.

ACCOUNTS AND BILLS.

137. When one person or company sells goods or renders service to another, it is customary for one or both of the parties to make a record of the transaction specifying the articles sold, the price and amount, the kind of service rendered, value, etc. Such a record is an **account.**

138. Persons who *owe* a debt are called **debtors.**

139. Persons to whom a debt *is owed* are called **creditors.**

140. An account of goods sold, given by the seller to the buyer, containing the quantities and prices of the articles, etc., is a **bill.**

141. When a bill, having been paid, is signed by the creditor, or some one authorized to sign for him, it becomes a **receipt.**

142. EXAMPLES.

Find the cost of each item in the following bills and the amounts of the bills: —

95. *Salem*, April 10, 1868.

Mr. JAMES CROCKER,

To FREEMAN COLE, Dr.*

March 3d.	To	13 bush. potatoes @ 60 cts.† . . .	$7.80
	"	8 lbs. grass seed @ 42 cts.	3.36
April 7th.	"	6 days' labor of self @ $3.00. .	18.00
	"	4 days' labor of boy @ $1.25. . .	5.00
	"	use of team 4 days @ $2.00 . .	8.00
			$42.16

Received Payment,

FREEMAN COLE.

96. *St. Louis*, July 9, 1868.

Mrs. E. B. STIMPSON,

Bought of STYLES & ROBY.

14 yds. calico @ 18 cts.	
34 yds. sheeting @ 22 cts.	
5 yds. broadcloth @ $3.50.	
12 yds. flannel @ 75 cts.	
8 handkerchiefs @ 42 cts.	
	$39.86

Received Payment,

STYLES & ROBY.

Per J. Flint

* Debtor. This means that Mr. Crocker is debtor to Mr. Cole.

† This means 13 bushels of potatoes at 60 cents a bushel.

Find the balance of the following account: —

97. *Herkimer*, Sept. 10, 1868.

Mr. M. J. DAY,

To TIMOTHY CLOVER, Dr.

To 100 bush. potatoes @ 62 cts. . . .
" 2 bush. beets @ 80 cts. . . . .
" 12 doz. eggs @ 17 cts. . . . . .
" 5 bush. turnips @ 40 cts. . . .

CR.*

By 20 lbs. rice @ 8 cts. . . . . .
" 96 lbs. sugar @ 13 cts. . . . . .
" 12 lbs. raisins @ 22 cts. . . . . .

Balance due T. Clover, $ 50.92

143. Perform the following examples and make out th bills.

98. December 24, 1868. Mr. Ezra Jones bought of Clara Day 2 work-boxes at $1.75 apiece, 8 yards ribbon at 10 cents a yard, 3 tops at 17 cents apiece, and 4 handkerchiefs at 23 cents apiece. What is the amount of Miss Day's bill? *Ans.* $5.73.

99. Mr. C. Dyer sold to James Hurd 2 Worcester's Dictionaries at $1.15 apiece, 9 spellers at 35 cents apiece, 18 Written Arithmetics at 88 cents apiece, and 50 Primary Arithmetics at 25 cents apiece. What was the amount of Dyer's bill? *Ans.* $33.79.

100. Thomas Winn sold to J. Smith 12 dozen cabbages at 12 cents apiece, 20 heads of lettuce at 6 cents a head, and 2 bushels of beets at 75 cents a bushel; he bought of J. Smith 3 pounds of steak at 24 cents a pound, and 5 pounds of veal at 15 cents a pound. What is the balance due Winn? *Ans.* $18.51.

☞ For Dictation Exercises, see "Manual and Key," pages 40–43.

* Creditor. This means that Mr. Day is credited for the goods delivered by him to Mr. Clover.

144. GENERAL REVIEW, No. 2.

1. How much must you add to 228 to make 1000?

2. What is the difference between 2832 + 1297 and 2832 — 1279?

3. Add the difference between 3291 and 1827 to the sum of 9836 and 2987.

4. If of a debt of $1762 there remains unpaid $887, how much has been paid?

5. If 5000 is a subtrahend and 278 the remainder, what is the minuend?

6. If at 28 cents each 368 fowls were bought for a certain sum, what was the sum?

7. If $22.68 is a dividend and $2.52 the quotient, what is the divisor?

8. If $132.30 is the product and $4.90 the multiplicand, what is the multiplier?

9. If $13.80 is the product and 23 the multiplier, what is the multiplicand?

10. If 3924 is divided by 18, and to the quotient 132 is added, this sum multiplied by 9, and that product diminished by 27, what is the result?

11. Of a debt of $273 there have been paid $48.72, $82.24, $21.50, and $36.28; if the balance of the debt be paid with boards at 2 cents a foot, how many feet will be required?

12. Mr. Thomas Winship owes Mr. Leonard Carpenter for 2780 feet of boards at 4 cents per foot, 628 feet of joist at 5 cents per foot, 11 thousand bricks at $12.50 per thousand, and 17 days of labor at $3.25 per day.

Make out and receipt the bill for the above, dating it at the present time, and at your place of residence.

☞ For Dictation Exercises upon this Review, see "Manual and Key" page 44.

PART II.

145. If we compare six objects with two objects of the same kind, we find that six equals *three twos;* we thus determine the **relation** of the numbers six and two.

What is the relation of the numbers four and two?

146. We have now illustrated a number (Art. 1), expressing numbers (Art. 2), combining numbers (Art. 41), and the relation of numbers (Art. 145).

The knowledge of numbers, of expressing and combining numbers, and of the relation of numbers, is **Arithmetic.**

Define Arithmetic.

NOTATION AND NUMERATION.

147. Topical Review in Notation and Numeration.

The pupil may illustrate the following topics to his class, using common objects when practicable, and giving definitions.*

1. A number. (Art. 1.)
2. Expressing numbers. (Art. 2.)
3. Combining numbers. (Arts. 40, 41.)
4. Relation of numbers. (Art. 145.)
5. Arithmetic. (Art. 146.)

* To the Teacher.—For method of conducting this and the following topical reviews, see "Manual and Key" to Illustrative Practical Arithmetic, pages 46 – 50.

6. Figures and Notation. (Art. 3.)
7. Units of first and second orders. (Art. 4.)
8. Expressing numbers from ten to ninety-nine. (Art. 5.)
9. Units of the third order, and expressing numbers from one hundred to nine hundred ninety-nine. (Arts. 7, 8.)
10. Units of fourth and fifth orders. (Arts. 11 – 14.)
11. Manner of expressing units generally, with table of units. (Art. 15.)
12. Groups. (Arts. 19, 20.)
13. Numeration. (Art. 21.)
14. Construct a Numeration Table. (Art. 22.)
15. Make a rule for reading a number expressed by figures (Art. 28), and compare it with the rule found in Art. 150.
16. Make a rule for expressing numbers in figures (Art. 30), and compare it with the rule found in Art. 151.

DECIMAL SYSTEM.

148. We have learned how the orders of units are formed and how they are expressed. (Arts. 1 – 30.)

We have seen (Art. 15) that 10 units = 1 ten, 10 tens = 1 hundred, 10 hundreds = 1 thousand, etc.

A succession of numbers, as 10, 10, 10, showing the number of units of each lower order taken to make a unit of the next higher, is a **scale.** A scale which is made up of tens, is a **scale of tens.**

Define a scale; a scale of tens.

A system of numbers whose units increase by a scale of tens is a **decimal system of numbers.**

Define a decimal system of numbers.

The system of numbers in general use is the decimal system.

149. Name the orders of units in the number 53, with the numbers of units of the several orders.

Ans. There are 3 units of the first order and 5 units of the second order, or 3 units and 5 tens. (Art. 32.)

Name the numbers of units of the several orders in 375.

Ans. There are 5 units of the first order, 7 units of the second order, and 3 units of the third order, or 5 units, 7 tens, and 3 hundreds.

The numbers of units of the several orders in a number are called the **terms** of the number.

Define the terms of a number.

EXERCISES.

Name the terms of the following numbers: —

1. 1 2 5. *Ans.* One hundred, two tens, five units.

2. 3 3 4.	4. 3 3 3 3.	6. 8 3 2 4.
3. 5 2 6.	5. 4 4 0 0.	7. 1 8 4 6 2.

150. Rule for reading numbers expressed in figures.

1. *Beginning at the right and proceeding towards the left, separate by commas the collection of figures into groups of three figures each.*

2. *Then name the number expressed in the left-hand group, together with the group, and thus proceed from left to right, omitting the name "units" after the units' group.*

NOTE. — In Part I. (Art. 22), the names of the groups are given to *trillions;* the names of the next five higher groups from *trillions* are *quadrillions, quintillions, sextillions, septillions,* and *octillions.*

EXERCISES.

Read the following: —

1.	2 2 4 6 8.	6.	1 4 6 8 0 0 0 1 2 4.
2.	8 9 0 0 1.	7.	8 3 4 0 8 0 6 0 0 0.
3.	8 3 0 4 7 0.	8.	1 0 4 6 8 3 2 4 4 1 6.
4.	5 1 0 0 0 0 0.	9.	7 0 3 0 0 0 4 5 0 7 2.
5.	1 7 4 5 0 3 7 5.	10.	1 0 1 8 4 0 7 8 7 6 4 0 0.

151. Rule for expressing numbers in figures.

Beginning with the group highest in order, express the number of that group; then, at the right, express the number of the next group, and thus proceed with each succeeding group, supplying vacant places in the expression with zeros.

EXERCISES.

Express the following in figures : —

11. Thirty-three million.
12. Eight hundred twenty-four million, six hundred thousand.
13. Nineteen million, seventy thousand.
14. Three hundred one million, seven hundred forty thousand.
15. One billion, one million, one thousand, one hundred.
16. Twenty-two billion, one hundred thirteen thousand, one hundred eighty-seven.
17. Four hundred billion, four hundred eighty thousand.
18. Seven trillion, four hundred twenty-five billion, two hundred twelve million.
19. Sixteen quadrillion, nine hundred thousand.
20. One quadrillion, thirty-six billion, two hundred.

ADDITION.

152. TOPICAL REVIEW IN ADDITION.

The pupil may illustrate the following topics to his class, using common objects, and giving definitions.*

1. Addition. (Arts. 42 – 44.)
2. Sum or amount. (Art. 45.)
3. Manner of adding units of a single order. (Art. 46.)
4. Method of proof of addition. (Art. 46.)
5. Meaning and use of the word "plus." (Art. 47.)
6. Use of the sign for addition. (Art. 48.)
7. Use of the sign of equality. (Art. 49.)
8. Manner of adding numbers in which the amount of units of each order is less than ten. (Art. 51.)
9. Manner of adding numbers in which the amount of units of any order exceeds ten. (Art. 52.)
10. Make a rule for addition (Arts. 51, 52), and compare it with the rule in Art. 154.

* See "Manual and Key," page 46.

153. EXERCISE.

A pupil (or the teacher) may assign to the class numbers consisting of hundreds, with tens and units. As these numbers are assigned, the pupils may express them upon their slates, and commence to add simultaneously, each giving a signal when he has finished. The pupil who finds the *true* answer *first*, may be presumed to add the best.

154. RULE FOR ADDITION.

1. *Express the numbers to be added so that units of the same order shall be expressed in the same column; draw a line beneath.*

2. *Add the units of the lowest order first; express the units of their sum under the line in the units' place, and reserve the tens, if there are any, to add with the tens expressed in the next column.*

3. *Proceed in the same way to add the tens, hundreds, etc., expressing the entire result of the last addition.*

155. EXAMPLES.

1. A teacher spent of his salary \$820 for the support of his family, \$32 for books, \$45 for religious and charitable purposes, and had \$43 left; what was his salary?

2. How many feet of fencing will be required to enclose a lot of land measuring on each of two sides 184 feet, on the third side 294 feet, and on the fourth side 278 feet?

3. A farmer raised 45 bushels of grain in one year, 235 bushels the next year, 784 bushels the third year, and the fourth year as much as he raised during the second and third; how many bushels did he raise in all?

4. How many square miles are there in the United States, there being in the New England States 63700, in the Middle Atlantic States 173600, in the Southern Atlantic States 186500, in the Central States 691500, in the Gulf States 381000, in the Pacific States 415000, in Alaska 371800, and in the other Territories 1472400?

5.	6.	7.
$16.00	$5.40	$8.13
3.85	6.37	4.48
8.07	52.25	16.72
17.63	25.25	9.19
4.17	48.00	5.84
38.00	3.75	126.08

SUBTRACTION.

156. Topical Review in Subtraction.

The pupil may now illustrate the following topics to his class, using common objects and giving definitions.

1. Subtraction. (Art. 53.)
2. Minuend, Subtrahend, and Remainder. (Arts. 54 – 56.)
3. Manner of expressing the operation. (Art. 59.)
4. Subtraction compared with addition. (Art. 59, Rem.)
5. Use of the sign of subtraction. (Art. 60.)
6. Manner of operating when each term of the subtrahend is less than the corresponding term of the minuend. (Art. 61.)
7. Proof for subtraction. (Art. 62.)
8. Manner of operating when any term of the subtrahend is greater than the corresponding term of the minuend. (Art. 64.)
9. Manner of operating when zeros occur in the expression for the minuend. (Art. 66.)
10. Make a rule for subtraction, and compare it with the rule in Art. 158.

157. Exercise.

A pupil (or the teacher) may assign to the class an example in subtraction, the minuend of which is expressed by 7 figures, and the subtrahend by 6, to see who will *first* find the correct answer.

158. Rule for Subtraction.

1. *Write the expression of the minuend, and underneath it that of the subtrahend, so that units of the same order shall be expressed in the same column; draw a line beneath.*

2. *Begin with the units of the lowest order to subtract, and proceed to the highest, taking each term of the subtrahend out of the corresponding term of the minuend; express the remainder under the line in its proper place.*

3. *If any term of the minuend is less than the corresponding term of the subtrahend, increase it by adding to it one unit of the next higher order changed to ten of the lower order, and then subtract.*

4. *Bear in mind, in subtracting the next term, that the term expressed above has been diminished by one.*

159. EXAMPLES IN ADDITION AND SUBTRACTION.

1. A man paid $6525 for a house and farm, $2750 being the price of the house; what was the price of the farm?

2. How old is a man who was born in 1829?

3. How many years were there from the invention of the printing-press in the year 1440 to the publication of the first newspaper in 1630?

4. In the year ending June, 1840, the United States yielded 144376927 bushels of coal, all but 77382732 bushels of which were mined in Pennsylvania; how many bushels of coal were mined in Pennsylvania?

5. In one year the three great salt-producing States, New York, Ohio, and Virginia, produced 11322088 bushels of salt; of this quantity Ohio produced 1744240 bushels, Virginia 2056513 bushels; how many bushels did New York produce?

6. Socrates was born in the year 468 B. C., and died in the year 399 B. C.; at what age did he die?

7. How many years was it from the death of Solomon, in the year 976 B. C., to that of Socrates?

8. How many years from the birth of Socrates to the birth of Shakespeare, who was born in the year 1564 A. D.?

9. Shakespeare died in 1616; how old was he when he died?

MULTIPLICATION.

160. Topical Review in Multiplication.

The pupil may illustrate the following topics to his class, using common objects, and giving definitions.

1. Multiplication. (Art. 69.)
2. Multiplicand, Multiplier, Product. (Arts. 70 – 72.)
3. Manner of expressing the operation. (Art. 73.)
4. Multiplication compared with addition. (Art. 73, Rem.)
5. Use of the sign of multiplication. (Art. 74.)
6. Manner of operating when the product of the units of any order exceeds ten. (Art. 76.)
7. When the multiplier is 10, 100, etc. (Art. 79.)
8. When zeros occur at the right of expressions for the multiplier. (Art. 81.)
9. When there are tens and units in the multiplicand and multiplier. (Art. 82.)
10. When zeros occur at the right of both expressions. (Art. 85, Note.)
11. Proof for multiplication. (Art. 83.)
12. Make a rule for multiplication, and compare it with the rule in Art. 162.

161. Exercise.

A pupil (or the teacher) may assign test examples in multiplication, in which the multiplicands shall contain, at least, four terms, and the multipliers from one to three terms.

162. Rule for Multiplication.

1. *Write the expression of the multiplicand and underneath it that of the multiplier; draw a line beneath.*

2. *If the multiplier consists of one term only, multiply the units of the multiplicand by the multiplier; express the units of the product under the line in the units' place, and reserve the tens, if there are any, to add with the next product.*

3. *Proceed in the same way to multiply the other terms of the multiplicand, expressing the whole of the last product.*

4. *If the multiplier consists of more than one term, proceed in the same manner to multiply by the other terms of the multiplier in order, writing the figures expressing each partial product so that units of the same order shall be expressed in the same column.*

5. *Add the partial products thus obtained, and the result will be the entire product.*

NOTE.—For contractions in Multiplication, see Appendix.

163. EXAMPLES IN ADDITION, SUBTRACTION, AND MULTIPLICATION.

1. A merchant bought 44 pieces of cloth, each piece containing 38 yards, and paid $ 5 a yard for it; what did he pay for the whole?

2. If 18 men can do a piece of work in 22 days, how many men can do it in one day?

3. How many hours will it take one person to do as much work as 37 will do in 2 days of 11 hours each?

4. A man, on receiving his week's wages, bought 7 pounds of beef at 13 cents a pound, 3 pecks of apples at 35 cents a peck, and 1 gallon of oil for 62 cents, and had $ 12.42 left; what were his week's wages?

5. A farmer cut 800 cords of wood, of which he sold 625 cords at $ 8 a cord, and the remainder at $ 11 a cord; what did he receive for the whole?

6. If a person receives a salary of $ 1000 a year, and spends $ 208 a year for board, $ 125 for clothing, and $ 250 for other expenses, what does he save in 5 years?

7. A man had $ 8000, and purchased 52 acres of land at $ 125 an acre, 2 yoke of oxen at $ 195 a yoke, and one plough for $ 17; how many dollars had he left?

DIVISION.

164. Topical Review in Division.

The pupil may illustrate the following topics to his class, using common objects and giving definitions.

1. Division. (Art. 86.)
2. Dividend, Divisor, Quotient. (Arts. 87 – 89.)
3. Remainder. (Art. 90.)
4. Manner of expressing the operation. (Art. 91.)
5. Division compared with multiplication. (Art. 91, Rem.)
6. Use of the sign of division. (Art. 92.)
7. Manner of operating in short division when each term of the dividend contains the divisor without a remainder. (Art. 93.)
8. When units of higher orders are changed to those of lower. (Arts. 95 – 97.)
9. Long division. (Arts. 100, 102.)
10. When the divisor is 10, 100, 1000, etc. (Art. 104.)
11. When the divisor is a number of 10's, 100's, etc. (Art. 106.)
12. Proof for division. (Art. 98.)
13. Meaning of 1 half, 1 third, etc. (Art. 109.)
14. Manner of finding one of the equal parts of a number, as 1 half, 1 third, etc. (Art. 110.)
15. Manner of operating when the quotient obtained is a whole number with one or more parts of units. (Art. 112.)
16. Make a rule for division, and compare it with the rule in Art. 166.

165. Exercise.

A pupil (or the teacher) may assign test examples in division.

166. Rule for Division.

1. *Write the expression of the dividend; at the left draw a curved line; at the left of this line express the divisor.*

2. *See how many numbers equal to the divisor are contained in the highest term or terms of the dividend.*

3. *Express the result for the first term of the quotient at the right in long division, beneath in short division.*

4. *Multiply the divisor by this term.*

5. *Take the product thus obtained out of that part of the dividend used.*

6. *Unite the next term of the dividend with the remainder, divide as before the number thus formed, and thus continue till all the terms of the dividend are used.*

7. *Should there be a final remainder, express it with the quotient.*

NOTE I. — If at any time the divisor is not contained in any partial dividend, write a zero in the quotient, and increase the partial dividend by uniting with it the next term.

NOTE II. — For contractions in Division, see Appendix.

167. EXAMPLES IN ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION.

1. In Camden County, New Jersey, there are 731 farms occupying 73571 acres of land; what is the average number of acres in each farm?

2. If by a machine 1500 sheets of paper can be folded in 1 hour, in how many hours can the sheets be folded for 6000 pamphlets of 2 sheets each?

3. In the circumference of the earth, which equals 360 degrees, there are 24898 miles; how many miles are there in a degree?

4. If light travels 184000 miles in a second, how many seconds is it in coming from the sun to the earth, a distance of 92000000 of miles? *

5. I bought three pieces of cloth, the first of which cost $62, the second $15 more than the first, and the third as much as the first and second; what did all cost?

6. The distance from the surface of the sun to its centre is 428000 miles, * and the distance of the moon from the earth † is 240000 miles. If the centre of the sun could be placed at the centre of the earth, how much beyond the orbit of the moon would the sun's surface extend?

7. On the 1st of October, 1867, the sum of $56927.62 was

* Loomis's Astronomy. † From centre to centre.

due depositors in the Freedmen's Savings Bank in Washington. During the month, $25489.36 was deposited, and $23952.32 was drawn out; what was due depositors at the end of the month? *Ans.* $58,464.66.

8. If a railroad car goes 32 miles in an hour, how far would it go in 3 days of 24 hours, allowing 3 hours each day for stopping?

9. Two boys start at the same place and trundle hoops in the same direction, one at the rate of 352 feet a minute, and the other at the rate of 278 feet a minute; how far apart will they be at the end of 15 minutes?

10. Two vessels start at the same place and sail in opposite directions, one at the rate of 60 miles a day, and the other at the rate of 23 miles a day; how far apart will they be at the end of 8 days?

11. Two locomotives start simultaneously, one at Buffalo, the other at Albany, 325 miles apart, and move towards each other, one at the rate of 28 miles an hour, and the other at the rate of 16 miles an hour; how far apart will they be at the end of 7 hours?

12. If 13 pounds of tea cost $16.25, what will 1 pound cost? what will 10 pounds cost?

13. If 5 boxes of pens cost $4.20, what will 12 boxes cost at the same rate?

14. If 2 men can do a certain piece of work in 18 days, in how many days can 1 man do the same work? in how many days can 9 men do it?

15. In how many days can 45 men weave as much cloth as 60 men can weave in 27 days? *Ans.* 36 days.

16. How many days would the same provision last 13 horses which lasts 12 horses 52 days?

17. How many horses at $150 each can be bought for the same sum that will pay for 2 acres of land at $600 an acre?

UNITED STATES MONEY.

168. Topical Review in United States Money.

The pupil may illustrate the following topics to his class: —

1. The units of United States Money with relative values. (Art. 115, 116.)
2. The manner of expressing these units in figures and reading the expréssions. (Art. 118.)
3. The manner of changing a number to units of lower orders. (Art. 121.)
4. The manner of changing a number to units of higher orders. (Art. 123.)
5. Addition of United States Money. (Art. 125.)
6. Subtraction. (Art. 127.)
7. Multiplication. (Art. 129.)
8. Division. (Art. 131.)
9. Coins. (Art. 135.)
10. Accounts, Debtor and Creditor, Bills of parcels, receipts, stamps. (Arts. 137 – 141.)

SYMBOLS OF OPERATION.

169. The signs of the various arithmetical operations have been previously explained; their use will now be further illustrated.

Example I. In an orchard there are 6 rows of trees, each row containing 10 apple-trees and 8 pear-trees; how many trees are there in the orchard?

Example II. In another orchard there are 10 apple-trees, also 6 rows of 8 pear-trees each; how many trees are there in this orchard?

Before doing the work of the above, we may indicate the operations thus:

Example I. $(10 + 8) \times 6$ or $\overline{10 + 8} \times 6$.
Example II. $10 + 8 \times 6$.

In Example I. the parenthesis ( ), or the vinculum $\overline{\quad\quad}$, indicates that the same operation is to be performed upon both 10 and 8 or upon 10 and 8 combined. In Example II. 6 is a multiplier of 8 alone.

When such expressions occur, *the numbers whose expressions are united by a parenthesis or a vinculum should first be combined; next those which are immediately affected by the sign of multiplication or division; after which the results should be added or subtracted as indicated by the signs.*

EXAMPLES.

170. Perform the work indicated in the following :—

1. $9 \times 2 \div 6 + 5.$ *Ans.* 8.
2. $[(12 - 2) \div 2] \times 4.$ *Ans.* 20.
3. $\dfrac{12 \times 2 + 8}{4} + 7.$ *Ans.* 15.
4. $(\overline{3 + 2} \times 6) \div 15.$ *Ans.* 2.
5. $150 \div 10 - [(4 + 2)] \times 2.$ *Ans.* 3.
6. $17 + 6 \times 63 \div 9.$
7. $(\overline{8 + 6} \div 2) \times 4.$
8. $132 - 12 \div 6 + \dfrac{4 \times 10 + 9}{7}.$

171. Indicate and perform the operations in the following examples :—

9. If a man worked 10 hours a day by daylight and 6 hours a day by gaslight, how many hours did he work in 5 days? $(10 + 6) \times 5$ *Ans.* 80 hours.

10. If a man worked 10 hours in 1 day by daylight and 6 hours a day for 5 days by gaslight, how many hours did he work in all? $10 + 6 \times 5$ *Ans.* 40 hours.

11. A person who had \$26 earned \$2 a day for 13 days; at \$4 a week for board, how many weeks' board could he pay with his money? *Ans.* 13 weeks' board.

12. A butcher offered to sell a piece of meat weighing 21 pounds at 15 cents a pound; if he sold 12 pounds of the inferior part of the piece at 9 cents a pound, at what price per pound must he sell the remainder, that he may lose nothing? *Ans.* 23 cents.

PROPERTIES OF NUMBERS.

172. All numbers with which we are now familiar, as three, four, twenty, etc., are made up of units, each of which is complete in itself. Such a unit is an **integral unit** or an **integer.**

A collection of integral units is an **integral number.**

What is an integral unit? An integral number?

173. If we have 6 blocks and place them in groups of 3 blocks each, we have two 3's of blocks.

Again, if we place them in groups of 2 blocks each, we have three 2's of blocks.

Because 6 is made up of a number (2) of 3's, or a number (3) of 2's, each of the numbers 2 and 3 is a **factor** of 6.

What is a factor of a number?

Ans. A factor of a number is a number employed either as a multiplicand or multiplier to make the number.*

NOTE I. — Because a factor of a number can exactly divide the number, or, when applied successively to the different parts of the number, exactly measures it, a factor is also called a **divisor** or **measure** of the number.

NOTE II. — To measure 15, 5 must be applied as a measure 3 times, 3 must be applied as a measure 5 times; hence such expressions as "3 times 5 are 15," "5 in 15, 3 times," "A's age is three times that of B," etc.; in which sense the use of the word times is correct.

NOTE III. — Because every integral number is made up of ones, one is a factor of every integral number, and because every integral number shows how many ones are united in itself, every integral number is a factor of itself.

* When the word "factor" is used in this Arithmetic, reference is made to integral factors only.

NOTE IV. — In naming the factors of a number, the number itself and one may be omitted.

174. Name the factors of 8; of 9; of 12.

Each of these numbers has other factors besides itself and one; such a number is a **composite** number.

What is a composite number?

175. Certain numbers, as 7, 11, 13, cannot be separated into equal groups of integers; each of these numbers contains no factor except itself and one, and is called a **prime** number.

What is a prime number?

Name all the prime numbers from 1 to 50; all the composite numbers to 50.

176. A factor that is a prime number is called a **prime factor.**

What is a prime factor?

Name the prime factors of 12.

177. What are the factors of 8? of 9?

These numbers have no factors which are alike. Such numbers are said to be **prime to each other.**

Are 5 and 6 prime to each other? are 6 and 15?

178. Methods of ascertaining when numbers contain certain factors.

1. Any number which is made up of 2's, as 6, 8, 10, etc., can be separated into two equal integral parts; because such a number can be separated into two such parts, it is an **even** number.

What is an even number? Name any two even numbers.

The figure expressing the units of every even number is 0, 2, 4, 6, or 8.

2. Any number which cannot be separated into two equal integral parts, as 3, 5, 7, is an **odd** number.

What is an odd number? Name any two odd numbers.

The figure expressing the units of every odd number is 1, 3, 5, 7, or 9.

3. Because every even number is made up of 2's, 2 is a factor of every even number.

4. For reasons given in the Key and Manual, 3 and 9 are respectively factors of any number the sum of whose digits* contains 3 or 9 as a factor.

Is 3 a factor of 258? Is 9?

5. Is 4 a factor of 100? of 200? of 224? of 226?

Because 4 is a factor of 100, it is a factor of any number of 100's; if then it is a factor of the tens with the units of any number it is a factor of the entire number.

Is 4 a factor of 368? of 516?

6. Is 5 a factor of 10? of 60? of 65? of 68?

Because 5 is a factor of 10, it is a factor of any number of 10's; if the figure expressing the units of a number is 0 or 5, 5 must be a factor of the entire number.

Is 5 a factor of 36? of 560? of 435?

7. Because 6 equals two 3's, 6 is a factor of any even number which contains 3 as a factor.

Is 6 a factor of 423? of 222?

8. Is 8 a factor of 1000? of 3000? of 3128? of 3125?

Because 8 is a factor of 1000, it is a factor of any number of 1000's; if then it is a factor of the hundreds with the tens and units of any number, it is a factor of the entire number.

Is 8 a factor of 2874? of 3536?

9. Because 10, 100, 1000, etc., are severally factors of themselves, they are respectively factors of any number of 10's, 100's, or 1000's, or of any number whose expression contains at the right, one, two, or three zeros.

Is 10 a factor of 8400? is 100? is 1000?

10. A composite number, as 12, is a factor of another number, as 36, when all the factors of the former number are factors of the latter number also.

* Digits are numbers represented by the nine figures 1, 2, 3, 4, 5, 6, 7, 8, 9, without regard to the order of their units.

179. PRIME NUMBERS TO 1201.

1	61	151	251	359	463	593	701	827	953	1069
2	67	157	257	367	467	599	709	829	967	1087
3	71	163	263	373	479	601	719	839	971	1091
5	73	167	269	379	487	607	727	853	977	1093
7	79	173	271	283	491	613	733	857	983	1097
11	83	179	277	389	499	617	739	859	991	1103
13	89	181	281	397	503	619	743	863	997	1109
17	97	191	283	401	509	631	751	877	1009	1117
19	101	193	293	409	521	641	757	881	1013	1123
23	103	197	307	419	523	643	761	883	1019	1129
29	107	199	311	421	541	647	769	887	1021	1151
31	109	211	313	431	547	653	773	907	1031	1153
37	113	223	317	433	557	659	787	911	1033	1163
41	127	227	331	439	563	661	797	919	1039	1171
43	131	229	337	443	569	673	809	929	1049	1181
47	137	233	347	449	571	677	811	937	1051	1187
53	139	239	349	457	577	683	821	941	1061	1193
59	149	241	353	461	587	691	823	947	1063	1201

180. COMPOSITE NUMBERS TO 917,

Which contain no prime factor less than 7 (*excepting* 1).

Nos.	Factors.	Nos.	Factors.	Nos.	Factors.	Nos.	Factors.	Nos.	Factors.
49	7^2,*	289	17^2,	469	7, 67	623	7, 89	779	19, 41
77	7, 11	299	13, 23	473	11, 43	629	17, 37	781	11, 71
91	7, 13	301	7, 43	481	13, 37	637	7^2, 13	791	7, 113
119	7, 17	319	11, 29	493	17, 29	649	11, 59	793	13, 61
121	11^2,	323	17, 19	497	7, 71	667	23, 29	799	17, 47
133	7, 19	329	7, 47	511	7, 73	671	11, 61	803	11, 73
143	11, 13	341	11, 31	517	11, 47	679	7, 97	817	19, 43
161	7, 23	343	7^3,	527	17, 31	689	13, 53	833	7^2, 17
169	13^2,	361	19^2,	529	23^2,	697	17, 41	841	29^2,
187	11, 17	371	7, 53	533	13, 41	703	19, 37	847	7, 11^2
203	7, 29	377	13, 29	539	7^2, 11	707	7, 101	851	23, 37
209	11, 19	391	17, 23	551	19, 29	713	23, 31	869	11, 79
217	7, 31	403	13, 31	553	7, 79	721	7, 103	889	7, 127
221	13, 17	407	11, 37	559	13, 43	731	17, 43	893	19, 47
247	13, 19	413	7, 59	581	7, 83	737	11, 67	899	29, 31
253	11, 23	427	7, 61	583	11, 53	749	7, 107	901	17, 53
259	7, 37	437	19, 23	589	19, 31	763	7, 109	913	11, 83
287	7, 41	451	11, 41	611	13, 47	767	13, 59	917	7, 131

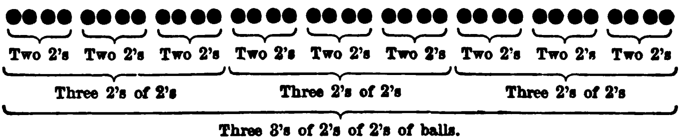
* The small figures placed at the right of this and other expressions for factors show how many factors equal to the one expressed are used to produce the given number. Thus, the factors of 49 are 7 and 7.

FACTORING NUMBERS.

181. ILLUSTRATIVE EXAMPLE. What are the prime factors of 36?

ILLUSTRATION.

(36 balls.)



OPERATION.

2) 36 = eighteen 2's.
2) 18 = nine 2's.
3) 9 = three 3's.
3 is prime.

Ans. 2, 2, 3, and 3.

Explanation. — As 36 is an even number, we know that 2 is one of the prime factors of 36. Taking 2 as a divisor, 36 is found to equal eighteen 2's; 18 equals nine 2's; 9 equals three 3's; 36, then, equals three 3's of 2's of 2's, or $2 \times 2 \times 3 \times 3$. The factors of 36 are, therefore, 2, 2, 3, and 3.

NOTE. — Small numbers, like the above, can be resolved into their factors without any written operation.

182. What are the prime factors of the following numbers?

1.	18.	4.	32.	7.	49.	10.	120.
2.	24.	5.	40.	8.	84.	11.	108.
3.	56.	6.	42.	9.	88.	12.	115.

Derive a rule for resolving a number into its prime factors.*

RULE. — To resolve a number into its prime factors: —

1. *Divide the given number by one of its prime factors.*

* The pupil should be encouraged to derive his own rules and compare them with the rules given in the book. He may then be allowed to retain his own language, if it is accurate and concise.

2. *Divide the quotient thus obtained by any one of* ITS *prime factors; and thus continue dividing till a quotient is obtained that is a prime number.*

3. *This quotient and the several divisors are the prime factors.*

NOTE. — The work may sometimes be shortened by dividing by a composite number, remembering afterwards to substitute the factors of that number for the number itself. Thus, in the Illustrative Example (Art. 181) we may divide by 4 instead of dividing by 2 twice.

183. EXAMPLES.

What are the prime factors of the following?

13.	114. *Ans.* 2, 3, 19.	17.	240.	21.	558.
14.	117. *Ans.* 3, 3, 13.	18.	253.	22.	570.
15.	135. *Ans.* 3, 3, 3, 5.	19.	300.	23.	612.
16.	168. *Ans.* 2, 2, 2, 3, 7.	20.	312.	24.	634.

184. Select the prime numbers expressed below, and find the factors of the composite numbers.

25.	613.	28.	707.	31.	821.	34.	1067.
26.	616.	29.	709.	32.	831.	35.	847.
27.	626.	30.	714.	33.	837.	36.	915.

☞ **For Dictation Exercises, see "Manual and Key," page 57.**

GREATEST COMMON FACTOR.

185. ILLUSTRATIVE EXAMPLE I. Separate the numbers 12 and 18 into their prime factors.

$$Ans. \begin{cases} 12 = \mathbf{2} \times \mathbf{3} \times 2 \\ 18 = \mathbf{2} \times \mathbf{3} \times 3 \end{cases}$$

On separating the numbers 12 and 18 into their prime factors, as above, we see that 2, 3, and consequently 6 (2 × 3), are factors of both 12 and 18; each of the numbers 2, 3, and 6 is therefore a **common factor** of the numbers 12 and 18.

What is a common factor of two or more numbers?

186. We see also that of these common factors 6 is the greatest; 6 is, therefore, the **greatest common factor** of 12 and 18.

What is the greatest common factor of two or more numbers?

187. Comparing the prime factors which are common to two or more numbers, (as **2** and **3** of 12 and 18,) with the prime factors of their greatest common factor (as **2** and **3** of 6), we shall find that these prime factors are the same; hence,

The greatest common factor of two or more given numbers *is the product of all the common prime factors of the numbers.*

NOTE.—To denote "greatest common factor," use G. C. F.

188. ILLUSTRATIVE EXAMPLE II. What is the G. C. F. of 18 and 24?

OPERATION.

$$18 = 2 \times 3 \times 3$$
$$24 = 2 \times 2 \times 2 \times 3$$
$$2 \times 3 = 6, \text{ the G. C. F.}$$

Explanation.—Separating 18 and 24 into their prime factors, we find 2 and 3 to be the only common prime factors; their product 6 is, therefore (Art. 187), the G. C. F. of 18 and 24.

RULE I.—To find the greatest common factor of two or more numbers: *Separate the numbers into their prime factors, and find the product of such prime factors as are common.*

NOTE.—We may sometimes shorten the work by separating one of the smaller numbers into its prime factors, and by trial finding which of these factors, if any, are factors of all the other numbers; the product of such factors will be the G. C. F.

189. EXAMPLES.

Find the G. C. F. of the following:—

37. 27, 36, and 60. *Ans.* 3.

38. 32, 48, and 56. *Ans.* 8.

39. 96, 104, and 14.

40. 81, 42, 72, and 21.

NOTE.—In example 40, 21 is a factor of 42; any factor of 21 will be a factor of 42, hence 42 may be disregarded in the operation.

Find the G. C. F. of the following:—

41.	35, 84, 70, and 28.	43.	18, 54, 33, and 24.
42.	25, 60, 36, and 12.	44.	45, 36, 24, and 20.

190. When numbers cannot be readily separated by the above process into their prime factors, the following method may be adopted:—

ILL. EX. III. Find the G. C. F. of 24 and 40.

OPERATION.

```
24 ) 40 ( 1
     24
    ----
    16 ) 24 ( 1
         16
        ----
         8 ) 16 ( 2
```

Explanation.—24 is the greatest factor of itself; if it is a factor of 40, it is the G. C. F. of 24 and 40; it is not a factor of 40, for there is a remainder of 16 after 24 is taken out of 40.

But since 40 is made up of a number of the G. C. F.'s, and since 24 is made up of a number of the G. C. F.'s, the remainder 16 must be made up of a number of the G. C. F.'s; hence the G. C. F. cannot be greater than 16.

16 is the greatest factor of itself; if it is a factor of 24, it is the G. C. F. of 16 and 24; it is not a factor of 24, for there is a remainder of 8 after 16 is taken out of 24.

But since 24 is made up of a number of the G. C. F.'s, and since 16 is made up of a number of the G. C. F.'s, the remainder 8 is made up of a number of the G. C. F.'s; hence the G. C. F. cannot be greater than 8.

8 is the greatest factor of itself; it is also a factor of 16; hence it is the G. C. F. of 8 and 16, hence of 16 and 24, and hence of 24 and 40.

RULE II.—To find the G. C. F. of two numbers: *Divide the greater number by the less, and the less number by the first remainder, the first remainder by the second, the second by the third, and so on till nothing remains. The last divisor is the G. C. F. sought.*

To find the G. C. F. of more than two numbers, *find the G. C. F. of any two of the numbers, and then of that factor and a third number, and so on till all the numbers are taken.*

191. EXAMPLES.

Find the G. C. F. of the following:—

45.	18 and 72.	*Ans.* 18.	49.	88 and 154.
46.	34 and 119.	*Ans.* 17.	50.	721 and 889.
47.	42 and 126.	*Ans.* 42.	51.	169 and 793.
48.	57 and 228.		52.	2431, 2639, and 2353.

53. What is the largest sized milk-can which is an exact measure of either 105, 224, or 182 gallons?

Ans. 7 gallons.

54. What is the length of one side of the largest squares that may form a quilt 91 inches long and 84 inches wide?

Ans. 7 inches.

55. A lady wishes to place knots of trimming at the same distance apart around the skirt of her dress and around her sack, the dress being 198 inches and the sack 171 inches around. What is the greatest distance from each other that she can place the knots?

☞ For Dictation Exercises, see "Manual and Key," page 58.

CANCELLATION.

192. The principle of factoring numbers may be employed in shortening arithmetical operations when the dividend and divisor contain factors which are alike. The following will illustrate:—

ILLUSTRATIVE EXAMPLE I. Mary received 14 cents for work each day for 6 days, and spent what she received for 3 yards of cloth; how many cents did she pay for each yard?

OPERATION.

$$\frac{14 \times \cancel{6}^{2}}{\cancel{3}} = 28$$

Explanation.—Each yard will cost 1 third of six 14's of cents; 1 third of 6 is 2, therefore 1 third of six 14's is two 14's. We may then strike out the factor 3 in the dividend and divisor, and take two 14's only.

Striking out of the dividend and divisor like factors is **cancelling.**

Define cancelling.

ILLUSTRATIVE EXAMPLE II. 10 yards of cloth worth 40 cents a yard were given for 5 boxes of figs containing 4 pounds each; what were the figs worth a pound?

OPERATION.

$$\frac{\overset{10}{\cancel{40}} \times \overset{2}{\cancel{10}}}{\cancel{5} \times \cancel{4}} = 20$$

Explanation.—At 40 cents a yard, 10 yards of cloth would be worth ten 40's of cents; 1 box of figs would be worth 1 fifth of ten 40's of cents, and 1 pound would be worth 1 fourth of 1 fifth of ten 40's of cents. 1 fifth of ten 40's is two 40's; we may, therefore, cancel the factor 5 in the dividend and divisor, and put 2 in place of the 10 in the dividend. We have now only to take 1 fourth of two 40's: 1 fourth of one 40 is 10, and 1 fourth of two 40's is two 10's; we cancel the 4's in the dividend and divisor, and have two 10's, which equal 20. Hence the figs were worth 20 cents a pound; thus,

Equal factors of the dividend and divisor may be rejected without affecting the result of the division.

NOTE.—In a similar way, it might be shown that equal factors may be introduced into the dividend and divisor, without affecting the result of the division.

N. B.—*All operations upon numbers should first be expressed, as far as possible, by signs, that the processes may be clearly indicated to the teacher, and that the work to be done may be shortened, if possible, by cancellation.*

193. EXAMPLES.

56. How many are $(72 \times 4) \div (6 \times 8)$? *Ans.* 6.

57. How many are $\frac{3 \times 7 \times 6 \times 10}{4 \times 5 \times 9}$? *Ans.* 7.

58. How many are $\frac{8 \times 6 \times 3 \times 4}{4 \times 2 \times 12}$? *Ans.* 6.

59. How many are $\frac{10 \times 12 \times 8}{3 \times 4 \times 5}$? *Ans.* 16.

60. If I receive \$156 for work, and spend 1 third of it for flour at \$13 a barrel, how many barrels shall I receive?

Ans. 4 barrels.

61. How many dozen eggs at 25 cents a dozen will pay for 50 pounds of flour at 8 cents a pound?

Ans. 16 dozen.

62. If 15 men consume a barrel of flour in 6 weeks, in how many weeks would 10 men consume it?

63. How many dresses of 12 yards each may be made from 4 pieces of cloth of 33 yards each?

LEAST COMMON MULTIPLE.

194. The numbers 3, 6, 9, 12, 15, etc., contain 3 as a factor.

A number which contains another number as a factor is a **multiple** of the number which it contains.

What is a multiple of a number?

195. Name some of the multiples of 4; of 6.

Ans.	Multiples of 4 . . .	4,	8,	**12**,	16,	20,	**24**,	28,	32,	**36**, etc.
	Multiples of 6 . . .	6		**12**,		18,	**24**,		30,	**36**, etc.

196. From the above we see that 12, 24, and 36 are multiples of each of the numbers 4 and 6; they are therefore **common multiples** of 4 and 6.

What is a common multiple of two or more numbers?

197. We see also that 12 is the least number that is a multiple of 4 and 6. It is therefore the **least common multiple** of 4 and 6.

What is the least common multiple of two or more numbers?

Name any five multiples of 7; four multiples of 11.

Name the multiples of 8 to 100.

Name any common multiple of 2, 3, and 5.

Name the least common multiple of 4, 6, and 3.

NOTE.—To denote "least common multiple," use L. C. M.

198. We have seen (Art. 195) that 12 is the least common multiple of 4 and 6.

$$\text{The prime factors of} \begin{cases} 4 \text{ are } 2 \text{ and } 2. \\ 6 \text{ are } 2 \text{ and } 3. \end{cases}$$

The prime factors of 12 are 2 and 2 and 3.

Comparing the prime factors of the L. C. M. 12 with the prime factors of 4 and 6, we find that 12 is the least number that contains all the prime factors of 4 and 6.

The least common multiple of two or more numbers will always be the least number that contains all the prime factors of each of the numbers.

199. ILLUSTRATIVE EXAMPLE I. Find the L. C. M. of 4, 6, and 9.

OPERATION.

$$4 = 2 \times 2$$
$$6 = 2 \times 3$$
$$9 = 3 \times 3$$
$$\text{L. C. M.} = 2 \times 2 \times 3 \times 3 = 36. \text{—} Ans.$$

Explanation.—We find the prime factors of 4 to be 2 and 2, of 6 to be 2 and 3, of 9 to be 3 and 3. To contain 4 as a factor, the L. C. M. required must contain the factors 2 and 2, which we note: to contain 6, the L. C. M. must contain the factors 2 and 3; the multiple of 4 contains a 2, so we need introduce only a 3; to contain 9, the L. C. M. must contain the factors 3 and 3; the multiple of 6 contains one 3, so we need introduce only one more 3. Therefore 36, the product of $2 \times 2 \times 3 \times 3$, is the L. C. M. sought.

RULE I. To find the L. C. M. of two or more numbers,

Separate the numbers into their prime factors. Find the product of all the different prime factors, taking each factor the greatest number of times that it is a factor of any number.

200. EXAMPLES.

Find the L. C. M. of the following:—

64. 8, 12, and 9. *Ans.* 72.
65. 8, 18, and 20. *Ans.* 360.
66. 10, 12, and 22.
67. 3, 5, 12, 20, and 45.

NOTE.—When one of the given numbers is a factor of another, it may be disregarded in the operation, as in example 67 above, where 3 and 5 may be omitted. Why?

Find the L. C. M. of the following:

68.	8, 14, and 32.	71.	3, 7, 6, and 4.
69.	4, 10, 12, and 18.	72.	6, 26, and 39.
70.	2, 5, 14, and 12.	73.	9, 11, and 50.

When several numbers are prime to each other, what must their L. C. M. equal?

201. The above is a good method of finding the L. C. M. of numbers that are easily separated into their prime factors. For larger numbers, the following method is recommended.

ILLUSTRATIVE EXAMPLE II. Find the L. C. M. of 36, 112, and 76.

OPERATION EXPRESSED.

$$\begin{array}{r|l} 2 & 36,\ 112,\ 76 \\ \hline 2 & 18,\ \ 56,\ 38 \\ \hline & \ \ 9,\ \ 28,\ 19 \end{array}$$

$$\text{L. C. M.} = 2 \times 2 \times 9 \times 28 \times 19 = 19152.$$

Explanation. — Here, by dividing, we find the factors that are common, 2 and 2; the product of these factors with those that are not common must be the L. C. M. sought.

RULE II. To find the L. C. M. of two or more numbers: —

1. *Express the given numbers in a line as dividends. Make any prime number which is a factor of two or more of the given numbers a divisor of those numbers.*

2. *Express the quotients and undivided numbers beneath as new dividends, and continue dividing as before, till the last quotients and undivided numbers are prime to each other.*

3. *The product of all the divisors, last quotients, and undivided numbers is the L. C. M. required.*

202. EXAMPLES.

Find the L. C. M. of

74.	34, 51, 68, and 85. *Ans.* 1,020.	77.	65, 75, 95, and 125. *Ans.* 92,625.
75.	25, 60, 75, and 100. *Ans.* 300.	78.	320, 450, and 640.
		79.	136, 144, and 284.
76.	30, 70, 90, and 140.	80.	272, 384, and 756.

81. What is the width of the narrowest box that will exactly pack ribbons either 2, 3, or 4 inches wide?

Ans. 12 inches.

82. What is the smallest sum of money that may be made up of either 2-cent, 3-cent, 5-cent, 10-cent, or 25-cent pieces?

Ans. $1.50.

83. Charles and Henry, wishing to ascertain how many slats there were in a certain fence, commenced counting them at the same place and counted in the same direction; Charles marked every tenth slat and Henry every twelfth. On which slats in order would both their marks be found?

Ans. Every 60th slat.

☞ For Dictation Exercises, see "Manual and Key," page 58.

203. TOPICAL REVIEW IN PROPERTIES OF NUMBERS.

The pupil may illustrate the following topics to his class, using common objects when practicable, and giving definitions and rules: —

1. Integral Number. (Art. 172.)
2. Factor. (Art. 173, with notes.)
3. Composite and prime numbers, and prime factor. (Art. 174 – 176.)
4. Odd and even numbers. (Art. 178.)
5. Divisibility of numbers by 2, 3, 4, 5, 6, 8, 9, 10, 100, 1000, &c. (Art. 178.)
6. Finding the prime factors of a number. (Art. 181, 182.)
7. Greatest common factor of two or more numbers. (Art. 185 – 188.)
8. Greatest common factor of two or more numbers, second method. (Art. 190.)
9. Cancellation. (Art. 192.)
10. Least common multiple. (Art. 194 – 199.)
11. Least common multiple, second method. (Art. 201.)

FRACTIONS.

204. If a unit, as an orange, is divided into two equal parts, each of these parts is one half of the unit.

If a unit is divided into three equal parts, each of the parts is one third of the unit.

One of the equal parts of a unit is a **Fraction** or **fractional unit.**

Define a fraction or *fractional unit.*

205. A collection of fractional units is a **fractional number.**

206. The unit of which the fraction is a part is the **unit of the fraction.**

Define the unit of the fraction.

NOTE TO THE TEACHER.—In its broadest sense, *any* part of a unit is a fraction, and any part of a unit when measured is found to be *one of the equal parts of a unit or a number of the equal parts united.*

But as in operating with fractions we consider the fractional units as separated, each fractional unit is a fraction, and two or more fractional units is a number of fractions or a fractional number.

It is believed that this view of fractions will greatly simplify the subject.

207. A knowledge of a fractional number, as two fourths of an apple, implies a knowledge, first, of the thing divided or the unit of the fraction; second, of the number of equal parts into which the unit of the fraction is divided, namely, "four"; and third, a knowledge of the number of parts taken, namely, "two."

Then in expressing a fraction or a fractional number, we must express the unit of the fraction, the number of equal parts into which the unit of the fraction is divided, and the number of fractional units taken.

To express the unit of the fractional number two fourths of an apple, we employ the word *apple*; to express the number of parts into which the unit has been divided we employ the figure 4 below a line, thus $\frac{}{4}$; to express the number of fractional units taken, we employ the figure $\frac{2}{}$ above the same line.

The full expression of this fractional number will then be $\frac{2}{4}$ apple, which is read "two fourths of an apple."

208. EXERCISES.

Express the following fractional numbers in figures: —

1. Three fourths of an apple.
2. Two thirds of a peach.
3. One eighth of an orange.
4. Seven twelfths of a yard.

Name the unit of the fraction of each of the above.

Name the fractional unit of each.

209. We have seen that to express a *fourth* of anything, as an apple, we write a figure 4 below a line, thus $\frac{}{4}$.

The number of equal parts, four, into which the unit of the fraction is divided, and which is expressed below the line, gives the name *fourth* to each of the fractional units; it is, therefore, the *namer* or **denominator** of each of the fractional units and of the fractional number.

Define denominator of fractional unit or fractional number.

What is the denominator of $\frac{1}{5}$? of $\frac{3}{8}$?

210. The number of parts taken, and which is expressed above the line, as 2 in $\frac{2}{4}$, is the **numerator** of the fraction or of the fractional number.

Define numerator of a fraction or fractional number.

What is the numerator of $\frac{1}{5}$? of $\frac{4}{7}$?

211. The numerator and denominator of a fraction or of a fractional number are called its **terms.**

What are the terms of a fraction or fractional number?

What are the terms of $\frac{2}{3}$ apple, and what does each term show?

Ans. Three is the denominator and two is the numerator; the denominator three shows that the unit of the fraction is divided into three parts, and the numerator two that two parts are taken.

212. Exercises.

Name the terms of each of the following, and tell what each term shows: —

1.	$\frac{3}{5}$.	3.	$\frac{1}{8}$.	5.	$\frac{2}{6}$.	7.	$\frac{10}{11}$.
2.	$\frac{3}{4}$.	4.	$\frac{3}{7}$.	6.	$\frac{4}{9}$.	8.	$\frac{8}{75}$

ADDITION.

213. 1. Add $\frac{1}{4}$ apple and $\frac{1}{4}$ peach.

Ans. We cannot add them.

2. Add $\frac{1}{3}$ apple and $\frac{1}{4}$ apple.

Ans. We cannot add them as now expressed.

3. Add $\frac{1}{4}$ apple and $\frac{1}{4}$ apple. *Ans.* $\frac{2}{4}$ apple.

By comparing the above examples we see that fractions which can be added must be like parts of the same or similar units.

Such fractions are **like fractions.**

Define like fractions.

Numbers whose units are like fractions are **like fractional numbers.**

Define like fractional numbers.

214. Examples.

Add the following: —

1. $\frac{1}{4}$ orange and $\frac{1}{4}$ orange.
2. $\frac{1}{5}$ melon and $\frac{3}{5}$ melon.
3. $\frac{2}{7}$ week and $\frac{4}{7}$ week.
4. \$ $\frac{1}{8}$, \$ $\frac{3}{8}$, and \$ $\frac{3}{8}$.
5. \$ $\frac{7}{10}$, \$ $\frac{9}{10}$, \$ $\frac{4}{10}$. *Ans.* \$ $\frac{20}{10}$.
6. \$ $\frac{7}{10}$, \$ $\frac{5}{10}$, \$ $\frac{3}{10}$. *Ans.* \$ $\frac{15}{10}$.

215. The fractional numbers, $ $\frac{20}{10}$ and $ $\frac{15}{10}$, the answers to examples 5 and 6 of Art. 214, exceed in amount the unit of the fraction, for $ $\frac{20}{10}$ equal $ 2; $ $\frac{15}{10}$ equal $ $1\frac{5}{10}$.

A number consisting of an integral with a fractional number, as $1\frac{5}{10}$, is a **mixed number.**

Define a mixed number.

216. TO CHANGE A FRACTIONAL NUMBER TO AN INTEGRAL OR TO A MIXED NUMBER.

ILLUSTRATIVE EXAMPLE. In $ $\frac{25}{4}$ how many dollars?

OPERATION.

$$4\,)\underline{\,2\,5\,}$$
$$6\tfrac{1}{4}$$

Explanation. — Since $\frac{4}{4}$ equal 1 unit, $ $\frac{25}{4}$ will equal as many dollars as there are 4's in 25; there are 6 fours in 25, and 1 remains; therefore there are $ $6\frac{1}{4}$ in $ $\frac{25}{4}$. *Ans.* $ $6\frac{1}{4}$.

From the above may be derived the following

RULE. — To change a fractional number to an integral or to a mixed number: — *Divide the numerator by the denominator.*

217. EXAMPLES.

Add the following, and change the fractional numbers to integral or to mixed numbers: —

7.	$ $\frac{3}{4}$, $ $\frac{3}{4}$, and $ $\frac{2}{4}$.	9.	$\frac{18}{35}$, $\frac{27}{35}$, and $\frac{33}{35}$.
8.	$ $\frac{7}{10}$, $ $\frac{5}{10}$, and $ $\frac{9}{10}$.	10.	$\frac{48}{101}$, $\frac{93}{101}$, and $\frac{86}{101}$.

218. In each example of Arts. 214 and 217 the fractions have like denominators.

If several fractions have like denominators, or if one denominator is common to them all, they have a **common denominator.**

Define common denominator.

219. All unlike fractional numbers that can be added may be changed to fractional numbers that have a common denominator.

In changing unlike to like fractional numbers or to those that have a common denominator, some of the fractional numbers must be expressed in larger or smaller terms.

220. TO EXPRESS FRACTIONAL NUMBERS IN LARGER OR SMALLER TERMS.

ILLUSTRATION.—If $\frac{1}{3}$ apple is divided into 4 equal parts, in $\frac{3}{3}$ apple or in the whole apple, there will be 12 such parts, consequently each of the parts will be $\frac{1}{12}$ apple; in $\frac{1}{3}$ apple there will be $\frac{1}{3}$ of 12 parts or $\frac{4}{12}$ apple; $\frac{1}{3}$ apple is thus shown to equal $\frac{4}{12}$ apple.

If we compare $\frac{1}{3}$ apple with $\frac{4}{12}$ apple it will be found that

To obtain	$\frac{1}{3}$ apple	the unit of the fraction is divided into	3	equal parts,	and 1 part	is taken.
	$\frac{4}{12}$ apple	the unit of the fraction is divided into	3×4	equal parts,	and 4 parts	are taken.

From the above it appears that in $\frac{1}{3}$ and $\frac{4}{12}$ the relation of the number of parts taken to the number of parts into which the units of the fractions are divided is the same (1 being 1 third of 3, and 4 being 1 third of 12).

INFERENCES.

(1.) *Any fractional number may be expressed in larger or smaller terms, if the relation of the terms is preserved.*

(2.) To express a fractional number in larger terms and preserve this relation, *both terms must be multiplied by the same number.*

(3.) To express a fractional number in smaller terms and preserve this relation, *both terms must be divided by the same number.*

(4.) *If both the terms of a fractional number are divided by their Greatest Common Factor, the terms will be prime to each other and the fractional number will be expressed in its smallest terms.*

221. TO CHANGE UNLIKE TO LIKE FRACTIONAL NUMBERS.

ILLUSTRATIVE EXAMPLE. Change $\frac{2}{3}$ apple, $\frac{3}{4}$ apple, and $\frac{1}{2}$ apple to fractional numbers having a common denominator, and add them.

OPERATION.

$\frac{2}{3} = \frac{2 \times \overline{4 \times 2}}{3 \times 4 \times 2} = \frac{16}{24}$ com. d.

$\frac{3}{4} = \frac{3 \times \overline{3 \times 2}}{4 \times 3 \times 2} = \frac{18}{24}$ com. d.

$\frac{1}{2} = \frac{1 \times \overline{4 \times 3}}{2 \times 4 \times 3} = \frac{12}{24}$ com. d.

$\frac{16}{24} + \frac{18}{24} + \frac{12}{24} = \frac{46}{24} = 1\frac{22}{24}$

Explanation.—The common denominator of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{2}$ must contain the denominators 3, 4, and 2 as factors; hence,

The common denominator must be a common multiple of the denominators.

The new denominator of each fractional number may be obtained by multiplying its denominator by the product of the denominators of the other fractional numbers; hence,

To obtain the new numerators we multiply the numerator of each fractional number by the product of the denominators of the other fractional numbers; we thus obtain $\frac{16}{24}$, $\frac{18}{24}$, *and* $\frac{12}{24}$, *which added equal* $\frac{46}{24}$, *or* $1\frac{22}{24}$.

222. EXAMPLES.

Change the following to fractional numbers having a common denominator, and add them:—

11. $\frac{1}{2}$ and $\frac{2}{3}$. *Ans.* $1\frac{1}{6}$.
12. $\frac{1}{2}$ and $\frac{2}{5}$. *Ans.* $\frac{9}{10}$.
13. $\frac{3}{5}$ and $\frac{1}{6}$. *Ans.* $\frac{23}{30}$.
14. $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$. *Ans.* $\frac{47}{60}$.
15. $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$. *Ans.* $2\frac{13}{60}$.
16. $\frac{1}{6}$, $\frac{2}{5}$, and $\frac{1}{7}$. *Ans.* $\frac{149}{210}$.
17. $\frac{5}{9}$, $\frac{3}{5}$, and $\frac{3}{4}$. *Ans.* $1\frac{163}{180}$.
18. $\frac{2}{3}$, $\frac{6}{7}$, and $\frac{7}{8}$. *Ans.* $2\frac{67}{168}$.
19. $\frac{4}{5}$, $\frac{4}{11}$, and $\frac{7}{13}$. *Ans.* $1\frac{502}{715}$.

223. If the common denominator of several fractional numbers is a multiple of their denominators, the least common denominator is the *least common multiple* of their denominators.

TO FIND THE LEAST COMMON DENOMINATOR.

ILLUSTRATIVE EXAMPLE. Change $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{2}{9}$ to fractional numbers having the least common denominator, and add them.

OPERATION.

$$\frac{3}{4} = \frac{3 \times \overline{3 \times 3}}{4 \times \overline{3 \times 3}} = \frac{27}{36}$$

$$\frac{5}{6} = \frac{5 \times \overline{2 \times 3}}{6 \times \overline{2 \times 3}} = \frac{30}{36}$$

$$\frac{2}{9} = \frac{2 \times \overline{2 \times 2}}{9 \times \overline{2 \times 2}} = \frac{8}{36}$$

$$\frac{27 + 30 + 8}{36} = \frac{65}{36} = 1\frac{29}{36}$$

Explanation.—We find the least common multiple (Art. 199) of the denominators 4, 6, and 9 to be 36.

To change $\frac{3}{4}$ to 36ths, we multiply the denominator 4 by 9: the numerator 3 must therefore be multiplied by 9, and $\frac{3}{4}$ equal $\frac{27}{36}$.

By a similar process $\frac{5}{6}$ and $\frac{2}{9}$ will be found equal respectively to $\frac{30}{36}$ and $\frac{8}{36}$; adding $\frac{27}{36}$, $\frac{30}{36}$, and $\frac{8}{36}$, we have $\frac{65}{36}$, or $1\frac{29}{36}$.

From what has been taught may be derived the following

RULE. — To change unlike fractional numbers to fractional numbers having the least common denominator : —

1. *For the common denominator take the least common multiple of the denominators.*

2. *For the new numerator of each fractional number, multiply the numerator by the same number by which you multiply its denominator for the common denominator.*

224. EXAMPLES.

Change the following to fractional numbers having the least common denominator, and add them : —

20. $\frac{1}{6}$ and $\frac{3}{4}$.	*Ans.* $\frac{11}{12}$.	23. $\frac{5}{12}$ and $\frac{7}{8}$.	*Ans.* $1\frac{7}{24}$.
21. $\frac{5}{6}$ and $\frac{4}{9}$.	*Ans.* $1\frac{5}{18}$.	24. $\frac{7}{10}$ and $\frac{8}{15}$.	*Ans.* $1\frac{7}{30}$.
22. $\frac{1}{4}$ and $\frac{9}{10}$.	*Ans.* $1\frac{3}{20}$.	25. $\frac{11}{12}$ and $\frac{5}{18}$.	*Ans.* $1\frac{7}{36}$.

NOTE. — Express the following fractional numbers in their smallest terms before adding. (See Art. 220. (4.))

26. $\frac{8}{10}$, $\frac{2}{5}$, and $\frac{12}{15}$.	*Ans.* 2.	29. $\frac{4}{16}$, $\frac{2}{9}$, and $\frac{6}{8}$.	*Ans.* $1\frac{2}{9}$.
27. $\frac{6}{16}$, $\frac{6}{48}$, and $\frac{20}{32}$.	*Ans.* $1\frac{1}{8}$.	30. $\frac{6}{12}$, $\frac{5}{30}$, and $\frac{4}{15}$.	*Ans.* $\frac{14}{15}$.
28. $\frac{36}{40}$, $\frac{6}{60}$, and $\frac{70}{100}$.	*Ans.* $1\frac{7}{10}$.	31. $\frac{8}{48}$, $\frac{15}{36}$, and $\frac{63}{84}$.	*Ans.* $1\frac{1}{3}$.

What rule can you give for the addition of fractional numbers?

225. EXAMPLES.

NOTE. — Add the integral and the fractional numbers of the following and similar examples separately.

32. $7\frac{1}{4} + 8\frac{2}{3}$?	*Ans.* $15\frac{11}{12}$.	37. $46\frac{19}{32} + 35\frac{27}{48}$?	
33. $9\frac{5}{8} + 12\frac{1}{6}$?	*Ans.* $21\frac{19}{24}$.	38. $22\frac{5}{14} + 8\frac{3}{4} + 9\frac{7}{16}$?	*Ans.* $40\frac{61}{112}$.
34. $21\frac{5}{9} + 15\frac{2}{3}$?	*Ans.* $37\frac{2}{9}$.		
35. $6\frac{3}{14} + 24\frac{17}{21}$?	*Ans.* $31\frac{1}{42}$.	39. $14\frac{3}{14} + 80\frac{19}{22} + 19\frac{13}{44}$?	
36. $27\frac{5}{18} + 19\frac{7}{24}$.		40. $69\frac{4}{15} + 46\frac{21}{25} + 7\frac{42}{54}$?	

☞ For Dictation Exercises, see "Manual and Key," page 65.

SUBTRACTION.

226. EXAMPLES.

Perform the subtraction indicated in the following: —

1.	$\frac{2}{3} - \frac{1}{3}$?	*Ans.* $\frac{1}{3}$.	4.	$\frac{17}{20} - \frac{9}{20}$?	*Ans.* $\frac{2}{5}$.
2.	$\frac{5}{8} - \frac{3}{8}$?	*Ans.* $\frac{1}{4}$.	5.	$\frac{31}{40} - \frac{19}{40}$?	*Ans.* $\frac{3}{10}$.
3.	$\frac{13}{16} - \frac{9}{16}$?	*Ans.* $\frac{1}{4}$.	6.	$\frac{43}{91} - \frac{17}{91}$?	*Ans.* $\frac{2}{7}$.

227. ILLUSTRATIVE EXAMPLE. If owning $\frac{1}{2}$ of an acre of land, I sell a part equal to $\frac{1}{3}$ of an acre, what part of the acre shall I still own?

OPERATION.

Minuend. $\frac{1}{2} = \frac{3}{6}$

Subtrahend. $\frac{1}{3} = \frac{2}{6}$

$\frac{3}{6} - \frac{2}{6} = \frac{1}{6}$ Remainder.

Explanation. — Changing the minuend and subtrahend to fractional numbers having a common denominator, and taking away the part $\frac{2}{6}$, we have $\frac{1}{6}$ for the remainder. *Ans.* $\frac{1}{6}$ acre.

What rule can you give for the subtraction of fractional numbers?

228. EXAMPLES.

Perform the subtraction indicated in the following: —

7.	$\frac{3}{5} - \frac{1}{3}$?	*Ans.* $\frac{4}{15}$.	10.	$\frac{15}{17} - \frac{7}{51}$?	*Ans.* $\frac{38}{51}$.
8.	$\frac{5}{8} - \frac{7}{12}$?	*Ans.* $\frac{1}{24}$.	11.	$\frac{56}{57} - \frac{8}{9}$?	*Ans.* $\frac{16}{171}$.
9.	$\frac{9}{10} - \frac{7}{8}$?	*Ans.* $\frac{1}{40}$.	12.	$\frac{100}{169} - \frac{17}{91}$?	*Ans.* $\frac{479}{1183}$.

NOTE. — In the following perform the subtraction of the integral and of the fractional numbers separately.

13.	$8\frac{2}{3} - 4\frac{1}{4}$?	*Ans.* $4\frac{5}{12}$.	17.	$75\frac{4}{9} - 18\frac{8}{75}$?
14.	$17\frac{5}{14} - 9\frac{5}{21}$?	*Ans.* $8\frac{5}{42}$.	18.	$160\frac{29}{81} - 108\frac{7}{54}$?
15.	$50\frac{8}{25} - 7\frac{4}{15}$?		19.	$71\frac{9}{10} - 35\frac{36}{40}$?
16.	$62\frac{18}{21} - 19\frac{22}{35}$?		20.	$92\frac{8}{11} - 18\frac{16}{23}$?

229. ILLUSTRATIVE EXAMPLE. If having \$ $3\frac{1}{4}$ I should spend \$ $\frac{7}{10}$, what should I have left?

$\frac{25}{11}$ $3\frac{1}{4} - \frac{7}{10} =$ $\frac{35}{20} - \frac{14}{20} = 2\frac{11}{20}$ a..

OPERATION.

	(2)	($\frac{5}{4}$)
Minuend.	3	$\frac{1}{4}$
Subtrahend.		$\frac{7}{10}$

$3\frac{1}{4} - \frac{7}{10} = 2\frac{11}{20}$ Remainder.

Explanation. — As $\frac{1}{4}$ is less than $\frac{7}{10}$, which we are to take away, we change one of the 3 (leaving 2) to fourths; 1 equals $\frac{4}{4}$; this with $\frac{1}{4}$ equals $\frac{5}{4}$. We then have the minuend expressed as $2\frac{5}{4}$; $2\frac{5}{4}$ less $\frac{7}{10}$ equals $2\frac{11}{20}$. *Ans.* \$$2\frac{11}{20}$.

TO CHANGE INTEGRAL OR MIXED NUMBERS TO FRACTIONAL NUMBERS.

230. In the above example an integer, 1, is changed to a fractional number, $\frac{4}{4}$. Any integral or mixed number may be changed to a fractional number.

ILLUSTRATIVE EXAMPLE. Change $8\frac{3}{4}$ to fourths.

OPERATION.

$8\frac{3}{4}$
4

$\frac{32}{4} + \frac{3}{4} = \frac{35}{4}$

Explanation. — Since 1 equals $\frac{4}{4}$, 8 will equal eight 4's of fourths, or $\frac{32}{4}$, which with $\frac{3}{4}$ equal $\frac{35}{4}$.

From the above operation we derive the following

RULE. — To change an integral or mixed number to a fractional number: *Multiply the integral number by the denominator of the required fractional number, and to the product add the numerator of the fractional number if there is one: the result will be the numerator of the fractional number required.*

EXAMPLES.

Perform the subtraction indicated in the following: —

21. $10\frac{2}{9} - 8\frac{5}{18}$? *Ans.* $1\frac{17}{18}$.
22. $17\frac{7}{12} - 9\frac{5}{8}$? *Ans.* $7\frac{23}{24}$.
23. $68\frac{8}{9} - 27\frac{11}{12}$? *Ans.* $40\frac{35}{36}$.
24. $127\frac{18}{25} - 87\frac{23}{30}$? *Ans.* $39\frac{143}{150}$.
25. $90\frac{8}{75} - 18\frac{47}{60}$?
26. $300\frac{67}{132} - 47\frac{191}{198}$?

231. MISCELLANEOUS EXAMPLES IN ADDITION AND SUBTRACTION.

1. A printer spent $22\frac{3}{4}$ hours in setting type; $8\frac{5}{6}$ hours in correcting proof, and $6\frac{2}{3}$ hours in distributing type; how many hours did he spend in all? *Ans.* $38\frac{1}{4}$ hours.

2. I bought a firkin of butter, the firkin with the butter weighing $39\frac{3}{4}$ pounds, the firkin alone weighing $5\frac{5}{8}$ pounds; what was the weight of the butter? *Ans.* $34\frac{1}{8}$ pounds.

3. A man bequeathed $\frac{14}{33}$ of his property to his wife, $\frac{5}{12}$ of it to his son, and the rest to the church; what part of it did he bequeath to the church? *Ans.* $\frac{21}{132}$.

4. A real-estate broker sold $4\frac{3}{7}$ acres of land to one person, $17\frac{5}{8}$ acres to another, and had $16\frac{17}{40}$ acres left; how many acres had he before he sold? *Ans.* $38\frac{67}{140}$ acres.

5. A farmer who had $17\frac{3}{5}$ tons of hay in his barn, and a quantity in his field, sold $24\frac{3}{4}$ tons; he then had $8\frac{1}{2}$ tons left; how many tons had he in his field before selling?

6. A farmer who had $25\frac{1}{4}$ acres of land to sow, has sowed $15\frac{2}{3}$ acres with a machine, and $4\frac{4}{5}$ acres by hand; how many acres remain to be sowed?

☞ For Dictation Exercises, see "Manual and Key," page 65.

MULTIPLICATION.

232. CASE I. When the multiplier is an integral number.

ILLUSTRATIVE EXAMPLE. If $\frac{2}{5}$ of a pine-apple is given to each of 2 boys, what part of the pine-apple will the 2 boys receive?

OPERATION.

$$\frac{2}{5} \times 2 = \frac{4}{5}$$

Explanation. — If each boy receives $\frac{2}{5}$ of a pine-apple, two boys will receive two $\frac{2}{5}$ of a pine-apple; two $\frac{2}{5}$ are $\frac{4}{5}$: therefore the 2 boys will receive $\frac{4}{5}$ of a pine-apple.

EXAMPLES.

1. What is the price of 3 railroad tickets at $\$\frac{3}{8}$ apiece? *Ans.* $\$\frac{9}{8}$ or $\$1\frac{1}{8}$.

2. What will 2 baskets cost at $ $\frac{3}{10}$ apiece?

NOTE. The operation of the above example may be expressed as follows $\frac{3 \times \cancel{2}}{\cancel{10}_{5}} = \frac{3}{5}$. Any common factors may then be cancelled before the multiplication is performed. (Art. 220 (4).) *Ans.* $ $\frac{3}{5}$.

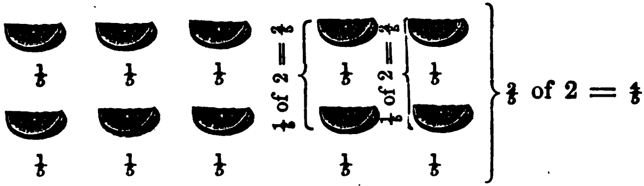
3. What cost 12 books at $ $\frac{2}{3}$ each?

4. What cost 10 loaves of cake at $ $\frac{7}{8}$ each?

233. CASE II. When the multiplicand is an integral number.

ILLUSTRATIVE EXAMPLE. If $ 1 will buy 2 melons what part of a melon will $ $\frac{1}{5}$ buy? $ $\frac{2}{5}$?

OPERATION ILLUSTRATED.



OPERATION EXPRESSED.

$$\frac{2 \times \cancel{5} \times 2}{5} = \frac{4}{5}$$

OPERATION ABBREVIATED.

$$\frac{2 \times 2}{5} = \frac{4}{5}$$

Explanation.—If $1 will buy 2 melons, $$\frac{1}{5}$ will buy $\frac{1}{5}$ of 2 melons, and $ $\frac{2}{5}$ will buy $\frac{2}{5}$ of 2 melons.

2 melons (divided into fifths) equal $\frac{10}{5}$ melons; $\frac{1}{5}$ of $\frac{10}{5}$ equals $\frac{2}{5}$ (see illustration), and $\frac{2}{5}$ of $\frac{10}{5}$ equals $\frac{2 \times 2}{5}$ or $\frac{4}{5}$; therefore $ $\frac{2}{5}$ will buy $\frac{4}{5}$ of a melon. *Ans.* $\frac{4}{5}$ melon.

Explanation abbreviated. $\frac{1}{5}$ of 2 melons equals $\frac{2}{5}$ of a melon, and $\frac{2}{5}$ of 2 melons equal $\frac{2 \times 2}{5}$ or $\frac{4}{5}$ of a melon.

EXAMPLES.

5. In 1 bushel there are 4 pecks; how many pecks are there in $\frac{2}{3}$ of a bushel? *Ans.* $2\frac{2}{3}$ pecks.

6. In 1 yard there are 3 feet; how many feet are there in $\frac{3}{4}$ of a yard? *Ans.* $2\frac{1}{4}$ feet.

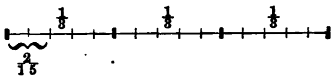
234. CASE III. When both multiplicand and multiplier are fractional numbers.

7. If $1 will buy $\frac{1}{3}$ yard of velvet, what part of $\frac{1}{3}$ yard will $$\frac{2}{5}$ buy? *Ans.* $\frac{2}{5}$ of $\frac{1}{3}$ yard.

NOTE. $\frac{2}{5}$ of $\frac{1}{3}$ is a form of denoting the multiplication of fractional numbers, $\frac{1}{3}$ being the multiplicand, $\frac{2}{5}$ the multiplier.

ILLUSTRATIVE EXAMPLE I. What part of a yard is $\frac{1}{5}$ of $\frac{1}{3}$ of a yard of ribbon? $\frac{2}{5}$ of $\frac{1}{3}$ of a yard?

OPERATION ILLUSTRATED.



OPERATION EXPRESSED.

$$\frac{1 \times \cancel{5} \times 2}{3 \times 5} = \frac{2}{15}$$

OPERATION ABBREVIATED.

$$\frac{1 \times 2}{3 \times 5} = \frac{2}{15}$$

Explanation. — $\frac{1}{3}$ of a yard (divided into fifths) equals $\frac{5}{15}$ of a yard; $\frac{1}{5}$ of $\frac{1}{3}$ of a yard, then, equals $\frac{1}{5}$ of $\frac{5}{15}$ or $\frac{1}{15}$ of a yard, and $\frac{2}{5}$ of $\frac{1}{3}$ of a yard will equal $\frac{1}{15} \times 2$ or $\frac{2}{15}$ of a yard; therefore $\frac{2}{5}$ of $\frac{1}{3}$ of a yard is $\frac{2}{15}$ of a yard.

Explanation abbreviated. — $\frac{1}{5}$ of $\frac{1}{3}$ of a yard equals $\frac{1}{15}$ of a yard, and $\frac{2}{5}$ of $\frac{1}{3}$ of a yard will equal $\frac{2}{15}$ of a yard.

EXAMPLES.

8. What part of an orange is $\frac{1}{3}$ of $\frac{1}{2}$ of an orange? $\frac{2}{3}$ of $\frac{1}{3}$? $\frac{1}{5}$ of $\frac{1}{2}$? $\frac{3}{5}$ of $\frac{1}{3}$? $\frac{1}{8}$ of $\frac{1}{9}$? $\frac{5}{8}$ of $\frac{1}{9}$?

9. What part of a day is $\frac{2}{3}$ of $\frac{1}{3}$ of it? $\frac{3}{5}$ of $\frac{1}{4}$? $\frac{2}{3}$ of $\frac{1}{5}$? $\frac{3}{7}$ of $\frac{1}{4}$? $\frac{2}{5}$ of $\frac{1}{8}$? $\frac{7}{8}$ of $\frac{1}{4}$?

235. ILLUSTRATIVE EXAMPLE II. If a man can earn $1 in $\frac{2}{5}$ of an hour, in what part of an hour can he earn $$\frac{2}{3}$?

OPERATION.

$$\frac{2 \times \cancel{3} \times 2}{5 \times 3} = \frac{4}{15}$$

OPERATION ABBREVIATED.

$$\frac{2 \times 2}{5 \times 3} = \frac{4}{15}$$

Explanation. — If he can earn $1 in $\frac{2}{5}$ of an hour, he can earn $$\frac{2}{3}$ in $\frac{2}{3}$ of $\frac{2}{5}$ of an hour. $\frac{2}{5}$ (divided into thirds) will equal $\frac{6}{15}$; $\frac{1}{3}$ of $\frac{6}{15}$ equals $\frac{2}{15}$ and $\frac{2}{3}$ of $\frac{6}{15}$ will equal $\frac{2}{15} \times 2$ or $\frac{4}{15}$; therefore he can earn $$\frac{2}{3}$ in $\frac{4}{15}$ of an hour.

Explanation abbreviated. — $\frac{1}{3}$ of $\frac{2}{5}$ equals $\frac{2}{15}$, and $\frac{2}{3}$ of $\frac{2}{5}$ will equal $\frac{2}{15} \times 2$ or $\frac{4}{15}$.

EXAMPLES.

10. What is $\frac{2}{5}$ of $\frac{3}{7}$? $\frac{5}{7}$ of $\frac{8}{9}$? *Ans.* $\frac{6}{35}$; $\frac{40}{63}$.

11. What will $\frac{2}{3}$ dozen eggs cost at $\$\frac{3}{8}$ a dozen?

12. At $\$\frac{4}{5}$ a pound, what will $\frac{2}{3}$ of a pound of soap cost?

13. At $\$\frac{3}{4}$ a day, what will a boy earn in $\frac{3}{4}$ of a day?

236. Recurring to what has been taught in the three cases which precede, noticing particularly the abbreviated operations, we may derive the following

RULE FOR THE MULTIPLICATION OF FRACTIONAL NUMBERS. — I. When either multiplier or multiplicand is an integral number (Cases I. and II.) *Take the product of the integral number and the numerator of the fractional number for the numerator of the answer; and take the denominator of the fractional number for the denominator of the answer.*

II. When both multiplier and multiplicand are fractional numbers (Case III.) *Take the product of the numerators for the numerator of the answer, and the product of the denominators for the denominator of the answer.*

237. EXAMPLES.

14. If a train moves at the rate of $\frac{5}{6}$ of a mile in a minute, how far will it move in 5 minutes? *Ans.* $4\frac{1}{6}$ miles.

15. If $\frac{7}{8}$ of a yard of cloth costs \$1, how many yards can be bought for \$6? *Ans.* $5\frac{1}{4}$ yards.

16. At $\$7\frac{3}{4}$ a ton, what is the cost of 5 tons of coal?

NOTE. — Multiply the fractional and the integral numbers separately, thus: $\frac{3}{4} \times 5 = \frac{15}{4} = 3\frac{3}{4}$; $7 \times 5 = 35$. $35 + 3\frac{3}{4} = 38\frac{3}{4}$. *Ans.* $\$38\frac{3}{4}$.

17. At $\$2\frac{3}{5}$ a bushel, what will 7 bushels of wheat cost?

18. If $5\frac{11}{16}$ yards of cloth are required to make one garment, how many yards will be required to make 100 like garments? *Ans.* $568\frac{3}{4}$ yd.

19. At \$2 a pound, what will be the cost of $\frac{5}{8}$ of a pound of calf-skin? *Ans.* $\$1\frac{1}{4}$.

20. What cost $\frac{5}{6}$ of a barrel of apples at $9 a barrel?

21. If in 1 fathom there are 6 feet, how many feet are there in $2\frac{1}{4}$ fathoms?

NOTE.—Employ the $\frac{1}{4}$ and the 2 as multipliers separately, thus: $6 \times \frac{1}{4} = 1\frac{1}{2}$; $6 \times 2 = 12$; $12 + 1\frac{1}{2} = 13\frac{1}{2}$. *Ans.* $13\frac{1}{2}$ feet.

22. At $5 a day, what are the wages of a man for $10\frac{3}{4}$ days? *Ans.* $$53\frac{3}{4}$.

23. At $$\frac{2}{5}$ a pound, what will $\frac{2}{3}$ of a pound of coffee cost? *Ans.* $$\frac{4}{15}$.

24. If $\frac{5}{8}$ of a bushel of rye will sow 1 acre, how much will sow $\frac{4}{5}$ of an acre? *Ans.* $\frac{1}{2}$ bush.

25. At $$\frac{7}{12}$ per yard, what will $\frac{1}{3}$ of a yard of cloth cost?

26. At $$2\frac{1}{5}$ a bag, what will $\frac{3}{4}$ of a bag of wheat cost?

NOTE.—First change $2\frac{1}{5}$ to fifths. *Ans.* $$1\frac{13}{20}$.

27. If a horse travels $5\frac{3}{4}$ miles in 1 hour, how far will he travel in $\frac{2}{3}$ of an hour? *Ans.* $3\frac{5}{6}$ miles.

28. What will $1\frac{1}{2}$ of a yard of cloth cost at $$5\frac{3}{5}$ a yard?

29. At $$8\frac{2}{5}$ a cord, what cost $3\frac{3}{4}$ cords of wood?

NOTE.—$8\frac{2}{5} = \frac{42}{5}$; $3\frac{3}{4} = \frac{15}{4}$; $\frac{\overset{21}{\cancel{42}} \times \overset{3}{\cancel{15}}}{\cancel{5} \times \underset{2}{\cancel{4}}} = \frac{63}{2} = 31\frac{1}{2}$.

30. If a franc equals $18\frac{3}{5}$ cents, what number of cents will $16\frac{3}{10}$ francs equal? *Ans.* $303\frac{9}{50}$ cts.

31. If 1 ton of hay cost $$25\frac{5}{8}$, what will $5\frac{7}{20}$ tons cost?

32. Add $3\frac{5}{6} \times 10$ with $4\frac{4}{9} \times 15$. *Ans.* 105.

33. Add $272\frac{1}{4} \times 75$ with $30 \times \frac{5}{9}$. *Ans.* $20{,}435\frac{5}{12}$.

34. Add $12 \times 16\frac{3}{8}$ with $11 \times 17\frac{3}{10}$ *Ans.* $386\frac{4}{5}$.

35. Add $\frac{9}{20} \times \frac{8}{15}$ with $5\frac{5}{8} \times \frac{4}{9}$. *Ans.* $2\frac{37}{50}$.

36. Add $\frac{3}{8}$ of $\frac{3}{5}$ of $3\frac{1}{3}$, $25\frac{2}{3} \times \frac{9}{11}$, and $30\frac{1}{4} \times 7\frac{4}{11}$. *Ans.* $244\frac{1}{2}$.

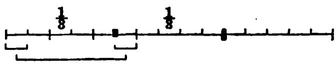
☞ For Dictation Exercises, see "Manual and Key," page 67.

DIVISION.

238. CASE I. When the divisor is an integral number.

ILLUSTRATIVE EXAMPLE I. If 5 pounds of coffee cost $\frac{2}{3}$ of a dollar, what will 1 pound cost?

OPERATION ILLUSTRATED.



$\frac{1}{5}$ of $\frac{2}{3}$ = $\frac{2}{15}$

Explanation. — If 5 pounds cost \$ $\frac{2}{3}$, 1 pound will cost 1 fifth of \$ $\frac{2}{3}$, which equals \$ $\frac{2}{15}$.

NOTE. — Though the above is an example in division, — finding one of the equal parts of a number (Art. 109), — the operation is the same as that illustrated in multiplication of fractions, Case III., Illustrative Example I. Hence it will be seen that dividing by 5 is equivalent to multiplying by $\frac{1}{5}$.

Dividing by 8 is equivalent to what? dividing by 16? by 24?

EXAMPLES.

1. If 6 keys cost \$ $\frac{3}{4}$, what will 1 key cost? *Ans.* \$ $\frac{1}{8}$.
2. If 4 nets cost \$ $\frac{3}{8}$, what will 1 net cost? *Ans.* \$ $\frac{3}{32}$.
3. What is 1 tenth of \$ $8\frac{1}{3}$? 1 twelfth of $62\frac{1}{2}$?

239. ILLUSTRATIVE EXAMPLE II. If 1 yard of cloth costs \$2, how many yards can be bought for \$ $\frac{3}{4}$?

OPERATION.

$$\frac{3}{4} \div 2 = \frac{3}{4} \div \frac{4 \times 2}{4} = \frac{3}{4 \times 2} = \frac{3}{8}$$

Ans. $\frac{3}{8}$ yard.

OPERATION ABBREVIATED.

$$\frac{3}{4 \times 2} = \frac{3}{8}$$

Explanation. — As many yards at \$ 2 a yard can be bought for \$ $\frac{3}{4}$ as there are 2's in $\frac{3}{4}$.

We first change 2 to fourths; 2 equals $\frac{8}{4}$; there are as many $\frac{8}{4}$ in $\frac{3}{4}$ as there are 8's in 3.* There are $\frac{3}{8}$ of 8 in 3. Hence $\frac{3}{8}$ of a yard can be bought.

* As 8 is a greater number than 3, it is not contained in 3; we will then see how many eighths of 8 are contained in 3. 1 eighth of 8 is 1; there are three ones in 3; therefore there are $\frac{3}{8}$ of 8 in 3.

NOTE.—It will be seen that in the abbreviated operation the number of fourths in the dividend is expressed by 3, and the number of fourths in the divisor by 4×2.

EXAMPLES.

4. At \$5 a box for oranges, what quantity of oranges can be bought for \$$\frac{4}{5}$?* *Ans.* $\frac{4}{25}$ box.

5. If the subscription price of a newspaper is \$3 a year, what part of a year will \$$\frac{3}{4}$ pay for? *Ans.* $\frac{1}{4}$ year.

6. At \$4 a bushel, what quantity of cranberries may be bought for \$$\frac{7}{8}$? *Ans.* $\frac{7}{32}$ bushel.

240. CASE II. When the divisor only is a fractional number.

ILLUSTRATIVE EXAMPLE. How many bushels of corn at \$$\frac{3}{4}$ a bushel can be bought for \$2?

OPERATION.

$$2 \div \frac{3}{4} = \frac{2 \times 4}{4} \div \frac{3}{4} = \frac{2 \times 4}{3} = \frac{8}{3} = 2\frac{2}{3}.$$

Ans. $2\frac{2}{3}$ bushels.

Explanation.—As many bushels can be bought for \$2 as there are $\frac{3}{4}$ in 2. We first change 2 to fourths. 2 equals $\frac{8}{4}$. There are as many $\frac{3}{4}$ in $\frac{8}{4}$ as there are 3's in 8, which is $2\frac{2}{3}$. *Ans.* $2\frac{2}{3}$ bushels.

OPERATION ABBREVIATED.

$$\frac{2 \times 4}{3} = \frac{8}{3} = 2\frac{2}{3}.$$

NOTE.—It will be seen that in the abbreviated operation the number of fourths in the dividend is expressed by 2×4, and the number of fourths in the divisor by 3.

EXAMPLES.

7. At \$$\frac{3}{4}$ apiece, how many railroad tickets may be bought for \$6? *Ans.* 8 tickets.

8. How many books, at \$$\frac{5}{8}$ each, may be bought for \$20? *Ans.* 32 books.

9. If \$$\frac{3}{16}$ will buy one peck of peas, how many pecks can be bought for \$4? *Ans.* $21\frac{1}{3}$ pecks.

* This example can be explained by analysis thus:—
If \$5 will buy 1 box of oranges, \$1 will buy $\frac{1}{5}$ of a box, and \$$\frac{4}{5}$ will buy $\frac{4}{5}$ of $\frac{1}{5}$ of a box, which equals $\frac{4}{25}$ of a box. *Ans.* $\frac{4}{25}$ box.

241. CASE III. When the dividend and divisor are both fractional numbers.

ILLUSTRATIVE EXAMPLE. If 1 yard of cloth costs \$ $\frac{2}{5}$, how many yards can be bought for \$ $\frac{1}{2}$?

OPERATION.

$$\frac{1}{2} \div \frac{2}{5} = \frac{1 \times 5}{2 \times 5} \div \frac{2 \times 2}{5 \times 2}$$

$$\frac{1 \times 5}{2 \times 2} = \frac{5}{4} = 1\frac{1}{4}.$$

Ans. $1\frac{1}{4}$ yards.

OPERATION ABBREVIATED.

$$\frac{1 \times 5}{2 \times 2} = \frac{5}{4} = 1\frac{1}{4}.$$

Explanation. — As many yards of cloth can be bought for \$ $\frac{1}{2}$ as there are $\frac{2}{5}$ in $\frac{1}{2}$. We first change $\frac{1}{2}$ and $\frac{2}{5}$ to fractional numbers having a common denominator. $\frac{1}{2}$ equals $\frac{5}{10}$, $\frac{2}{5}$ equal $\frac{4}{10}$. There are as many $\frac{4}{10}$ in $\frac{5}{10}$ as there are 4's in 5, which is $1\frac{1}{4}$. *Ans.* $1\frac{1}{4}$ yards.

NOTE. — It will be seen that in the abbreviated operation the number of tenths in the dividend is expressed by 1×5, and the number of tenths in the divisor by 2×2.

EXAMPLES.

10. If $\frac{3}{8}$ of a yard of silk will make one cravat, how many cravats can be made by using $\frac{3}{4}$ of a yard?

Ans. 2 cravats.

11. At \$ $\frac{1}{4}$ a bushel, how many bushels of turnips can be bought for \$ $\frac{2}{3}$? *Ans.* $2\frac{2}{3}$ bush.

12. How many pounds of rice, at \$ $\frac{7}{21}$ a pound, can be bought for \$ $\frac{3}{4}$?

242. From the three cases in division of fractional numbers we derive the following Rules: —

RULE I. FOR DIVISION OF FRACTIONAL NUMBERS. — *Change the dividend and divisor to fractional numbers having a common denominator, and then divide the numerator of the dividend by the numerator of the divisor.*

NOTE. — Recurring to the abbreviated operations, we may derive

RULE II. 1. When the divisor is an integral number (Case I): *Multiply the denominator of the dividend by the integral number.*

2. When the divisor is a fractional number (Cases II. and III.): *Multiply the dividend by the denominator of the divisor and divide that product by the numerator of the divisor.*

243. EXAMPLES.

13. If $8\frac{3}{4}$ acres of land be divided into 4 equal lots, what will each lot contain? *Ans.* $2\frac{3}{16}$ acres.

14. When the dividend is $\frac{3}{4}$ of an acre, and the divisor 2 acres, what is the quotient?

15. At $2 a barrel, how many barrels of apples can be bought for $$5\frac{1}{3}$?

NOTE. — Change $5\frac{1}{3}$ to thirds. $\frac{16}{3} \div 2 = \frac{\overset{8}{\cancel{16}}}{3 \times \cancel{2}} = 2\frac{2}{3}$. *Ans.* $2\frac{2}{3}$ barrels.

16. At $3 a cord, how many cords of wood can be bought for $$8\frac{1}{4}$? *Ans.* $2\frac{3}{4}$ cords.

17. If a man can walk 4 miles an hour, in what time can he walk $16\frac{1}{3}$ miles? $1\frac{1}{3}$ miles? $\frac{1}{5}$ of a mile?

Ans. $4\frac{1}{12}$ hours; $\frac{1}{3}$ hour; $\frac{1}{20}$ hour.

18. At $2 a day for work, what part of a day's work can be secured for $$\frac{2}{3}$? *Ans.* $\frac{1}{3}$ day.

19. At $$\frac{7}{16}$ an hour, how many hours of labor can be paid for with $4? with $84? *Ans.* $9\frac{1}{7}$ hours; 192 hours.

20. If $\frac{5}{8}$ of a pound of cotton will make 1 yard of muslin, how many yards will 546 pounds make?

21. At $$2\frac{1}{2}$ a ream, how many reams of paper may be bought for $15? (Change $2\frac{1}{2}$ to halves.) *Ans.* 6 reams.

22. In 1 rod there are $5\frac{1}{2}$ yards; how many rods are there in 407 yards? *Ans.* 74 rods.

23. At $$\frac{1}{5}$ a quire, how many quires of paper can be bought for $$\frac{1}{2}$? *Ans.* $2\frac{1}{2}$ quires.

24. If I can weave $\frac{5}{8}$ of a yard of cloth in 1 hour, in what time can I weave $\frac{2}{3}$ of a yard? *Ans.* $1\frac{1}{15}$ hours.

25. How many lots of land, each containing $\frac{1}{15}$ of an acre, are there in $\frac{11}{12}$ of an acre?

26. How many pairs of gloves, at $$\frac{5}{6}$ a pair, can be bought for $$13\frac{1}{3}$? (Change $13\frac{1}{3}$ to thirds.) *Ans.* 16 pairs.

27. How many feet of wood at $\$\frac{5}{12}$ per foot can be bought for $\$33\frac{1}{3}$? *Ans.* 80 feet.

28. If a family consume $\frac{5}{8}$ of a barrel of flour in a month, in how many months will they consume $4\frac{1}{6}$ barrels?

29. If $2\frac{1}{4}$ quires of paper are required to make a book, how many books may be made by using $1678\frac{1}{2}$ quires?

30. What is the value of $\frac{2\frac{2}{5}}{\frac{3}{4}}$ $(= 2\frac{2}{5} \div \frac{3}{4})$? *Ans.* $3\frac{1}{5}$.

31. What is the value of $\frac{3\frac{3}{7}}{2\frac{5}{8}}$? *Ans.* $1\frac{15}{49}$.

32. Add $30 \div 3\frac{1}{4}$ with $48\frac{1}{4} \div \frac{3}{15}$. *Ans.* $250\frac{25}{52}$.

33. Add $25\frac{1}{2} \div 4\frac{2}{5}$ with $18\frac{3}{4} \div 1\frac{3}{5}$. *Ans.* $17\frac{181}{352}$.

34. Add $\frac{\frac{2}{3}}{5}$ with $7\frac{1}{5} \div \frac{\frac{2}{3} \text{ of } 7\frac{1}{2}}{\frac{2}{3}}$. *Ans.* $1\frac{7}{75}$.

☞ For Dictation Exercises, see "Manual and Key," page 69.

244. To find the Whole when a Part is given.

ILLUSTRATIVE EXAMPLE I. In $\frac{1}{4}$ of a year there are three months; how many months are there in 1 year?

Explanation.—If in $\frac{1}{4}$ of a year there are 3 months, in $\frac{4}{4}$, or 1 year, there are four 3's of months, or 12 months; therefore in 1 year there are 12 months. *Ans.* 12 months.

EXAMPLES.

35. In $\frac{1}{3}$ of a rod there are $5\frac{1}{2}$ feet; how many feet are there in 1 rod? *Ans.* $16\frac{1}{2}$ feet.

36. A man in business lost $\$758$ in 1 year; this was equal to $\frac{1}{10}$ of the money he put in trade; how much money did he put in trade?

37. I gained $\$3478$ by selling a lot of cotton; if this is equal to $\frac{1}{8}$ of what the cotton cost me, what did it cost me?

38. $\$46$ is $\frac{1}{8}$ of what number?

39. $30\frac{1}{4}$ yards is $\frac{1}{9}$ of what number?

ILLUSTRATIVE EXAMPLE II. If $\frac{2}{5}$ of a pound of beef cost 6 cents, what will 1 pound cost?

OPERATION.

$$\text{Cents.}\ \frac{\overset{3}{\cancel{6}} \times 5}{\cancel{2}}$$

Explanation.—If $\frac{2}{5}$ of a pound cost 6 cents, $\frac{1}{5}$ of a pound will cost $\frac{1}{2}$ of 6 cents, and $\frac{5}{5}$ or 1 pound will cost $\frac{1}{2}$ of 6 cents multiplied by 5, which equals 15 cents. *Ans.* 15 cents.

40. If $\frac{5}{11}$ acre of land is sold for \$75, for what would 1 acre be sold at the same rate? *Ans.* \$165.

41. The cost of fencing a lot of land was \$100; if this was $\frac{4}{9}$ of the cost of the land, what did the land cost?

42. 768 men is $\frac{16}{25}$ of what number?

43. 3 bushels is $\frac{2}{7}$ of what number?

44. 8 days is $\frac{7}{10}$ of what number?

45. When $\frac{3}{4}$ of a pound of butter costs \$$\frac{3}{10}$, what does 1 pound cost? *Ans.* \$$\frac{2}{5}$.

46. If $2\frac{1}{4}$ dozen needles can be bought for \$$\frac{5}{8}$, how many dozens can be bought for \$1? *Ans.* $3\frac{3}{5}$ dozen.

47. If $\frac{7}{8}$ of a yard of carpeting costs \$$4\frac{2}{3}$, what is the cost per yard?

48. A farmer sold a lot of land so as to gain $\frac{1}{8}$ as much as it cost, how many eighths of what it cost did he receive? If he received \$810, what did it cost him? *Ans.* \$720.

49. A grocer sold sugar for \$23.21; if this is $\frac{1}{10}$ more than it cost him, what did it cost him? *Ans.* \$21.10.

50. If when a book is sold for \$1.75, there is a gain equal to $\frac{3}{4}$ of the cost, what was the cost?

51. After spending $\frac{1}{3}$ of his money, a person had \$18 left? how much money had he at first? *Ans.* \$27.

52. A jockey sold a horse at a loss equal to $\frac{1}{4}$ of what he paid; if he sold the horse for \$120, what did he pay for him?

53. By selling beef at $\$\frac{1}{12}$ per pound, I lost $\frac{1}{8}$ of what it cost me; what did it cost me per pound? *Ans.* $\$\frac{2}{21}$.

54. By selling a hammer for $\$1\frac{1}{20}$, I lost $\frac{3}{10}$ of what it cost me; what did it cost me?

55. I sold a lot of timber at $\$12.83\frac{1}{3}$ per thousand feet, and thereby gained a sum equal to $\frac{3}{8}$ of what it cost; what did it cost?

245. To find what Part one Number is of Another.

ILLUSTRATIVE EXAMPLE. A boy who had 40 cents gave away 4 cents; what part of his money did he give away?

Explanation. — He gave away such a part of his money as 4 cents is of 40 cents; 1 cent is $\frac{1}{40}$ of 40 cents, and 4 cents is $\frac{4}{40}$ or $\frac{1}{10}$ of 40 cents; therefore he gave away $\frac{1}{10}$ of his money.

From the preceding example we may derive the following

RULE. — To find what part one number is of another: — *Divide the number which is the part by that with which it is compared.*

EXAMPLES.

56. Of a wall 80 rods in length, 16 rods were laid in one day; what part of the wall was laid in 1 day? *Ans.* $\frac{1}{5}$.

57. What part of 20 is 4? 15? 13? $2\frac{1}{2}$?

NOTE. $2\frac{1}{2} = \frac{5}{2}$; $20 = \frac{40}{2}$; $2\frac{1}{2}$ is the same part of 20 that $\frac{5}{2}$ is of $\frac{40}{2}$, or that 5 is of 40. *Ans.* $\frac{1}{8}$.

58. What part of 6 is $1\frac{1}{2}$? $4\frac{1}{2}$? $\frac{1}{2}$? $\frac{1}{3}$? $\frac{3}{4}$?

59. What part of 40 is $13\frac{1}{3}$? $26\frac{2}{3}$? $5\frac{5}{7}$? $\frac{1}{7}$?

60. What part of $7\frac{1}{2}$ is 5? *Ans.* $\frac{2}{3}$.

61. What part of $33\frac{1}{3}$ is 25? $8\frac{1}{3}$? $6\frac{1}{4}$?

62. What part of $18\frac{3}{4}$ is $12\frac{1}{2}$? $6\frac{2}{3}$? 5? $\frac{3}{4}$?

63. What part of $\frac{4}{5}$ is $\frac{2}{3}$? of $\frac{5}{8}$ is $\frac{2}{5}$? of 1 is $\frac{4\frac{1}{8}}{5}$?

64. What part of $\$1$ is 20 cents? 75 cents? $12\frac{1}{2}$ cents? $62\frac{1}{2}$ cents? $6\frac{5}{30}$

65. What part of $1 is $16\frac{2}{3}$ cents? $33\frac{1}{3}$ cents? $18\frac{3}{4}$ cents? $83\frac{1}{3}$ cents?

66. If a piece of work can be performed in 18 days, what part of the work can be performed in 12 days? in $4\frac{1}{2}$ days? in $\frac{2}{3}$ of a day? *Ans.* $\frac{2}{3}$; $\frac{1}{4}$; $\frac{1}{27}$.

67. A and B hired a carriage together, for which they paid $8. A occupied 2 seats, B occupied 3 seats; what should each pay? *Ans.* A, $$3\frac{1}{5}$; B, $$4\frac{4}{5}$.

68. When butter which cost 25 cents per pound is sold for 35 cents, to what part of the cost is the gain equal? *Ans.* $\frac{2}{5}$.

246. TOPICAL REVIEW IN FRACTIONAL NUMBERS.

The pupil may illustrate the following topics to his class, using common objects when practicable, and giving definitions and rules: —

1. Fraction or fractional unit; fractional number; unit of the fraction. (Arts. 204 – 206.)

2. Manner of expressing a fractional number. (Art. 207.)

3. Denominator, numerator, terms. (Arts. 209 – 211.)

4. Like fractions. (Art. 113.)

5. Mixed numbers and changing a fractional number to an integral or mixed number. (Arts. 115, 116.)

6. Common denominator. (Art. 218.)

7. Expressing fractional numbers in larger or smaller terms. (Arts. 219, 220.)

8. Changing unlike to like fractional numbers. (Art. 221.)

9. Least common denominator. (Art. 223.)

10. Make a rule for Addition of fractional numbers.

11. Make a rule for Subtraction of fractional numbers.

12. Changing an integral or mixed number to a fractional number. (Art. 230.)

13. Multiplication of fractional numbers. Cases I., II., III., with general rule. (Arts. 232 – 236.)

14. Division of fractional numbers. Cases I., II., III., with general rule. (Arts. 238 – 242.)

15. To find the whole when a part is given. (Art. 244.)

16. To find what part one number is of another. (245.)

247. GENERAL REVIEW, No. 3.

1. What are the prime factors of 265? of 2000?
2. What is the greatest common factor of 924 and 660?
3. What is the least common multiple of 18, 24, and 420?
4. Express $\frac{8}{24}$, $\frac{24}{80}$, and $\frac{187}{289}$ in their smallest terms.
5. What mixed number is equal to $\frac{5}{21} + \frac{11}{12} + \frac{7}{15}$?
6. Add $28\frac{3}{10}$, $72\frac{5}{12}$, and $29\frac{15}{20}$.
7. What is $\frac{8}{15}$ less $\frac{7}{25}$?
8. What is $9\frac{7}{58}$ less $2\frac{1}{2}$?
9. Multiply 10 by $\frac{3}{12}$.
10. Multiply $\frac{1}{2}$ of $\frac{3}{4}$ by $\frac{3}{5}$.
11. What is the cost of $2\frac{1}{2}$ yards of silk at $\$2\frac{7}{8}$ a yard?
12. Divide $3\frac{3}{5}$ acres into 12 equal lots.
13. At $2 per pound, how much tea can be bought for $\$\frac{4}{5}$?
14. At $\$\frac{5}{12}$ per pound, how many pounds of coffee can be bought for $6?
15. Divide $\frac{2}{5}$ by $\frac{3}{8}$; $4\frac{2}{7}$ by $1\frac{1}{20}$.
16. What is the value of $\dfrac{4\frac{3}{4} + 6\frac{2}{5}}{\frac{2}{15} \text{ of } \frac{9}{10}}$?
17. After A had lost $\frac{2}{11}$ of his money, and B had gained a sum equal to $\frac{2}{11}$ of his, each had $1108.80; what had each at first?

☞ For Dictation Exercises upon this Review, see "Manual and Key," page 70; for Miscellaneous Exercises, see pages 71 – 72.

248. MISCELLANEOUS EXAMPLES.

1. What will $17\frac{1}{2}$ yards of cloth cost at $2.50 a yard?
Ans. $43.75.

2. At 40 cents a pound, what cost 4 firkins of butter, weighing respectively $25\frac{1}{2}$ pounds, 47 pounds, $33\frac{1}{4}$ pounds, and $35\frac{3}{8}$ pounds? *Ans.* $56.45.

3. If a firkin of butter weighs $43\frac{1}{2}$ pounds, what must I pay for $\frac{2}{3}$ of it at 38 cents a pound? *Ans.* $ 11.02.

4. If 10 barrels of pork weigh 1 ton, what part of a ton will $\frac{4}{7}$ of a barrel weigh?

5. A man who was offered $ 95 an acre for a farm of 137 acres, afterwards sold $48\frac{1}{2}$ acres of it at $ 125 an acre, and the remainder at $ 87 an acre; did he gain or lose by waiting, and how much? *Ans.* gained $ 747.

6. What is the value of a boat if $\frac{3}{7}$ of it is worth $ 42?

7. If a barrel of flour lasts a family of 4 persons $3\frac{1}{2}$ months, how long would it last if 3 members were added to the family? *Ans.* 2 months.

8. I bought 4 bunches of asparagus at $16\frac{2}{3}$ cents a bunch, and 5 pounds of beef at 33 cents a pound, and gave in payment $ 2.50; what change was due me?

9. What number is that of which if you take away $12\frac{1}{2}$, the remainder will be $15\frac{2}{7}$? *Ans.* $27\frac{11}{14}$.

10. What number is that to which if you add $21\frac{7}{11}$, the sum will be $36\frac{2}{3}$?

11. If a man walked from Portland to Chicago, a distance of 1000 miles, in $29\frac{1}{2}$ days, what was the average distance walked per day?

12. In how many days would a crew of 6 sailors consume a barrel of beef (200 pounds), if each sailor consumed $1\frac{7}{8}$ pounds a day?

13. The difference between $\frac{1}{2}$ of a certain number and $\frac{1}{4}$ of it is 13; what is the number? *Ans.* 52.

14. The difference between $\frac{5}{8}$ of a certain number and $\frac{3}{8}$ of it is $4\frac{1}{3}$; what is the number?

15. $\frac{1}{2}$ of $\frac{7}{8}$ of $3\frac{1}{2} \div (3 - \frac{2}{5})$ is equal to what number?

Ans. $\frac{245}{416}$.

16. What is the value of $\frac{1\frac{2}{3}}{2\frac{1}{3}} \times \frac{5}{3\frac{1}{3}} \times \frac{7}{5\frac{1}{3}} \times \frac{10\frac{2}{3}}{14}$? *Ans.* $1\frac{1}{14}$

17. Owning $\frac{1}{3}$ of a mill, I sold $\frac{1}{4}$ of my share for \$1525; what was the value of the whole mill at the same rate?

18. How many barrels of apples at \$$4\frac{1}{4}$ a barrel will pay for 12 tons of hay at \$$38\frac{1}{3}$ a ton? *Ans.* $108\frac{4}{17}$ barrels.

19. How many days will it take a boy to pay for a suit of clothes worth \$23 and a hat worth \$1.50, if he earns $87\frac{1}{2}$ cents a day?

20. If $\frac{1}{5}$ of my property is in real estate, $\frac{1}{4}$ in government bonds, $\frac{1}{3}$ in railroad stock, and the balance, \$728, in greenbacks, how much property have I in all? *Ans.* \$3,360.

21. I sold my cow for \$72, by which I gained $\frac{1}{7}$ of her cost; what did she cost? 63

22. A horse and carriage together are worth \$475; the horse is worth 5 $\frac{2}{5}$ as much as the carriage; what is the worth of the horse? of the carriage? \$$339\frac{2}{7}$. Carriage \$$135\frac{5}{7}$

23. A can mow a field in 2 days, and B in 5 days; what part of the field can each mow in 1 day? what part can both mow in 1 day? In how many days can both mow it working together? *Ans.* $1\frac{3}{7}$ days.

24. C can do a piece of work in 2 days, and D in 6 days; in what time can both do it? *Ans.* $1\frac{1}{2}$ days.

25. If Charles, James, and John can remove a pile of stones in 2 hours, and Charles and John can remove it in 4 hours, in how many hours can James remove it working alone? *Ans.* 4 hours.

26. If $\frac{3}{4}$ of a pound of beef cost 15 cents, what cost 6 pounds? $8\frac{7}{8}$ pounds? *Ans.* \$1.20; \$$1.77\frac{1}{2}$.

27. If $\frac{2}{3}$ of a box of oranges cost \$$4\frac{2}{5}$, what will $\frac{5}{6}$ of a box cost? *Ans.* \$$5\frac{1}{2}$.

28. When $3\frac{3}{4}$ dozen pens at 33 cents a dozen will pay for $2\frac{3}{4}$ pounds of figs, what are the figs worth a pound?

Ans. 45 cents.

29. If I receive 36 cents a yard for cloth by selling it at $\frac{6}{7}$ of its cost, what should I have received by selling it at $\frac{7}{8}$ of its cost?

30. If a quantity of hay lasts 14 horses $2\frac{2}{7}$ weeks, how long will it last 10 horses? *Ans.* $3\frac{1}{5}$ weeks.

31. By a pipe of a certain capacity a cistern can be emptied in $3\frac{7}{15}$ hours; in what time can it be emptied by a pipe of $\frac{7}{5}$ the capacity? *Ans.* $2\frac{10}{21}$.

32. If a man can do a piece of work in $46\frac{1}{4}$ days by working $8\frac{1}{2}$ hours a day, in what time can he do it by working $12\frac{3}{4}$ hours a day? *Ans.* $30\frac{5}{6}$ days.

33. If $7\frac{5}{8}$ tons of hay cost $213.50, what will $5\frac{2}{3}$ tons cost when the price is $\frac{3}{4}$ as much?

34. To what must you add the difference between $13\frac{2}{3}$ and $36\frac{3}{4}$ that the amount may be $43\frac{5}{8}$? 20 $\frac{27}{36}$

35. I paid this year for my coal $8.50 a ton, which was $\frac{5}{6}$ as much as I paid last year; what I paid last year was $\frac{6}{7}$ of what I paid the year before; what did I pay the year before?

36. I sold my house and farm of $62\frac{1}{2}$ acres for $7000, the house being worth $\frac{1}{3}$ as much as the farm; what did I receive per acre for the farm?

EXAMPLES FOR ADVANCED PUPILS.

37. I bought a sack of coffee containing 144 pounds at $33\frac{1}{3}$ cents a pound; I paid $2\frac{1}{2}$ cents a pound for roasting it; allowing for a loss of $\frac{1}{32}$ in weight by roasting, at what price per pound must I sell it to gain a sum equal to $\frac{1}{4}$ of what I gave? *Ans.* $46\frac{66}{279}$ cents.

38. A merchant sold a lot of sugar for $1500, and thereby gained a sum equal to $\frac{1}{2}$ of the cost; if he had sold the sugar for $1200, would he have gained or lost, and what part of the cost?

39. A owes a sum equal to $\frac{2}{3}$ of his income for a year; by saving $\frac{5}{18}$ of his income annually for 5 years, he can pay his debt and have $1300 left; what is his yearly income?

DECIMALS.

249. If a unit, as an apple, is divided into 10 equal parts, each of the parts is $\frac{1}{10}$ of the unit.

If $\frac{1}{10}$ of a unit is divided into 10 equal parts, each of the parts is $\frac{1}{100}$ of the unit.

If $\frac{1}{100}$ of a unit is divided into 10 equal parts, each of the parts is $\frac{1}{1000}$ of the unit.

Continuing these divisions, we have a series of fractions whose denominators are 10 or a number made by taking 10's only as factors.

Such fractions are **decimal fractions.**

— *Define a decimal fraction.*

NOTE I. — All fractions except decimal fractions are called **common fractions.**

NOTE II. — Decimal fractions are generally called **decimals.**

250. It will be seen by Art. 249 that a series of decimal fractions constitutes a *scale of tens.*

Now as each of a series of integral units, as 1 hundred, 1 ten, and 1 unit, is expressed by writing the figure 1 in its appropriate place, thus, 100, 10, 1:

So each of this series of fractional units may be expressed by writing the figure 1 in its appropriate place; thus, to express

$\frac{1}{10}$	write	.1	$\frac{1}{10000}$	write	.0001
$\frac{1}{100}$	"	.01	$\frac{1}{100000}$	"	.00001
$\frac{1}{1000}$	"	.001	$\frac{1}{1000000}$	"	.000001

Also,	to	express	$\frac{3}{10}$	write	.3
"	"	"	$\frac{7}{100}$	"	.07
"	"	"	$\frac{7}{1000}$	"	.007

NOTE. — We call the dot which is placed at the left of the decimal expression a **decimal point.**

251. The manner of expressing decimals and integral numbers together is illustrated in the following

TABLE.

	Integral Numbers.							Fractional Numbers.									
Names of Units.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	*Decimal Point.*	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.	Hundred-millionths.	Billionths.
Expression.	7	6	8	3	1	2	.	7	2	4	6	9	8	1	3	5
No. of Place.	6th	5th	4th	3d	2d	1st		1st	2d	3d	4th	5th	6th	7th	8th	9th

EXERCISES UPON THE TABLE.

252. Name the fractional units in order from tenths to ten-millionths ; from ten-millionths to tenths.

253. In which place at the right of the decimal point are expressed

1. tenths ?
2. hundredths ?
3. ten-thousandths ?
4. thousandths ?
5. millionths ?
6. ten-millionths ?

254. Express in figures

7. 1 hundredth.
8. 7 ten-thousandths.
9. 3 thousandths.
10. 8 millionths.
11. 4 units with 1 tenth.
12. 1 unit with 4 hundredths.
13. 14 units with $\frac{3}{10}$, $\frac{7}{100}$, and $\frac{5}{1000}$.
14. 5 hundred with $\frac{6}{10}$, $\frac{9}{1000}$, and $\frac{1}{10000}$.
15. 40 units with $\frac{4}{100}$, $\frac{5}{1000}$, and $\frac{7}{100000}$.

255. Express .0111 as common fractions, and show what fractional number they equal.

In .0111 there are $\frac{1}{100}$, $\frac{1}{1000}$, and $\frac{1}{10000}$, which, expressed in ten-thousandths, the lowest order of units given, equal $\frac{100}{10000}$, $\frac{10}{10000}$, and $\frac{1}{10000}$; these united equal $\frac{111}{10000}$.

From the above we may derive the following

RULE. — For expressing decimals as common fractions.

1. *Strike off the decimal point and any zeros at the left of the expression of the decimals.*

2. *Underneath the result express the denominator.*

256. If it is required to read the number expressed by a collection of figures, as

.0 4 9 6 8 ,

we first numerate and point the collection as if it expressed an integral number; thus,

. 0 4 , 9 6 8 ;

then, pointing to the figures 0, 4, 9, 6, 8, in succession, we numerate from left to right tenths, hundredths, etc., to find the name of the lowest order of units expressed, which is *hundred-thousandths*.

The number expressed above is then read, "four thousand nine hundred sixty-eight *hundred-thousandths* ($\frac{4968}{100000}$).

From the above we derive the following

RULE. — To read numbers expressed as decimals: *Read the number as if it were an integral number, giving to it the name of the lowest order of units in the number.*

EXERCISES.

257. Read the following; also express the decimals as common fractions.

1.	.011	6.	.0007006
2.	.0527	7.	1.11 (See Note I.)
3.	.000111	8.	408.408
4.	.020768	9.	4000.0004 } (See Note II.)
5.	.0000768	10.	.4004 }

NOTE I. — 1.11 may be expressed as a mixed or as a fractional number, thus, $1\frac{11}{100}$ or $\frac{111}{100}$.

NOTE II. — To distinguish 4000.0004 from .4004 employ the word "units" after 4000 in reading the former expression. Employ the word "units" in all similar cases.

Read the following:

11.	300.003	14.	425000.000341
12.	.303	15.	1300.00005
13.	1200.0056	16.	.00064782

258. If it is required to express a number, as one hundred seventy-five ten-thousandths, in figures, we first express the number one hundred seventy-five as we express an integral number, thus,

1 7 5

As the lowest order of units will be expressed by the place of the right-hand figure 5, pointing to the figures 5, 7, and 1 in succession, we numerate from the right towards the left, thus, "ten-thousandths, thousandths, hundredths, tenths." We then supply the place of tenths with a zero, and prefix a decimal point to the expression, thus,

. 0 1 7 5

From the above may be derived the following

RULE. — To express decimals in figures:

1. *Express the number as an integral number is expressed.*

2. *Numerating from right to left, determine and fix the place of the decimal point, supplying vacant places with zeros.*

EXERCISES.

259. Express the following in figures: —

17. Thirteen *hundredths.*
18. Twenty-eight *thousandths.*
19. One hundred eleven *ten-thousandths.*
20. 4 thousand 208 *ten-thousandths.* *Ans.* .4208.
21. 27 thousand 62 *hundred-thousandths.* *Ans.* .27062.
22. 768 thousand 704 *millionths.*
23. 90 thousand 962 *ten-millionths.* *Ans.* .0090962.
24. 800 thousand 806 *hundred-millionths.*
25. 76 million 384 thousand 367 *billionths.*
26. 78 with $\frac{25}{100}$.

27. 460 with $\frac{273}{10000}$.
28. 3 hundred units with $\frac{5}{1000}$.
29. 305 thousand with $\frac{1184}{10000}$.
30. 467 thousand units with $\frac{842}{1000000}$.
31. 9 thousand 7 hundred units with $\frac{88}{1000000}$.
32. 600 thousand with $83\frac{1}{3}$ *thousandths*.

ADDITION.

260. ILL. EX. Add .7, .28, .035, and 1.111.

OPERATION.

$$\begin{array}{r} .700 \\ .280 \\ .035 \\ 1.111 \\ \hline 2.126 \end{array}$$

Explanation. — Since some of these numbers are already expressed as thousandths, one thousand is the least common denominator which can be used for expressing the numbers decimally.

If to the expression .7 two zeros be annexed, and to the expression .28 one zero be annexed, all the numbers will be expressed as thousandths. They may then be added, and the result will be 2.126.

NOTE I. — In practice the zeros may be omitted.

NOTE II. — Instead of changing the decimals in the illustrative example to fractional numbers having the least common denominator and adding the numerators, we might add the units of each order separately, commencing with the lowest and proceeding to the highest.

EXAMPLES.

1. .11 + .8 + 15.7 = what number? *Ans.* 16.61.
2. 4.111 + .09 + 5.2 = what number? *Ans.* 9.401.
3. 11.375 + .0005 + .87 = what? *Ans.* 12.2455.
4. Add .75, with 4.8 and .89. *Ans.* 6.44.
5. Add 8.625, with 17.6 and .56. " 26.786
6. Add 46.7 with 5.0625 and .875. " 52.6375
7. Add 4.68 with 94 *thousandths*, 13 and 8762 *ten-thousandths*, and 92984 *hundred-thousandths*. *Ans.* 19.58004.
8. Add $9\frac{8}{10}$, $8\frac{33}{100}$, $\frac{94}{1000}$, and $\frac{1875}{10000}$. " 18.4115
9. Add $498\frac{762}{1000}$, $67\frac{706}{1000}$, $5\frac{2497}{10000}$, and $5\frac{4887}{1000000}$.

261. Ill. Ex. Add 3 dollars 7 dimes, with 5 dimes 8 cents, and 15 cents 8 mills.

OPERATION.

$$\begin{array}{r} \$\,3.7 \\ .58 \\ \underline{.158} \\ \$\,4.438 \end{array}$$

Explanation. — It will be seen that the division of United States money (Art. 116) into dimes, cents, and mills corresponds with the division of units decimally into tenths, hundredths, and thousandths.

The unit of United States money is 1 dollar; the lower orders of units are parts of the dollar.

Examples.

10. Add 4 dollars 50 cents, with 3 dollars 6 dimes, and 10 dollars 17 cents 8 mills. *Ans.* \$ 18.278.

11. A bookseller sold a Bible for 3 dollars 62 cents 5 mills, a hymn-book for 87 cents 5 mills, a blank-book for 75 cents, and a quire of paper for 6 dimes 8 cents; what did he receive for all the articles?

262. Ill. Ex. Change $\frac{4}{5}$, $\frac{3}{4}$, and $\frac{5}{8}$ to decimals, and add them.

OPERATION.

$$\begin{array}{lcr} \frac{4}{5} = \frac{8}{10} & = & .8 \\ \frac{3}{4} = \frac{75}{100} & = & .75 \\ \frac{5}{8} = & & \\ 8\,)\,\underline{5.000} & & \\ \quad .625 & = & \underline{.625} \\ & & 2.175 \end{array}$$

Explanation. $\frac{4}{5}$ may be expressed as 10ths, thus, .8. $\frac{3}{4}$ cannot be expressed wholly as 10ths, but it may be expressed as 100ths, thus, .75. $\frac{5}{8}$ cannot be expressed wholly as 10ths or as 100ths; if we divide each of the $\frac{5}{8}$ into ten equal parts, we shall have $\frac{50}{8}$ tenths, which equals .6, with $\frac{2}{8}$ tenths remaining.

If we divide $\frac{2}{8}$ tenths into ten equal parts, we shall have $\frac{20}{8}$ hundredths, which equals .02, with $\frac{4}{8}$ hundredths remaining.

If we divide $\frac{4}{8}$ hundredths into 10 equal parts, we shall have $\frac{40}{8}$ thousandths, which equals .005: hence $\frac{5}{8}$ equals .625.

Adding the decimals, we have for their sum 2.175.

263. From the manner of changing $\frac{5}{8}$ to a decimal, we derive the following

RULE. — **For expressing common fractions as decimals:**

1. *Annex one or more zeros to the expression for the numerator, and divide the number thus expressed by the denominator; continue annexing zeros and dividing until there is no remainder, or until the desired order of units is reached.*

2. *Point off a number of decimal places equal to the number of zeros annexed.*

NOTE. — *When there are not as many places as there are zeros annexed, zeros must be prefixed to the decimal expression.*

264. EXAMPLES.

Express decimally and add the following: —

12. $\frac{1}{4} + \frac{1}{8} + \frac{3}{8}$. *Ans.* .75.

13. $\frac{2}{5} + \frac{13}{20} + \frac{5}{16}$. *Ans.* 1.3625.

14. $\frac{3}{20} + \frac{1}{20} + \frac{7}{25}$. *Ans.* .48.

15. $\frac{21}{40} + \frac{3}{80} + \frac{1}{16}$. *Ans.* .625.

16. What is the sum of \$ $35\frac{1}{4}$, \$ $82\frac{7}{100}$, and \$ $65\frac{7}{8}$?

17. What is the sum of $.5\frac{1}{4}$, $.18\frac{3}{4}$, and $.56\frac{4}{5}$?

265. ILL. EX. A painter bought canvas for three pictures; one contained $1\frac{7}{16}$ yards, another $2\frac{2}{3}$ yards, and the other $3\frac{5}{11}$ yards; how many yards did all contain?

OPERATION.

$$
\begin{aligned}
1\tfrac{7}{16} &= 1.4375 \\
2\tfrac{2}{3} &= 2.6666+ \\
3\tfrac{5}{11} &= \underline{3.4545+} \\
&\ 7.5586+
\end{aligned}
$$

Remarks. — I. $\frac{7}{16}$ may be wholly expressed decimally; but $\frac{2}{3}$ and $\frac{5}{11}$ cannot be thus expressed; however far the division is carried, there will still be a remainder. The order of units to which the decimals should be carried in such cases must depend upon the degree of accuracy required. In examples in this book they will end with ten thousandths, when no other direction is given.

II. When a fractional number is not fully expressed decimally, the sign + may be annexed to the expression, to show that something more is required to express the entire number.

III. In expressing $\frac{2}{3}$ decimally, the 6 repeats again and again; so in expressing $\frac{5}{11}$ decimally, the figures 4 and 5 repeat again and again. The fractional numbers thus expressed are called **repeating** or **circulating fractions**. (See "Key and Manual.")

266. EXAMPLES.

18. A merchant sold four lots of paper, containing severally $18\frac{3}{4}$, $16\frac{2}{3}$, $67\frac{1}{9}$, and $41\frac{3}{7}$ pounds; what did all together weigh? *Ans.* 143.9562+ pounds.

19. How many acres of land in four lots, the first containing $9\frac{7}{80}$ acres, the second $26\frac{5}{11}$ acres, the third $7\frac{1}{15}$ acres, and the fourth $104\frac{5}{6}$ acres? *Ans.* 147.4419 acres +.

20. Add together $5\frac{4}{9}$ hours, $\frac{1}{15}$ of an hour, $9\frac{7}{13}$ hours, and $18\frac{1}{120}$ hours.

21. Add together $.5\frac{3}{8}$, $\frac{14}{25}$, $\frac{9}{17}$, $\frac{23}{37}$, and $.02\frac{1}{4}$. *Ans.* 2.2710+.

☞ For Dictation Exercises, see "Manual and Key," page 74.

SUBTRACTION.

267. ILL. EX. I. If having .9 of a pine-apple, I give away .7 of the pine-apple, what part will remain? *Ans.* .2.

ILL. EX. II. If having .75 of an acre of land, I sell a part equal to .1875 of an acre, what part of an acre will remain?

OPERATION.

Minuend.	.7500
Subtrahend.	.1875
Remainder.	.5625

Explanation. — If to .75 two zeros be annexed, the minuend and subtrahend will both be expressed as fractional numbers having the least common denominator, ten thousand.

Taking away the part .1875, the remainder will be .5625.

NOTE I. — In practice the zeros may be omitted.

NOTE II. — Instead of changing the decimals in the illustrative example to like fractions having the least common denominator and then subtracting, we might perform the subtraction with the units of each order separately, commencing with the lowest and proceeding to the highest.

268. EXAMPLES.

Perform the following: —

	(1.)	(2.)	(3.)
Minuend.	.74	.361	4.07
Subtrahend.	.087	.1735	3.6666+
Remainder.	.653	.1875	.4034—

4. 63.804 less 38.285 = ?
5. 48.327 less .00876 = ?
6. 2000.7 less 2.0007 = ?
7. 82 less 37.639 = ?
8. 7.5 less 4.9307 = ?
9. 16 less .15009 = ?

10. Owing $475, I paid $218.75; what remained due?

11. The income of a man was $1000 for a year, and his expenses were $863.94; how much did he save?

12. Express $\frac{3}{4}$ and $\frac{5}{8}$ decimally, and find their difference. *Ans.* .125.

13. Express $\frac{3}{16}$ and $\frac{2}{5}$ decimally, and find their difference.

14. Owning 240.625 cords of wood, I sold $162\frac{5}{32}$ cords; how much remained?

☞ For Dictation Exercises, see "Manual and Key," page 75.

MULTIPLICATION.

269. CASE I. When the multiplier is an integral number.

ILL. EX. What will 3 pounds of sugar cost at $.16 a pound?

OPERATION.

Multiplicand.	$.16
Multiplier.	3
Product.	$.48

Explanation. — Three pounds, at $.16 per pound, will cost three $.16, which equals $.48. *Ans.* $.48.

EXAMPLES.

1. What will 5 fans cost at $.25 each?
2. At $.375 each, what will 17 melons cost?
3. What will 50 dozen combs cost at $1.08 per dozen?

270. CASE II. When the multiplier is a fractional number.

ILL. EX. I. At $ 9 a barrel for flour, what will .7 of a barrel of flour cost?

OPERATION.	
$ .9 .7 $ 6.3	*Explanation.* — The multiplier .7 denotes that 7 tenths of the multiplicand $ 9 are to be taken. 1 tenth of 9 equals 9 tenths (Art. 233), (expressed by placing the decimal point at the left of 9), and 7 tenths of 9 equal .9 × 7. *Ans.* $ 6.3 or $ 6.30.

4. What will .6 of a barrel of flour cost at $ 8 a barrel? *Ans.* $ 4.8.

5. At $ 75 per acre, what cost .67 of an acre of land?

6. If in 1 ton there are 2000 pounds, how many pounds are there in .375 tons? *Ans.* 750 pounds.

271. ILL. EX. II. At $ .28 per yard for cloth, what will .6 of a yard cost?

OPERATION.	
.028 .6 .168	*Explanation.* — The multiplier .6 denotes that 6 tenths of the multiplicand, $ .28, are to be taken. 1 tenth of 28 hundredths equals 28 thousandths, (expressed by moving the decimal point one place farther to the left), and 6 tenths of 28 hundredths equal .028 × 6. *Ans.* $ .168.

7. What cost .3 yard of silk at $ .6 per yard? *Ans.* $ .18.

8. If I franc equals $ .186 what will 18.5 francs equal?

272. From the preceding cases, noticing that there are as many decimal places in the expression for the product as there are in the expressions for the multiplicand and multiplier counted together, we derive the following

RULE. — For multiplying decimals: *Multiply as in integral numbers, and point off as many places for decimals in the expression for the product as there are decimal places in the expressions for the multiplicand and multiplier counted together; supply the deficiency of places, if there is any, by prefixing zeros before pointing.*

EXAMPLES.

9. $.27 \times .9 = ?$
10. $3.29 \times .62 = ?$
11. $.586 \times 1.065 = ?$

Find sum of last three answers. *Ans.* 2.90689.

12. $5.07 \times .0078 = ?$
13. $1001 \times .1001 = ?$
14. $.001 \times 1000 = ?$

Find sum of last three answers. *Ans.* 101.239646.

15. 5.5 yards equal 1 rod; how many yards are there in .75 of a rod?

16. What is the product if .0756 be a multiplicand and .076 a multiplier?

17. At $ 15.875 per M., what cost 8.2 M.'s of bricks?

18. What cost 227.6 acres of land at $ 87.50 per acre?

19. What cost 1.25 tons of hay at $ 17.50 a ton?

20. A farmer sells 75 pounds of butter at $ .375 a pound; he receives in payment 2 barrels of flour at $ 8.56 a barrel, and 5 yards of cloth at $ 1.80 a yard; what was the balance due, and to whom?

21. Multiply 376 by .1, .01, .001, .00001, and add the products. *Ans.* 41.73976.

How do you express the product when the multiplier is .1, .01, .001, etc.?

22. Multiply 268.5 by .01, .0001, .000001, .1, and add the products.

☞ For Dictation Exercises, see "Manual and Key," pages 75 – 77.

DIVISION.

273. CASE I. When the divisor is an integral number.

ILL. EX. I. If .8 of a melon is divided equally between 2 boys, what part of a melon will each boy receive?

OPERATION.

$$2\,)\,\underline{.8}$$
$$.4$$

Explanation. — If .8 of a melon is divided equally between 2 boys, each boy will receive 1 half of .8 of a melon, or .4 of a melon. *Ans.* .4 melon.

ILL. EX. II. How many 2's are there in .18?

OPERATION.

$$\begin{array}{r} 2\ )\ .18 \\ \text{1 tenth of 2} = .2 \quad \underline{\qquad} \\ \text{1 hundredth of 2} = .02 \quad .09 \end{array}$$

Explanation. — As 2 is a greater number than .18, it is not contained in it. We will see, then, how many tenths of 2 are contained in .18; 1 tenth of 2 is .2. There are no tenths of 2 in .18; this we express by putting a zero in the tenth's place of the quotient.

We will see next how many hundredths of 2 are contained in .18; 1 hundredth of 2 is .02. There are 9 hundredths of 2 in .18; this we express by putting a figure 9 in the hundredth's place of the quotient.

We see, by the above examples, that when the divisor is an integral number, *the expression for the quotient contains as many decimal places as there are decimal places in the expression for the dividend.*

EXAMPLES.

1. If 45 men share equally in .315 of a gold-mine, what part of the mine will each man have? *Ans.* .007.

2. If a dividend is 2.727 tons, and the divisor is 27 tons, what is the quotient? *Ans.* .101.

274. ILL. EX. III. A piece of land which contained 14.8 acres was divided into 13 equal building-lots; how many acres did each lot contain?

OPERATION.

$$\begin{array}{r} 13\ )\ \underline{14.8000} \\ 1.1384+ \end{array}$$

Explanation. — In this example zeros may be annexed to the expression for the dividend, and the division carried as far as desired; in pointing off places for decimals in the expression for the quotient, the zeros annexed should be considered a part of the expression for the dividend.

EXAMPLES.

3. At $ 7 a barrel for flour, how much flour can be bought for $ 43.9? *Ans.* 6.2714+ barrels.

4. If 768.37 rods of fencing are required to fence a square lot, how many rods will be required to fence each side?

275. CASE II. When the divisor is a fractional number.

ILL. EX. How many .03's are there in 1.326 ?

OPERATION.

‸03.) 1‸32.6
44.2

Explanation. — Here the divisor is a number of hundredths. The dividend expressed in hundredths is 132.6 hundredths.

There are as many 3 hundredths in 132.6 hundredths as there are 3's in 132.6. We therefore move the decimal point in the expression for the divisor to the right of the expression, and move the decimal point in the expression for the dividend as many places to the right, and then divide, pointing off as many decimal places in the expression for the quotient as there are decimal places in the altered expression for the dividend.

EXAMPLES.

5. If a man walks at the rate of a mile in .4 of an hour, how far will he walk in 3.432 hours ? *Ans.* 8.58 miles.

6. At $ .0625 a paper, how many papers of pins can be bought for $ .625 ? *Ans.* 10 papers.

7. If .001 is a dividend and 2.5 a divisor, what is the quotient ? *Ans.* .0004.

8. How many handkerchiefs at $ .25 each can be bought for $ 18 ? *Ans.* 72 handkerchiefs.

276. From the two cases illustrated above we derive the following

RULE FOR DIVISION OF DECIMALS. — I. When the divisor is an integral number, *divide as integral numbers are divided, and point off as many decimal places in the expression for the quotient as there are decimal places in the expression for the dividend.*

II. When the divisor is a fractional number, *move the decimal point in the expression for the divisor to the right of the expression, and move the decimal point in the expression for the dividend as many places to the right, and then divide. Point off as many decimal places in the expression for the quotient as there are decimal places in the altered expression for the dividend.*

277. EXAMPLES.

9. $2.196 \div 8 = ?$ *Ans.* .2745.

10. $.0279 \div 31 = ?$ *Ans.* .0009.

11. $8.025 \div .25 = ?$ *Ans.* 32.1.

12. $.0098 \div .014 = ?$

13. $3.879 \div .009 = ?$

14. $.576 \div .06 = ?$

Find sum of last three answers. *Ans.* 441.3.

15. $.0098 \div 200 = ?$

16. $.00001 \div 100 = ?$

17. $3 \div 4.8 = ?$

18. $.069 \div 7.5 = ?$

Find sum of last four answers. *Ans.* .6342491.

19. $38.702 \div 1.98 = ?$

20. $4.48 \div 52.1 = ?$

21. $307.28 \div 1.008 = ?$

Find sum of last three answers. *Ans.* 324.4735+

22. Divide 7.49 by .1, .001, .01, and add the quotients. *Ans.* 8,313.9.

How do you express the quotient when the divisor is .1, .01, .001, etc.?

23. $30872 \div 1.79 = ?$

24. $.30872 \div .179 = ?$

25. $30.872 \div .0179 = ?$

26. $308.72 \div 17.9 = ?$

Find sum of last four answers. *Ans.* 18,990.5915+

27. How many feet of boards at \$.16$\frac{2}{3}$ per foot can be bought for \$70.38? (Change .16$\frac{2}{3}$ and 70.38 to thirds.) *Ans.* 422.28 ft.

28. At \$8.33$\frac{1}{3}$ a cord, how much wood can be bought for \$13.20? *Ans.* 1.584 cords.

29. If \$1 on interest for 1 year gains \$.073, how much must be kept on interest for the same time to gain \$42.34?

30. I paid \$.375 for 1 pound of butter; how much could be bought for \$2.1? for \$.50?

31. In 1 rod there are 5.5 yards; how many rods are there in 27.225 yards?

32. If the 5-cent piece of 1866 is .02 of a meter in diameter, how many 5-cent pieces placed in a straight line will extend 100 meters?

☞ For Dictation Exercises, see "Manual and Key," pages 77, 78.

278. Topical Review in Decimal Fractions.

The pupil may illustrate the following topics to his class, giving definitions and deriving rules:—

1. Decimal fractions. (Art. 249.)
2. Expressing the orders of units, with the Table. (Arts. 250, 251.)
3. Expressing decimals as common fractions. (Art. 255.)
4. Reading decimals. (Art. 256.)
5. Expressing decimals in figures. (Art. 258.)
6. Addition of decimals. (Art. 260.)
7. United States money. (Art. 261.)
8. Changing common fractions to decimals. (Arts. 262, 263.)
9. Repeating or circulating fractions. (Art. 265.)
10. Subtraction of decimals. (Art. 267.)
11. Multiplication of decimals. Cases I., II., with general rule. (Arts. 269 – 272.)
12. Division of decimals. Cases I., II., with general rule. (Arts. 273 – 276.)

279. General Review, No. 4.

1. Add $\frac{7}{10}$, $\frac{628}{1000}$, $43\frac{71}{10000}$, $\frac{422}{100000}$, and $7\frac{2}{11}$ together.
2. Change .0025, also $.043\frac{3}{5}$ to common fractions.
3. What is $.2\frac{3}{8}$ less .02229?
4. Multiply 3.25 by .0008.
5. What is the cost of 80 bricks at $ 2.20 per hundred?
6. Divide .072 by .18.
7. Divide $83\frac{1}{3}$ by .2.
8. Divide .87 by $1.06\frac{2}{11}$.
9. At $ 12.50 per thousand feet, how many thousand feet of boards can be bought for $ 1725?
10. Divide 32.22 by 10; divide the quotient by 100; multiply this quotient by 10; multiply this product by 1000; multiply this product by .1; and add the five results.

Ans. 357.99642.

☞ For Dictation Exercises upon this Review, see "Manual and Key," page 78; for Miscellaneous Exercises, see page 79.

280. Miscellaneous Examples.

1. What will 2.4 barrels of flour cost at \$12.87½ per barrel, 5⅞ pounds of beef at \$.25 per pound, and 4.5 pounds of tea at \$1.56 per pound? *Ans.* \$39.38⅞.

2. What will a person have left of \$20 who buys 6 volumes of poems at \$1.25 per volume, and 3¼ quires of paper at \$1.62 per quire? *Ans.* \$7.23½.

3. What is the difference between 5000 and .005?

4. In how many hours can a man travel 40.475 miles, travelling at the rate of 5¾ miles an hour?

5. Divide .1 of .01 by .5; multiply that quotient by 100; multiply that product by .1 of 1000, and add the three results. *Ans.* 20.202.

6. What cost 787 laths at \$.31¼ a hundred and 17360 feet of boards at \$18.5 a thousand?

281. Examples for Advanced Pupils.

7. Owning .1 of a cotton-mill, I sold .7½ of my share for \$1500; what was the value of .43¾ of the mill at the same rate?

8. If when cheese is worth \$14.5 per hundred pounds, I give 60 pounds for 1.33⅓ tons of coal, what are 9.2 tons of the coal worth?

9. The product of three numbers is $\frac{2.5}{4}$; two of the numbers are 8.03, and .06; what is the third?

10. I shipped to Havana 1500 barrels of flour, which was sold at \$12.18¾ per barrel, and received in payment 36000 pounds of sugar at \$.06¼ per pound; and the balance in coffee at \$.15 per pound; how much coffee did I receive?

11. Of my money .2 is in gold, .56 of the remainder in silver, and the balance, which is \$2200, in bank-notes; how much money have I in all?

12. .3 of .85 of 1.084 is .65 of what number?

13. Multiply 124 by 10; multiply that product by $\frac{1}{10}$ of .01; divide that by $\frac{1}{1000}$ of 10, add the results, and divide the amount by .003⅓.

COMPOUND DENOMINATE NUMBERS.

282. ILLUSTRATION.—If we wish to ascertain the length of anything, as a board, we apply to its length some known length, as 1 inch (——1 inch.——), and find how many such lengths it contains; if it contains 40 such lengths, we say that the board is 40 inches long. The board can thus be measured.

Anything that can be measured is **quantity.**

283. The unit which is applied to a quantity in measuring it is a **unit of measure.**

What is the unit of measure of 3 quarts? 2 hours?

Define quantity; unit of measure.

284. In the expression 1 inch, the unit is named by the word *inch.* A number whose units are named is a **denominate number.**

NOTE.—A number whose units are not named, as 5, is a *general number.*

Define denominate number; general number.

285. In the number 6 pounds, the units are all of one kind or denomination, pounds; such a number is a **simple number.**

286. The unit of measure, 1 inch, employed in measuring the board mentioned above, would be inconveniently small if employed as the unit of measure for great lengths. If a larger unit, as 1 ft. (12 inches), were employed in measuring the board, the length of the board would be expressed by a mixed number, $3\frac{1}{3}$ feet.

To avoid the inconvenience of operating with large numbers or with mixed numbers, we express a part of the length in feet and the rest in inches (3 ft. 4 in.), and thus have a number made up of unlike denominate numbers, used to express one kind of quantity. Such a number is a **compound number.**

Define a simple number; a compound number.

WEIGHT AND ITS MEASURES.

287. When we hold a ball in the hand, we perceive that it presses downward; if we release our hold upon the ball, it falls towards the ground.

The tendency of bodies to press downwards, or to fall to the ground, is **weight.**

The weight of a body is measured by the force that is required to keep it from falling to the ground.

288. To ascertain the weight of articles that do not require great accuracy in weighing, we employ the units *ton, pound,* and *ounce.* These are the units of

AVOIRDUPOIS WEIGHT.*

TABLE.

16 ounces (oz.)	= 1 pound,	marked	lb.
2000 pounds	= 1 ton,	"	T.

NOTE.—The English or *long ton* of 2240 pounds is sometimes used for weighing gross articles, as iron and coal at the mines, and is the ton used at the custom-houses for weighing English goods which are imported into this country.

The long ton is divided into 20 hundred weight (cwt.) of 112 lbs. each, and the hundred weight is divided into 4 quarters of 28 lbs. each.

EXAMPLES.

1. How many ounces in 1 pound? in 2 pounds?

2. In 4 pounds 6 ounces how many ounces?

Explanation.—As in 1 pound there are 16 ounces, in 4 pounds there are four 16's of ounces, which with 6 ounces equal 70 ounces.

3. In 10 pounds how many ounces?

* NOTE TO THE TEACHER.—The units of measure spoken of in this and the following tables should be shown to the pupils, and, when practicable, they should be required to use them in weighing and measuring.

4. In 4 tons how many pounds? 8000 lb

5. In 34 ounces how many pounds?

Explanation. — As 16 ounces equal 1 pound, in 34 ounces there are as many pounds as there are 16's in 34. There are two 16's in 34 and 2 remain; therefore, in 34 ounces there are 2 pounds and 2 ounces remain.

6. In 40 ounces how many pounds? 2 lb + 8 oz

7. How many tons in 3200 pounds? in 9000 pounds?

289. In the above examples, numbers expressing a given quantity have been changed to numbers whose units are larger or smaller without changing the quantity expressed; such a process is called **reduction.**

What is reduction?

290. To change a number to units of a lower denomination.

ILL. EX. In 2 T. 4 lb. 5 oz. how many ounces?

OPERATION.

```
2 T. 4 lb. 5 oz.
2000
--------
4004 lb.
  16
--------
24024
4004
--------
64069 oz.
```

Explanation. — In 1 ton there are 2000 pounds, in 2 tons there are two 2000 pounds (or 2000 twos of pounds, see Art. 78), which equal 4000 pounds; 4000 pounds with 4 pounds are 4004 pounds. In 1 pound there are 16 ounces; in 4004 pounds there are four thousand four 16's of ounces (or sixteen 4004's of ounces), which equal 64064 ounces; 64064 ounces with 5 ounces are 64069 ounces; therefore in 2 T. 4 lb. 5 oz. there are 64069 ounces.

From the above we derive the following

RULE. — To change a number to units of a lower denomination: — *Multiply the number of the highest denomination by the number which it takes of the next lower denomination to make one of that higher, and to the product add the given number of the next lower denomination. Multiply that sum in like manner, and thus proceed till the number is changed to the required denomination.*

EXAMPLES.

8. Change 5 T. 700 lb. to pounds. *Ans.* 10,700 lbs.

9. Change 3 T. 90 lb. 4 oz. to ounces. *Ans.* 97,444 oz.

10. How many ounces are there in 1 ton? in 1 long ton?

11. Change 3 long tons to pounds.

12. Change 2 long tons 5 cwt. 2 qr. to pounds.

291. To change a number to units of higher denominations.

ILL. EX. Change 84589 ounces to units of higher denominations.

OPERATION.

```
  16 ) 84589 oz.
2000 ) 5286 lb. + 13 oz.
       2 T. + 1286 lb.
Ans. 2 T. 1286 lb. 13 oz.
```

Explanation. — As 16 ounces equal 1 pound, in 84589 oz. there are as many pounds as there are 16's in 84589, or 5286 pounds, and 13 ounces remain.

As 2000 pounds equal 1 ton, in 5286 pounds there are as many tons as there are 2000's in 5286, or 2 tons and 1286 pounds remain; making the entire result 2 T. 1286 lb. 13 oz.

From the above we derive the following

RULE. — To change a number to units of higher denominations: — *Divide the given number by the number which it takes of its denomination to equal one of the next higher, and note the remainder. Divide, as before, the quotient thus obtained, and thus proceed till the required denomination is attained. The last quotient, with the several remainders, will be the required result.*

EXAMPLES.

Change to units of higher denominations: —

13. 7328 pounds. *Ans.* 3 T. 1328 lb.

14. 4032 ounces. *Ans.* 252 lb.

15. 10376 ounces.

16. In 4720 pounds how many tons and what remains?

17. In 38769 pounds how many *long* tons and what remains?

292. In weighing gold, silver, precious stones, and such other articles as require great accuracy in weighing, we employ the *pound, ounce, pennyweight,* and *grain.* These are the units of

Troy Weight.

TABLE.

24 grains (gr.)	= 1 pennyweight,	marked	pwt.
20 pennyweights	= 1 ounce,	"	oz.
12 ounces	= 1 pound,	"	lb.

Examples.

18. How many ounces in 12 lb. 5 oz.? *Ans.* 149 oz.

19. How many grains in 9 oz. 4 pwt.? *Ans.* 4,416 gr.

Change to numbers of higher denominations:—

20.	492 oz. *Ans.* 41 lbs.	23.	3218 pwt.
21.	1682 gr.	24.	7984 gr.
22.	3691 pwt.	25.	93621 gr.

26. A gold eagle weighs 258 grains; how many ounces will 5 eagles weigh, and how many grains remain?

Note.—In mixing medicine, apothecaries use Troy weight, but instead of using the pennyweight they divide the Troy ounce (marked ℥ or oz.) into 8 drams (ʒ or dr.) of 3 scruples (℈ or sc.) of 20 grains (gr.) each.

293. Comparison of Weights.

Troy.		Avoirdupois.
175 lb.	=	144 lb.
175 oz.	=	192 oz.
7000 gr.	=	1 lb.

Note.—We see by the above table of comparison, that one pound avoirdupois is heavier than a pound Troy, but that an ounce avoirdupois is lighter than an ounce Troy.

Examples.

27. How many pounds Troy in 1 pound avoirdupois? in 3 pounds? in $7\frac{1}{2}$ pounds? *First Ans.* $1\frac{31}{144}$ lb. Troy.

28. How many pounds avoirdupois in 8 pounds Troy? in $5\frac{2}{3}$ pounds? *First Ans.* $6\frac{102}{175}$ lb. Av.

29. Change 6 lb. 0 oz. 2 pwt. 18 gr. to avoirdupois weight.

NOTE. — Change to grains and divide by 7000. *Ans.* $4\frac{3313}{3500}$ lb.

30. Change 4 lb. 3 oz. 1 pwt. to avoirdupois weight.

31. Which is heavier, and how many grains heavier, a pound of feathers or a pound of gold?

EXTENSION AND ITS MEASURES.

294. If we examine a body, as a block of wood, we find that it has the property of occupying room or space. This property is **extension.**

295. We find, also, that the block extends in three directions, that of length, of breadth, and of thickness; such a body is a **solid body.**

296. The space that a solid body occupies is called a **solid.**

297. The outside of a solid body or of a solid is **surface.** A surface has *length and breadth.*

298. The boundary of a surface is a **line.** A line has *length.*

Measures of extension include measures of *length,* measures of *surface,* and measures of *solids.*

MEASURES OF LENGTH.

299. In measuring length we employ the *mile, rod, yard, foot, inch,* and *line.* These are the units of

LONG MEASURE.

TABLE.

12 lines (l.)	=	1 inch,	marked	in.
12 inches	=	1 foot,	"	ft.
3 feet	=	1 yard,	"	yd.
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet	=	1 rod,	"	r. or rd.
320 rods or 5280 feet	=	1 mile,	"	m.

$69\frac{1}{6}$ miles nearly = 1 degree (°) of longitude at the equator.
360 of which degrees = the distance round the earth.

ILL. EX. I. Change 4 m. 9 rd. 4 yd. to feet.

OPERATION.

4 m. 9 rd. 4 yd.
320
1289 rd.
$5\frac{1}{2}$
$6444\frac{1}{2}$
6445
$7093\frac{1}{2}$ yd.
3
$21,280\frac{1}{2}$ ft. *Ans.*

ILL. EX. II. Change 56500 in. to units of higher denominations.

OPERATION.

12) 56500
3) 4708 ft. . . . . . . + 4 in.
$5\frac{1}{2}$) 1569 yd. . . + 1 ft.
2   2
11) 3138
285 rd. + $1\frac{1}{2}$ yd.
$\frac{1}{2}$ yd. = 1 ft. 6 in.
Ans. 285 rd. 1 yd. 2 ft. 10 in.

EXAMPLES.

34. Change 7 m. 21 rd. to rods.

35. Change 9 m. 31 rd. 2 yd. to yards. *Ans.* $16,012\frac{1}{2}$ yds.

36. Change 54762 in. to units of higher denominations.

37. What cost a mile of horse-railroad at $ 1.33 a foot?

38. How many miles through the earth from pole to pole, the distance being 41704788 feet? *Ans.* 7,898 m. 3,348 ft.

SURVEYORS', MARINERS', AND CLOTH MEASURES.

300. Surveyors in measuring use a chain called *Gunter's chain,* which is 4 rods or 66 feet long. This chain is divided into 100 *links* of 7.92 inches each.

EXAMPLES.

39. How many inches equal 1 link? how many links 1 chain? how many chains 1 mile? *Last Ans.* 80 ch.

40. In a distance of 7 chains 13 links how many rods with how many feet and inches are there?
Ans. 28 rd. 8 ft. 6.96 in.

41. On measuring the length of a road, it was found to be 12 chains 87 links long; what was its length in rods with feet and inches?

301. Short distances, at sea, are measured by the *cable-length* of 720 feet.

Longer distances at sea are measured by the *nautical* or *geographical mile,* each mile being $\frac{1}{60}$ of a degree of latitude, and averaging 6086.34 feet or 1.15+ common miles.

3 nautical miles = 1 sea league.

EXAMPLES.

42. How many feet equal 1 cable-length? how many cable-lengths equal 1 common mile? *Last Ans.* $7\frac{1}{3}$ c. l.

43. In 8 cable-lengths, how many feet? *Ans.* 5,760 ft.

44. How many cable-lengths in 3789 feet?

45. Change 6821 feet to miles with cable-lengths, etc.

46. How many leagues in 784 nautical miles?

302. Some goods, as cloth, ribbons, etc., are measured by the length in yards, without regard to width. The yard is divided into *halves, quarters, eighths,* and *sixteenths.*

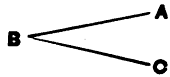
47. What cost $12\frac{1}{2}$ yards of cloth at \$.$37\frac{1}{2}$ a yard?

48. What cost $\frac{3}{16}$ of a yard of velvet at \$ 7.50 a yard?

☞ **For Dictation Exercises, see "Manual and Key," page 103.**

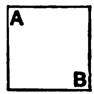
MEASURES OF SURFACE.

303. The difference in direction of two lines, as of A B and B C, is an **angle.**



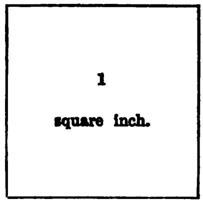
304. A surface that does not change its direction, as the surface of a slate, is a **plane surface.**

305. A plane surface, as in A B, that is bounded by four equal straight lines, and whose angles are all equal, is a **square.**



Define a plane surface; an angle; a square.

306. If each side of a square is 1 inch in length, the square contains 1 *square inch;* if each side is 1 foot in length, it contains 1 *square foot,* etc.



Extent of surface, or **area,** is measured by applying to it as a measuring unit some known square, as a square inch or a square foot.

307. To find the relative size of units of square measure.

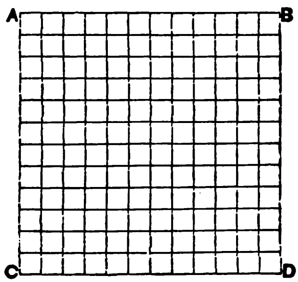


ILLUSTRATION.—Let the figure A B C D represent 1 square foot; each side of the figure will represent 12 inches of length. Dividing each side into 12 equal parts and drawing lines from the points of division, as in the figure, it will be seen that the square foot may be divided into 144 smaller squares, each of which is 1 inch long and 1 inch wide, and therefore contains 1 square inch; hence 1 square foot equals 144 square inches. In the same way the relative size of other units of surface may be found.

308. In measuring surface we employ the *square mile, acre, square rod, square foot,* and *square inch.* These are the units of

SQUARE MEASURE.

TABLE.

144 square inches (sq. in.)	= 1 square foot,	marked	sq. ft.
9 square feet	= 1 square yard,	"	sq. yd.
$30\frac{1}{4}$ square yards or $272\frac{1}{4}$ square feet	= 1 square rod,	"	sq. r.
160 square rods	= 1 acre,	"	A.
640 acres	= 1 square mile,	"	sq. m.

ILL. EX. Change 5376 sq. ft. to units of higher denominations.

OPERATION.

$$\begin{array}{rl} 9\,) & \underline{5\;3\;7\;6} \\ 30\tfrac{1}{4}\,) & \quad 5\;9\;7 \text{ sq. yd.} \;\; . \; . \; . \; . \; . \;\; + \; 3 \text{ sq. ft.} \\ \underline{4} & \underline{\qquad\quad 4} \\ 121\,) & \underline{2\;3\;8\;8} \\ & \quad 1\;9 \text{ sq. rd.} + 22\tfrac{1}{4} \text{ sq. yd.} \\ & \qquad\qquad\qquad \tfrac{1}{4} \text{ sq. yd.} = 2\tfrac{1}{4} \text{ sq. ft.} \\ \hline \textit{Ans.} & 19 \text{ sq. rd.} \quad 22 \text{ sq. yd.} \quad 5\tfrac{1}{4} \text{ sq. ft.} \end{array}$$

EXAMPLES.

49. How many feet in 70 sq. rds.? *Ans.* $19{,}057\frac{1}{2}$ sq. ft.

50. How many acres are there in a township of 36 square miles? How many quarter sections of 160 acres each?

NOTE.—Government lands are divided by parallels and meridians into townships of 36 sq. miles or sections each, and each section into quarter sections of 160 acres each. (See "Manual and Key," page 96.)

51. Change 5283 sq. ft. to units of higher denominations. *Ans.* 19 sq. rd. 12 sq. yd. $2\frac{1}{4}$ sq. ft.

52. Change 4287 sq. ft. to units of higher denominations.

53. Change 3792 sq. yds. to units of higher denominations.

54. What must I pay for 1 quarter section of land in Nebraska at $1.50 an acre? *Ans.* $240.

55. What must I pay for 1 acre 37 sq. yards of land at 6 cents a square foot?

309. A plane surface, as A B, bounded by four straight lines, and whose angles are all equal, is a **rectangle.**

310. To find the area of a rectangle.

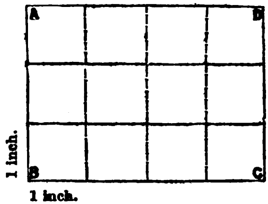


ILLUSTRATION. — Suppose the length of the rectangle A B C D to be 4 inches, and its breadth 3 inches. By dividing the lines A B and C D into 3 equal parts, and drawing lines from the points of division as in the figure, the rectangle will be divided into 3 smaller rectangles, each of which is 4 inches long and 1 inch wide, and which must therefore contain 4 square inches; if one of these rectangles contains 4 square inches, 3 will contain three 4's of square inches, or 12 square inches.

In the same way it can be shown that the number of square units in any rectangle or square *equals the number of units in the length multiplied by the number of like units in the breadth.*

EXAMPLES.

56. How many square inches are there in a sheet of paper 8 inches long and 5 inches wide?

57. How many square feet are there in a walk 20 feet long and $2\frac{1}{2}$ feet wide?

58. How many acres in a rectangular field that is 16 rods long and 15 rods wide?

311. In 1 square chain how many square rods? (See Art. 300.) 16

NOTE I. — As 1 chain equals 4 rods, 1 square chain equals 16 square rods, and 10 square chains equal 160 square rods, or 1 acre.

NOTE II. — In measuring land, surveyors take the dimensions in chains. From these dimensions they obtain the area in square chains, and divide the number by 10. The quotient equals the number of acres.

59. On surveying a rectangular piece of land, it was found to be 40 chains long and $12\frac{1}{2}$ chains wide; how many acres did it contain? *Ans.* 50 A.

312. ILL. EX. If a floor contains 72 square feet of surface, and its length is 9 feet, what is its breadth?

Explanation. — As the number of square feet in the floor equals the number of feet of length multiplied by the number of feet of breadth, it follows that if the number of square feet, 72, is divided by the number of feet of length, 9, the quotient will equal the number of feet of breadth; $72 \div 9 = 8$: therefore the room is 8 feet wide.

60. What must be the width of a walk that is 15 feet long to contain 130 square feet?

61. Mr. Lee bought 1 acre of land which bordered on the street 100 feet; how far back did it extend, the land being rectangular? *Ans.* $435\frac{3}{5}$ ft.

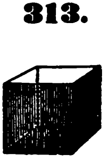
62. Which contains more, and how many square feet more, a piece of land 36 feet long and 28 feet wide, or a piece 2 rods square (that is, 2 rods long and 2 wide)?

63. What is the difference, in square rods, between a piece of land that contains 4 square rods and a piece that is 4 rods square?

☞ For Dictation Exercises, see "Manual and Key," page 104.

MEASURES OF SOLIDS.

313. A solid bounded by six equal squares is called a **Cube**. The squares are called the **faces** of the cube, and, together, make its **surface**. The bounding lines are called **edges**. If each of its edges is 1 inch long, the cube is 1 cubic inch; if 1 foot long, it is 1 cubic foot, etc.



314. A solid is measured by applying to it, as a measuring unit, some known cube, as a cubic inch, a cubic foot, etc.

315. To find the relative size of units of solid measure.

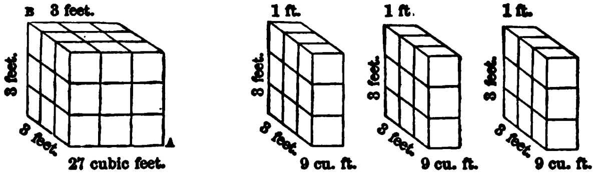


ILLUSTRATION. — Let A B represent 1 cubic yard; this cubic yard is 3 feet long, 3 feet high, and 3 feet thick. It may then be divided into three smaller solids, each of which is 3 feet long, 3 feet high, 1 foot thick, and which, as seen at the right, contains 9 cubic feet. If each of these 3 solids contains 9 cubic feet, the three solids composing the cubic yard contain three 9's of cubic feet, or 27 cubic feet. In the same way the relative size of other units of solid measure may be found.

316. In measuring solids we employ the *cubic yard, cubic foot,* and *cubic inch.* These are the units of

CUBIC MEASURE.

TABLE.

1728 cubic inches	=	1 cubic foot, marked cu. ft.
27 cubic feet	=	1 cubic yard, " cu. yd.

NOTE. — Light bulky freight is generally estimated in tons by the space it occupies, instead of its actual weight; but the number of feet assigned to a ton varies from less than 50 feet to 150 feet.

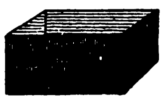
EXAMPLES.

64. In 4 cu. yd. 12 cu. ft. how many cubic feet?

65. In 32 cu. ft. 724 cu. in. how many cubic inches?

66. Change 86872 cu. in. to units of higher denominations. *Ans.* 1 cu. yd. 23 cu. ft. 472 cu. in.

67. Change 728987 cu. in. to cubic yards, etc.



317. A solid that is bounded by six rectangles is a **rectangular solid.**

318. To find the contents of a rectangular solid.

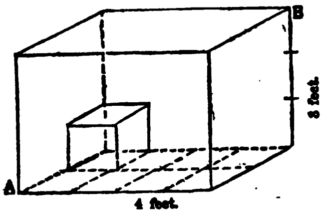


ILLUSTRATION.—Let A B represent a rectangular solid 4 feet long, 2 feet wide, and 3 feet high. If the solid is 4 feet long and 2 feet wide, its lower face or base must contain $4 \times 2 = 8$ square feet. If upon these square feet the solid extends 1 foot high, it will contain 8 cubic feet resting upon the base. But the solid is 3 feet high, and must, therefore, contain three 8's of cubic feet, or $4 \times 2 \times 3$ cubic feet $= 24$ cubic feet.

In the same way it may be shown that the number of cubic units in any rectangular solid *equals the product of the number of units in the length and breadth multiplied by the number of like units in the height.*

EXAMPLES.

68. How many cubic inches are there in a rectangular block of marble whose length is 18 inches, whose breadth is 12 inches, and height 8 inches? *Ans.* 1,728 cu. in.

69. How many packages, containing 24 cubic inches each, may be packed in a chest that measures, on the inside, 1 foot 10 inches in length, 10 inches in breadth, and 6 inches in height?

319. ILL. EX. What must be the length of a box inside which is 2 feet deep and $2\frac{1}{2}$ feet wide to contain 15 cubic feet?

Explanation.—As the number of units of cubic measure in any rectangular solid equals the product of the number of units in the length and breadth multiplied by the number of like units in the height, it follows that *if the number of units of cubic measure is divided by the product of the number of units in two of the dimensions, the quotient will equal the number of like units in the third dimension.*

$15 \div (2 \times 2\frac{1}{2}) = 3$: therefore the box must be 3 feet long.

70. What must be the depth of a bin that will contain 72 cubic feet, the length being 6 feet and breadth 4 feet (inside measure)? *Ans.* 3 ft.

71. How long must a bin be to contain 84 cubic feet, if its height is 4 feet and its breadth 2 feet (inside measure)?

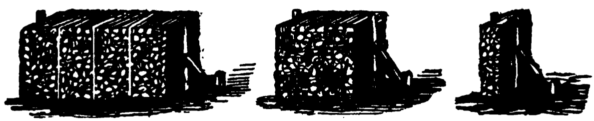
72. A cellar is to be dug 30 feet long, 20 feet wide, and 10 feet deep; to what depth must it be dug that 50 cubic yards of earth may be removed? *Ans.* $2\frac{1}{4}$ feet.

73. How many cubic yards will be removed when the entire cellar is dug? *Ans.* $222\frac{2}{9}$ cu. yd.

74. If a stick of timber contains 50 cubic feet, and its width is $1\frac{1}{3}$ feet, and thickness $1\frac{1}{4}$ feet, what is its length?

320. In measuring fire-wood and some other articles, we employ the *cord, cord foot,* and *cubic foot.* These are the units of

WOOD MEASURE.



1 cord. ½ cord. 1 cd. ft.

321. A pile of wood 4 feet wide, 4 feet high, and 8 feet long, contains 1 *cord;* a pile 4 feet wide, 4 feet high, and 1 foot long, contains 1 *cord foot.*

TABLE.

16 cubic feet	= 1 cord foot,	marked	cd. ft.
8 cord feet or 128 cubic feet	= 1 cord,	"	cd.

EXAMPLES. 4 cd. ft.

75. In 1 half-cord how many cord feet? how many cubic feet? 64 cu. ft.

76. How many cords of wood are contained in a pile 4 feet wide, 4 feet high, and 20 feet long? 72 feet long? 100 feet long? *Ans.* 2½ cd.; 9 cd.; 12½ cd.

77. What must be the length of a pile of wood 4 feet wide and 4 feet high to contain 12 cords? *Ans.* 96 ft.

78. What must be the length of a pile of wood 4 feet wide and 6 feet high to contain 10 cords?

79. How many cords in a pile of wood 40 feet long and 10 feet high, the wood being cut the usual length? How many cords, the wood being cut 4 inches shorter than the usual length? *Second Ans.* $11\frac{11}{24}$ cd.

☞ For Dictation Exercises, see "Manual and Key," page 104.

MEASURES OF CAPACITY.

322. To find the capacity of vessels for containing liquids, and in measuring liquids themselves, we employ the *gallon, quart, pint,* and *gill.* These are the units of

LIQUID MEASURE.

TABLE.

4 gills (gi.)	= 1 pint,	marked	pt.
2 pints	= 1 quart,	"	qt.
4 quarts	= 1 gallon,	"	gal.

NOTE I.—The capacity of casks is found by measurement, and they are marked accordingly. They are called *hogsheads* (hhds.), *pipes, butts,* and *tuns,* without much discrimination. In estimating the capacity of reservoirs, 31½ gal. = 1 barrel (bbl.); 63 gal. = 1 hhd.

NOTE II.—In mixing liquid medicines, apothecaries use the pint (O) divided into 16 fluid ounces (f ℥) of 8 fluid drachms (f ʒ) of 60 minims each.

NOTE III.—A pint of water weighs about a pound avoirdupois.

NOTE IV.—A minim is about ¼ of a drop.

EXAMPLES.

80. How many pint bottles will be required to contain 2 gal. 3 qt. 1 pt. of bitters? *Ans.* 23 bottles.

81. How many gills are there in 5 gal. 1 qt. 1 pt. 3 gi.?

82. Change 3872 gills to units of higher denominations.

83. What cost 18 gal. 2 qt. of milk at 7 cents a quart?

84. How many ounce-vials can be filled with a quart of tincture? *Ans.* 32 vials.

323. To find the capacity of vessels for holding dry articles, as corn, beans, etc., and to measure such articles, we use the *bushel, peck, quart, pint,* and *gill.* These are the units of

DRY MEASURE.

TABLE.

4 gills (gi.)	= 1 pint,	marked	pt.
2 pints	= 1 quart,	"	qt.
8 quarts	= 1 peck,	"	pk.
4 pecks	= 1 bushel,	"	bu.

EXAMPLES.

85. In 3 bu. 3 pk. of chestnuts how many quarts?

86. What cost 8 bu. 2 pk. 5 qt. of nuts, at $.10 a quart?

87. Change 39265 pints to units of higher denominations.

324. COMPARISON OF LIQUID AND DRY MEASURES.

Liquid Measure.		Cu. in.	Dry Measure.		Cu. in.
1 quart	=	$57\frac{3}{4}$	1 quart	=	$67\frac{1}{5}$
1 gallon	=	231	1 bushel	=	$2150\frac{2}{5}$

EXAMPLES.

88. I have a tin box that contains 1 cubic foot; how many quarts of currants will it hold? *Ans.* $25\frac{5}{7}$ qts.

89. How many quarts of water will it hold?

Ans. $29\frac{213}{231}$ qts.

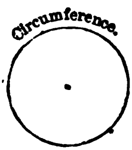
90. How many gallons of water will a cistern hold that is on the inside $4\frac{1}{2}$ feet long, 3 feet wide, and 3 feet deep?

91. How many bushels of potatoes can I put into a bin that is on the inside 7 ft. long, $2\frac{1}{2}$ ft. wide, and 2 ft. high?

CIRCULAR AND ANGULAR MEASURE.

325. A surface that does not change its direction, as the surface of a slate, is a **plane surface.**

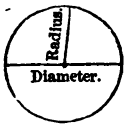
Fig. 1.



326. A plane surface, as in Fig. 1, bounded by a line, every part of which is equally distant from a point within, called the centre, is a **Circle.**

327. The bounding line is called the **Circumference** of the circle. Any part of the circumference is called an **Arc.**

Fig. 2.



328. A straight line passing from the centre of the circle to the circumference is called a **Radius** (plural, *radii*).

329. A straight line passing from any point in the circumference of a circle, through the centre, to an opposite point in the circumference, is called a **Diameter.**

Define plane surface; circle; circumference; arc; radius; diameter.

330. The circumference of a circle is divided into 360 equal parts called *degrees*, each degree into 60 *minutes*, and each minute into 60 *seconds*. These are the units of

CIRCULAR MEASURE.

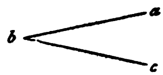
TABLE.

60 seconds (″)	=	1 minute,	marked	′.
60 minutes	=	1 degree,	"	°.
360 degrees	=	1 circumference,	"	circ.

331. Half a circumference is a **semi-circumference**, one fourth is a **quadrant**, and one sixth a **sextant.**

NOTE.—The Zodiac, the apparent path of the sun in the heavens, is divided into 12 equal parts called **signs**. Hence a sign equals 30 degrees.

Fig. 3.



332. *What is an angle?* (See Art. 303.) The point at which the lines forming an angle meet, as *b* in the annexed figure, is the **vertex** of the angle. The angle in the annexed figure may be read, "the angle *a b c*," or simply "the angle *b*."

Fig. 4.

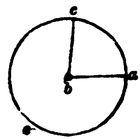
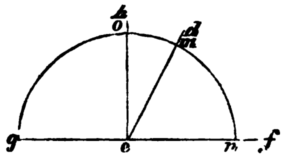


ILLUSTRATION.—Let *a c e* be a circle whose centre is at *b*. From the centre *b* draw the lines *b a* and *b c* to the circumference of the circle; the arc *a c* measures the angle *a b c*. The centre of a circle being placed at the vertex of any angle, the angle is measured by the arc included between the lines which include the angle.

Fig. 5.



In Fig. 5, what arc measures the angle *g e d*? *g e h*? *d e f*? If the arc *m n* contains 65°, what is the size of the angle *d e f*?

NOTE I.—An angle of 90°, or one that is measured by 1 fourth of the circumference of a circle, as *g e h*, is a **right angle**. The sides of a right angle, as *g e* and *e h* of the angle *g e h*, are said to be **perpendicular** to each other.

NOTE II.—As arcs are measurements of angles, the table for angular measure is the same as the table for circular measure.

EXAMPLES.

92. Change 18° 54′ 8″ to seconds. *Ans.* 68,048″.

93. Change 135° 0′ 16″ to seconds.

94. Change 6058 seconds to units of higher denominations.

95. Through how many degrees does the minute-hand of a clock move in 10 minutes of time?

TIME MEASURE.

333. In measuring time, we employ the *century, year, month, week, day, hour, minute,* and *second.* These are the units of

TIME MEASURE.

TABLE.

60 seconds (s.)	=	1 minute,	marked	m.
60 minutes	=	1 hour,	"	h.
24 hours	=	1 day,	"	d.
7 days	=	1 week,	"	w.
52 weeks 1 day, or 365 days	=	1 common year,	"	c. y.
366 days	=	1 leap year,	"	l. y.
100 years	=	1 century,	"	C.

334. A **day** is the length of time it takes the earth to turn upon its axis so as to bring a given meridian a second time under the sun.

A **year** is the length of time it takes the earth to revolve around the sun.

335. The earth revolves around the sun in 365 days 5 hours 48 minutes and 50 seconds nearly, but we call 365 days a year. It will thus be seen that what we call a year is nearly 6 hours less than the true year, and 4 such years nearly one day less than 4 true years. To rectify this error, 366 days are allowed to one year in every 4 years. The year of 366 days is called a **leap year**.

The addition of a day in every fourth year is too much by a number of minutes, which in one hundred years amounts to about three fourths of a day; to balance this error every one hundredth year for three hundred of every four hundred years is not a leap year.

Hence any year is a leap year, *when the number denoting the year is divisible by* 4 *and not by* 100, *and when it is divisible by* 400.

336. A year is divided into four **seasons**, of three calendar months each, and commences with January, the second winter month.

The *succession of the seasons, quarters*, and *months*, and the number of days in each month, are shown by the following diagram:—

EXAMPLES.

ILL. EX. I. In 5 y. 123 d. how many days?

OPERATION.

```
   5 y. 123 d.
 365
1825
 124
Ans. 1,949 days.   (See note.)
```

ILL. EX. II. In 52708 hours how many years, etc.?

OPERATION.

```
 24 ) 52708
365 )  2196 d + 4 hrs.
          6 y + 6 d.
                1 d.  (See note.)
                5 d.
Ans. 6 y. 5 d. 4 h.
```

NOTE.—As 1 year at least of every 4 years is a leap year, in changing years to days, for every 4 years changed, 1 day must be added to the number of days; and in changing days to years 1 day of the given number of days must be deducted for every 4 years obtained.

96. Change 9 y. 29 d. 2 h. to hours, allowing for 2 leap years. *Ans.* 79,586 hours.

97. Change 22 y. 122 d. to days, allowing for 5 leap years.

98. Change 12 y. 67 d. 5 h. to hours, allowing for 3 leap years.

99. Change 4167 days to numbers of higher denominations. (See note.) *Ans.* 11 y. 149 d.

337. ILL. EX. What is the time in years, months, and days from Nov. 12, 1869, to March 4, 1872?

NOTE I.—From Nov. 12, 1869, to Nov. 12, 1871, is 2 years; from Nov. 12, 1871, to Feb. 12, 1872, is 3 months; from Feb. 12 to Feb. 29 is 17 days; and from Feb. 29 to March 4 is 4 days. *Ans.* 2 y. 3 m. 21 d.

Thus to find the difference in time between two dates, *we first find the entire years between the two dates, then the entire calendar months remaining, and then count the remaining days.*

100. How many years, months, and days from May 10, 1865, to Jan. 1, 1868? *Ans.* 2 y. 7 m. 22 d.

101. How many years, months, and days from Oct. 27, 1870, to Dec. 5, 1871? *Ans.* 1 y. 1 m. 8 d.

102. From July 11, 1868, to April 19, 1870?

103. What time passed between the declaration of American Independence, July 4, 1776, and the signing of the treaty of peace, Sept. 3, 1783? *Ans.* 7 y. 1 m. 30 d.

104. How long from the declaration of American Independence to Jan. 1, 1864, when the Proclamation for the Emancipation of slaves took effect? *Ans.* 87 y. 5 m. 28 d.

105. Napoleon Bonaparte assumed the title of Emperor, May 18, 1804, and Louis Napoleon, Nov. 22, 1852; what time elapsed between these events?

106. General Grant was born April 27, 1822, and inaugurated President of the United States March 4, 1869; what was his age when inaugurated?

107. How many days from Jan. 1, 1868, to April 1, 1868?

108. How many days from June 17, 1869, to Sept. 8, 1869?

NOTE II. — From June 17 to Aug. 17 is 30 + 31 = 61 days; from Aug. 17 to Aug. 31 is 14 days, and from Aug. 31 to Sept. 8 is 8 days. 61 + 14 + 8 = 83 days. *Ans.* 83 days.

109. How many days from Oct. 3, 1811, to March 12, 1812?

338. The exact number of days from one date to another can be easily obtained by the use of the following table.

A TABLE SHOWING THE NUMBER OF DAYS

FROM ANY DAY OF	TO THE CORRESPONDING DAY OF THE FOLLOWING											
	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
January	365	31	59	90	120	151	181	212	243	273	304	334
February	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August	153	184	212	243	273	304	334	365	31	61	92	122
September	122	153	181	212	242	273	303	334	365	30	61	91
October	92	123	151	182	212	243	273	304	335	365	31	61
November	61	92	120	151	181	212	242	273	304	334	365	30
December	31	62	90	121	151	182	212	243	274	304	335	365

NOTE. — In leap years, if the last day of February is included in the time, a day must be added to the number obtained from the table.

110. How many days are there from March 4, 1869, to July 4, 1869? (See table above.)

111. How many days are there from Feb. 22 to Oct. 25, 1872? from Jan. 1 to Nov. 10?

112. How many days from Oct. 10, 1866, to Jan. 1, 1870? *Ans.* 1,179 d.

339. Miscellaneous Tables.

NUMBERS.

12 units	=	1 dozen.
12 dozen	=	1 gross.
12 gross	=	1 great gross.
20 units	=	1 score.

PAPER.

24 sheets	=	1 quire.
20 quires	=	1 ream.
2 reams	=	1 bundle.
5 bundles	=	1 bale.

BOOKS.

A book formed of sheets folded	in 2 leaves, is a folio.
	in 4 leaves, is a quarto.
	in 8 leaves, is an octavo.
	in 12 leaves, is a duodecimo or 12mo.
	in 16 leaves, is a 16mo.
	in 18 leaves, is an 18mo.
	in 24 leaves, is a 24mo.
	in 32 leaves, is a 32mo.
	in 64 leaves, is a 64mo.

LENGTH.

3 inches = 1 palm.
9 inches = 1 span.
4 in. = 1 hand, used in measuring the height of horses.
6 ft. = 1 fathom, used in measuring depths at sea.

CAPACITY.

1 barrel of flour	=	196 lbs.
1 barrel of beef, pork, or fish	=	200 lbs.
1 cental of grain or 1 quintal of fish	=	100 lbs.
1 bushel of potatoes, wheat, or beans	=	60 lbs.
1 bushel of rye or corn	=	56 lbs.

Examples.

113. How many buttons in a great gross of buttons? 144

114. How many sheets of paper are there in a ream?

115. How many loaves of bread, each containing 13 ounces of flour, may be made of a barrel of flour?

116. How many barrels of beef will serve a regiment of 974 men one day, allowing $1\frac{1}{2}$ lbs. to each man?

☞ For Dictation Exercises, see "Manual and Key," pages 104, 105.

REDUCTION OF DENOMINATE FRACTIONS.

340. **To change a fractional number of a higher denomination to units of lower denominations.**

ILL. EX. I. Change $\frac{3}{8}$ lb. Troy to units of lower denominations.

OPERATION.

$$\frac{3}{8}\text{ lb.} = \frac{3 \times \cancel{12}^{\,3}}{\cancel{8}_{\,2}}\text{ oz.} = \frac{9}{2}\text{ oz.} = 4\tfrac{1}{2}\text{ oz.}$$

$$\frac{1}{2}\text{ oz.} = \frac{1 \times \cancel{20}^{\,10}}{\cancel{2}}\text{ pwt.} = 10\text{ pwt.}$$

Ans. 4 oz. 10 pwt.

Explanation. $\frac{3}{8}$ lb. equals $\frac{3}{8}$ of 12 oz., or $\frac{3 \times 12}{8}$ oz. (Art. 78), which equals $\frac{9}{2}$ oz., or $4\frac{1}{2}$ oz.

$\frac{1}{2}$ oz. equals $\frac{1}{2}$ of 20 pwt., or 10 pwt.

ILL. EX. II. Change .375 lb. Troy to units of lower denominations.

OPERATION.

$$\begin{array}{r} .375\text{ lb.} \\ 12 \\ \hline 4.500\text{ oz.} \\ 20 \\ \hline 10.0\text{ pwt.} \end{array}$$

Explanation. — .375 lb. equals .375 of 12 oz., or (.375 × 12) oz. (Art. 78), which equals 4.5 oz.

.5 oz. equals .5 of 20 pwt. or (.5 × 20) pwt., which equals 10 pwt. *Ans.* 4 oz. 10 pwt.

From the preceding operations we derive the following

RULE. — **To change a fractional number of one denomination to units of lower denominations:** ***Multiply the fractional number by the number which it takes of the next lower denomination to make one of that; change the product thus obtained to an integral or mixed number, if possible. If a fractional number occurs in the result, proceed with it as before, and thus continue as far as required.***

EXAMPLES.

Change to units of lower denominations

116. $\frac{2}{11}$ mile. *Ans.* 58 rd. 1 yd.
117. $\frac{9}{22}$ sq. rd. *Ans.* 12 sq. yd. 3 sq. ft. 54 sq. in.
118. .25 cu. yd. *Ans.* 6 cu. ft. 1296 cu. in.
119. $\frac{7}{12}$ cd. *Ans.* 4 cd. ft. $10\frac{2}{3}$ cu. ft.
120. $\frac{35}{64}$ bu. *Ans.* 2 pk. 1 qt. 1 pt.

Change to units of lower denominations

121. .9 gal.	125. .32 lb. Troy.	129. .362 mile.
122. $\frac{2}{9}$ day.	126. .75 l. year.	130. $\frac{8}{9}$ c. year.
123. $\frac{5}{8}$ °.	127. .875 c. year.	131. $\frac{3}{8}$ T.
124. $\frac{5}{9}$ cu. yd.	128. .125 bu.	132. $\frac{5}{7}$ long ton.

341. The number 5 is one of the two equal parts of 10; $3\frac{1}{3}$ is one of the three equal parts of 10. One of the equal parts of a number is an **aliquot part** of the number.

NOTE. — The pupil may give the aliquot parts left blank in the following table. He will find it very useful in future operations to commit to memory these aliquot parts.

TABLE OF ALIQUOT PARTS.

Of a year.	Of a month.	Of 100.	Of 100.	Of 200.	Of 200.
y. m.	m. d.	100 units.	100 units.	200 units.	200 units.
$\frac{1}{2} = 6$	$\frac{1}{2} = 15$	$\frac{1}{2} = 50$	$\frac{1}{12} = 8\frac{1}{3}$	$\frac{1}{2} = 100$	$\frac{1}{12} =$
$\frac{1}{3} =$	$\frac{1}{3} =$	$\frac{1}{3} = 33\frac{1}{3}$	$\frac{1}{16} = 6\frac{1}{4}$	$\frac{1}{4} =$	$\frac{1}{24} = 8\frac{1}{3}$
$\frac{1}{4} =$	$\frac{1}{5} =$	$\frac{1}{4} =$	$\frac{1}{20} =$	$\frac{1}{8} =$	$\frac{1}{5} =$
$\frac{1}{6} =$	$\frac{1}{6} =$	$\frac{1}{5} =$	$\frac{1}{50} =$	$\frac{1}{16} = 12\frac{1}{2}$	$\frac{1}{10} =$
$\frac{1}{8} = 1\frac{1}{2}$	$\frac{1}{10} =$	$\frac{1}{6} = 16\frac{2}{3}$		$\frac{1}{3} = 66\frac{2}{3}$	$\frac{1}{20} =$
$\frac{1}{9} =$	$\frac{1}{15} =$	$\frac{1}{8} = 12\frac{1}{2}$		$\frac{1}{6} =$	$\frac{1}{100} =$
$\frac{1}{12} =$		$\frac{1}{10} =$			

☞ For Dictation Exercises in reduction of fractions, see "Manual," page 105.

342. To change units of lower denominations to a fractional number of a higher denomination.

ILL. EX. I. 9 h. 36 m. is what part of a day?

OPERATION.

$$36 \text{ min.} = \tfrac{36}{60} \text{ h.} = \tfrac{3}{5} \text{ h.}$$

$$9\tfrac{3}{5} \text{ h.} = \tfrac{48}{5} \text{ h.} = \frac{\overset{2}{\cancel{48}} \times 1}{5 \times \cancel{24}} \text{ d.} = \tfrac{2}{5} \text{ d.}$$

Ans. $\frac{2}{5}$ of a day.

Explanation. — 36 min. equal $\frac{36}{60}$ or $\frac{3}{5}$ of an hour. $\frac{3}{5}$ h. with 9 h. equals $9\frac{3}{5}$ h. or $\frac{48}{5}$ h. $\frac{48}{5}$ hours equals $\frac{48}{5}$ of $\frac{1}{24}$ of a day, or $\frac{2}{5}$ of a day.

ILL. EX. II. Change 22 h. 4 m. 48 sec. to the decimal of a day.

OPERATION.	
60) 48 sec.	*Explanation.* 48 sec. equal $\frac{48}{60}$, or .8 of a minute, which with 4 min. equals 4.8 min. 4.8 min.
60) 4.8 m.	equal $\frac{4.8}{60}$ of an hour, or .08 of an hour, which with 22 h. equals 22.08 h.
24) 22.08 h.	22.08 h. equal $\frac{22.08}{24}$ of a day, or .92 of a day.
.92 d.	*Ans.* .92 of a day.

From the above operations we derive the following

RULE. — To change units of lower denominations to a fractional number of a higher denomination: *Change the number of the lowest denomination given to a fractional number of the next higher. Unite the fractional number with the number of that higher denomination. Change, as before, the number thus formed, and thus continue as far as required.*

EXAMPLES.

133. 1 pk. 2 qt. is what part of a bushel? *Ans.* $\frac{5}{16}$ bu.

134. 120 lb. 8 oz. is what part of a ton? *Ans.* $\frac{241}{4000}$ T.

135. 4 yd. 0 ft. 9 in. is what part of a rod? *Ans.* $\frac{17}{22}$ rd.

136 228 d. 3 h. is what part of a common year?

137. 13 d. 13 h. 20 m. is what part of a leap year?

138. Change 75° 58′ 18″ to the decimal of a circle.
Ans. .2110 circle+.

139. Change 7 cd. ft. 14 cu. ft. to the decimal of a cord.
Ans. .9843 cd+.

140. Change 9 rd. 4½ yd. to the decimal of a mile.
Ans. .0306 m+.

141. Change 17 sq. yd. 6 sq. ft. to the decimal of a sq. rd.

142. Change 3 h. 27 m. to the decimal of a week.

143. What will 3 qt. 1 pt. of kerosene oil cost at 52 cents a gallon? $ .45½.

144. At $ 200 a pound, what is the value of 6 oz. 5 pwt. of gold? *Ans.* $ 104.16⅔.

☞ For Dictation Exercises upon this subject, see "Manual and Key," page 105.

ADDITION.

343. Ill. Ex. John picked 1 bu. 3 pk. 2 qt. of peas, 2 pk. 5 qt., and 3 bu. 3 pk. 6 qts.; what quantity of peas did he pick in all?

OPERATION.

bu.	pk.	qt.
1	3	2
	2	5
3	3	6
6 bu.	1 pk.	5 qt.

Explanation. — We express these numbers so that units of the same kind shall be expressed in the same column. Adding the quarts, we have 13 quarts, which equal 1 pk. with 5 qt.; expressing the 5 qt. under the line, we add the 1 pk. with the pecks of the given numbers, and have 9 pecks, which equal 2 bu. with 1 pk. Expressing the 1 pk., we add the 2 bu. with the bushels of the given numbers, and have 6 bu., and for the answer, 6 bu. 1 pk. 5 qt.

In what respects are the operations in the addition of compound numbers and of simple numbers alike? How do they differ?

Make a rule for the addition of compound numbers.

EXAMPLES.

(1.)

	T.	lb.	oz.
	2	896	12
	1	928	9
	2	364	14
Ans.	6	190	3

(2.)

Long T.	cwt.	qr.	lb.
5	15	2	26
3	12	1	20
2	18	3	27
12	7	0	17

(3.)

lb.	oz.	pwt.	gr.
3	4	7	22
1	2	14	7
3	9	0	2
8	4	2	7

(4.)

m.	rd.	yd.	ft.	
2	132	4	2	
1	242	3	2	
	300	1	0	
5	35	$3\frac{1}{2}$	1	
		$\frac{1}{2}$ =	1	6 in.
5	35	3	2	6

(5.)

A.	sq. rd.	sq. yd.	sq. ft.	
40	100	4	2	
24	142	15	3	
413	127	23	8	
479	50	$12\frac{3}{4}$	4	
		$\frac{3}{4}$ =	6	108 in.
479	50	13	1	108

(6.)				(7.)			(8.)		
rd.	yd.	ft.	in.	sq. yd.	sq. ft.	sq. in.	cu. yd.	cu. ft.	cu. in.
1	3	2	9	4	6	120	4	22	1048
2	1	2	11	5	8	150	9	18	173
8	5	1	4	5	3	172	4	19	806

(9.)			(10.)				(11.)			
cd.	cd. ft.	cu. ft.	gal.	qt.	pt.	gi.	bu.	pk.	qt.	pt.
3	6	4	3	2	1	3	5	3	2	1
7	4	8	1	3	0	2	8	2	3	0
2	5	6	5	1	0	1	3	2	1	0

12. How much land is there in 4 pastures, the first containing 3 A. 20 sq. rd.; the second, 5 A. 18 sq. rd. 12 sq. yd.; the third, 27 A.; and the fourth, 128 sq. rd. 28 sq. yd.?

Ans. 36 A. 7 sq. rd. 9¾ sq. yd.

13. If I have travelled 2 m. 32 rd., 6 m. 120 rd. 4 yd., and 9 m. 300 rd. 3 yd., what distance have I travelled in all?

14. What is the sum of 213 d. 10 h. 40 m., 5 y. 145 d. 2 h. 32 m., and 3 y. 152 d. 3 h. 10 m.?

☞ For Dictation Exercises in Addition, see "Manual and Key," page 106.

SUBTRACTION.

344. ILL. EX. A person who had 7 bu. 2 pk. 2 qt. of grapes sold 3 pk. 4 qt.; what quantity of grapes had he left?

OPERATION.

bu.	pk.	qt.
(6)	(5)	(10)
7	2	2
	3	4
6	2	6

Explanation.—We write the expression of the minuend and, underneath, that of the subtrahend, so that units of the same kind shall be expressed in the same column.

As the number of quarts in the minuend is less than the number in the subtrahend, we change 1 of the 2 pecks to quarts. 1 pk. = 8 qt.; 8 qt. + 2 qt. = 10 qt. If 4 of the 10 quarts are taken 6 quarts remain. As 1 of the 2 pecks has been changed to quarts, we

have but 1 peck left. As 1 peck is less than the pecks in the subtrahend, we change one of the 7 bushels to pecks. 1 bu. = 4 pk. 4 pk. + 1 pk. = 5 pk. If 3 of the 5 pecks are taken, 2 pecks remain. As there are no bushels in the subtrahend, 6 bushels remain, and the entire remainder is 6 bu. 2 pk. 6 qt.

In what respects are the operations in the subtraction of compound numbers and of simple numbers alike? How do they differ?

Make a rule for the subtraction of compound numbers.

EXAMPLES.

(1.)

	T.	lb.	oz.
	3	127	4
	1	800	8
Ans.	1	1326	12

(2.)

m.	rd.	yd.	ft.	in.
8	47	4	2	
	285	5	1	
7	81	4	2	6

(3.)

°	′	″
30	5	8
12	48	16

(4.)

Long T.	cwt.	qr.	lb.
1	5	2	8
	18	1	25

(5.)

sq. yd.	sq. ft.	sq. in.
18	2	9
5	2	18

(6.)

cu. yd.	cu. ft.	cu. in.
87	17	140
18	22	144

7. Mr. Snow had 27 cords of wood, and sold 19 cords 5 cord feet of it; how much wood had he left?

8. Mr. Gill sold 2 A. 7 sq. rd. 20 sq. ft. of a piece of land containing 18 A. 12 sq. ft.; how much land had he left? *Ans.* 15 A. 152 sq. rd. 264¼ sq. ft.

9. How much of a day remains when 2 h. 19 min. 7 sec. of it have passed?

10. How much longer must a man live to be a century old who has lived 87 y. 97 d. 18 h.?

11. If a boy who had 9 bushels of berries to pick, has picked 4 bu. 2 pk. 1 qt. 1 pt. of them, what quantity has he yet to pick?

12. If 5 gal. 2 qt. 1 pt. of molasses are drawn out of a hogshead of 100 gallons, how much molasses remains in the hogshead?

☞ For Dictation Exercises in Subtraction, see "Manual and Key," page 106.

345. To find the Difference of Latitude and Longitude between two Places.

TABLE OF LATITUDES AND LONGITUDES.*

Place.	State or Country.	Longitude from Greenwich.				Latitude.			
			°	′	″		°	′	″
Albany,	N. Y.,	W.	73	44	39	N.	42	39	50
Boston,	Mass.,	W.	71	3	30	N.	42	21	27
Canton,	China,	E.	113	14		N.	23	7	
Calcutta,	India,	E.	88	19	2	N.	22	35	5
Cape Horn,	S. America,	W.	67	16	8	S.	55	58	40
Cape of Good Hope,	Africa,	E.	18	29		S.	32	24	3
Charleston,	S. C.,	W.	79	55	38	N.	32	46	33
Chicago,	Ill.,	W.	87	37	47	N.	42	0	0
Cincinnati,	Ohio,	W.	84	27	–	N.	39	5	54
Constantinople,	Turkey,	E.	28	59		N.	41	0	16
London,	England,	W.		5	48	N.	51	30	48
Mexico,	Mexico,	W.	103	45	30	N.	19	25	45
Montreal,	L. C.,	W.	73	35		N.	45	31	
New Orleans,	La.,	W.	90			N.	29	57	30
New York,	N. Y.,	W.	74	0	3	N.	40	42	43
Paris,	France,	E.	2	20	22½	N.	48	50	12
Philadelphia,	Pa.,	W.	75	9	54	N.	39	58	24
Portland,	Me.,	W.	70	14	34	N.	43	39	54
Quebec,	L. C.,	W.	71	12	18	N.	46	49	12
San Francisco,	Cal.,	W.	122	26	48	N.	37	47	53
St. Petersburg,	Russia,	E.	30	19		N.	59	56	30
Washington,	D. C.,	W.	77	0	15	N.	38	53	20

Examples.

What is the difference of latitude between

1. Calcutta and Boston? *Ans.* 19° 46′ 22″.
2. London and Paris?
3. Quebec and Washington?
4. Cape of Good Hope and St. Petersburg? †
   Ans. 92° 20′ 33″.
5. Cape Horn and San Francisco?

* From the American Almanac and New American Cyclopædia.

† The difference of latitude between places on opposite sides of the equator is found by *adding* the latitudes. The difference of longitude between places on opposite sides of the first meridian is found by *adding* the longitudes. If their sum exceeds 180°, the difference of longitude equals 360° minus that sum.

What is the difference of longitude between

6. Albany and Cincinnati? *Ans.* 10° 42′ 21″.

7. New York and Paris? *Ans.* 76° 20′ 25$\frac{1}{2}$″.

8. Philadelphia and Constantinople?

9. Charleston and Chicago?

10. San Francisco and Canton? *Ans.* 124° 19′ 12″.

346. To add and subtract denominate fractions.

ILL. EX. Add $\frac{2}{3}$ rd. with $\frac{5}{6}$ yd.

Explanation. — These fractional numbers, being of different denominations, must first be changed to numbers of the same denomination. This may be done either by changing both fractional numbers to numbers of lower denominations (Art. 340), by changing $\frac{5}{6}$ of a yard to part of a rod (Art. 342), or by changing $\frac{2}{3}$ of a rod to yards. Here the operation is by the last method.

OPERATION.

$$\frac{2}{3}\text{ rd.} = \frac{11 \times 2}{2 \times 3}\text{ yd.} = 3\tfrac{2}{3}\text{ yd.}$$
$$\tfrac{5}{6}\text{ yd.}$$
$$Ans.\ 4\tfrac{1}{2}\text{ yd.}$$

EXAMPLES.

11. Add $\frac{1}{2}$ of a day and $\frac{2}{3}$ of an hour. *Ans.* 12$\frac{2}{3}$ h.

12. Add $\frac{5}{8}$ of a gallon and 2$\frac{1}{3}$ pints.

13. Add $\frac{5}{12}$ of a common year and 5$\frac{1}{3}$ days.

14. If of a journey of $\frac{2}{3}$ of a mile a man has 7$\frac{1}{2}$ rods yet to travel, how far has he travelled? *Ans.* 205$\frac{5}{6}$ rd.

15. John sawed $\frac{1}{3}$ of a cord of wood before tea and $\frac{1}{2}$ of a cord foot after tea, how many cord feet did he saw in all?

16. How much more did he saw before tea than after?

17. Add .25 sq. rd. and 3.36 sq. yd. *Ans.* 10.92$\frac{1}{4}$ sq. yd.

18. Add .62 lb. and 5 oz. (Av. weight).

19. How much more is there in a lump of gold which weighs $\frac{1}{2}$ of a pound than in one which weighs 19.5 pwts.?

☞ For Dictation Exercises, see "Manual and Key," page 107.

MULTIPLICATION.

347. ILL. EX. How much milk is there in 5 cans, containing 2 gallons 3 quarts each?

OPERATION.

	gal.	qt.
	2	3
		5
Ans.	13 gal.	3 qt.

Explanation. — 3 quarts multiplied by 5 equals 15 qt. or 3 gal. with 3 qt. We express the 3 quarts, and reserve the 3 gallons to add to the next product. 2 gal. multiplied by 5 equals 10 gal.; 10 gal. with 3 gal. are 13 gal. *Ans.* 13 gal. 3 qt.

Compare multiplication of compound numbers with multiplication of simple numbers, and make a rule for the multiplication of compound numbers.

EXAMPLES.

1. If 1 cubic foot of water weighs 62 lbs. 8 oz., how much will 35 cubic feet weigh? *Ans.* 1 T. 187 lb. 8 oz.

2. If a person in going once to school walks 19 rods 4 yards, how far does he walk in going twice and returning? *Ans.* 78 rd. 5 yd.

3. How much glass is there in 24 windows of 12 panes each, if each pane contains 2 sq. ft. 8 sq. in.?

4. How far does a locomotive run in a day, that makes 4 trips of 27 m. 16 rd. 4 yd. each way?

5. How many revolutions are made in 1 hour by a wheel that turns 300° 40′ in every second of time? *Ans.* 3,006⅔.

6. In how many days can I stitch 12 dozen garments, allowing 2 h. 15 m. to each, and 8 working hours to a day?

7. How many cubic yards of earth are there in 15 loads each containing 1 cu. yd. 20½ cu. ft.?

NOTE. — A cubic yard of earth is generally considered a load

8. Multiply 5 oz. 3 pwt. 6 gr. by 21.

☞ For Dictation Exercises in Multiplication, see "Manual and Key," page 107.

DIVISION.

348. ILL. EX. If 14 cd. 6 cd. ft. 12 cu. ft. of wood are divided equally among 5 persons, what will each person receive?

OPERATION.

	cd.	cd. ft.	cu. ft.
5)	14	6	12
	2	7	12

Explanation.—Dividing what we can of the dividend, and have cords as the result, we have 2 cords in the quotient and a remainder of 4 cords, which equal 32 cord feet. 32 cd. ft. + 6 cd. ft. = 38 cd. ft. Dividing as before, we have 7 cord feet in the quotient and a remainder of 3 cord feet, which equal 48 cubic feet. 48 cu. ft. + 12 cu. ft. = 60 cu. ft. 1 fifth of 60 cu. ft. = 12 cu. ft.; therefore each person will receive 2 cd. 7 cd. ft. 12 cu. ft.

Compare division of compound numbers with division of simple numbers, and make a rule for the division of compound numbers.

EXAMPLES.

1. Divide 4 lb. 5 oz. 18 pwt. 2 gr. by 6.
Ans. 8 oz. 19 pwt. $16\frac{1}{3}$ gr.

2. Divide 5 T. 290 lb. by 8. *Ans.* 1,286 lb. 4 oz.

3. What is 1 eleventh of 29° 42′ 32″?

4. What is 1 twenty-fifth of 1 sq. m. 35 A. 2 sq. rd. 49 sq. ft.? *Ans.* 27 A. $23\frac{3}{5}\frac{1}{0}$ sq. ft.

5. If a piece of land containing 48 A. 150 sq. rd. 8 sq. yd. is divided into 100 house-lots of equal size, what is the size of each lot? *Ans.* 78 sq. rd. $9\frac{31}{200}$ sq. yd.

6. How far will a boy walk in a minute who walks 4 miles in an hour? *Ans.* 21 rd. 1 yd. $2\frac{1}{2}$ ft.

7. If a person consumes $10\frac{1}{2}$ gallons of milk in 4 weeks, how much does he consume in 1 day?

8. How many 3-pint cans will be required to contain 9 gal. 1 qt. 1 pt. of preserved peaches? *Ans.* 25.

NOTE. — Change 9 gal. 1 qt. 1 pt. to pints before dividing.

9. If a boy walks 3 m. 40 rd. in an hour, how long will it take him to walk 25 m. 19 rd.? *Ans.* $8\frac{18}{1060}$ hours.

10. How many bins, each containing 4 bu. 3 pk., will be required to hold 242 bu. 1 pk.?

11. Among how many persons may 35 bushels of corn be divided, that each one may receive 2 bu. 1 pk., and how many pecks will remain?

12. What is 1 eighteenth of 20 y. 5 d. 12 h. 24 min.?

☞ For Dictation Exercises in Division, see "Manual and Key," page 108.

LONGITUDE AND TIME.

349. As there are 360° of longitude in the circumference of the earth, and as the earth turns upon its axis once in 24 hours, it follows that $\frac{1}{24}$ of 360°, or 15° of longitude, must pass under the sun in 1 hour, and $\frac{1}{60}$ of 15°, or 15′, must pass under it in 1 minute of time, and $\frac{1}{60}$ of 15′, or 15″, must pass under it in 1 second of time; therefore a difference upon the earth's surface

of 15° of longitude makes a difference of 1 hour in time,
of 15′ " " " 1 minute in time,
of 15″ " " " 1 second in time.

From the above we derive the following

RULE.—To find the difference of longitude between any two places when the difference of time is given: *Multiply the numbers expressing the difference of time between the two places given in hours, minutes, and seconds, by* 15. *The resulting products will indicate the number of degrees, minutes, and seconds of longitude between the two places.*

NOTE.—As the earth turns from west to east, the sun reaches the meridian of any place east of us *before* it reaches ours, and it reaches the meridian of any place west of us *after* it reaches ours. Hence at any given time of day with us, it is later in the day at all places east of us and earlier in the day at all places west of us.

EXAMPLES.

1. When the difference of time between two places is 5 hours 27 minutes, what is the difference of longitude?

Ans. 81° 45′.

2. At a certain time at Cincinnati the time at Buffalo is 22 min. 8 sec. later; what is the difference of longitude between the two places? *Ans.* 5° 32′.

3. What is the longitude of Buffalo? (Art. 345.)

Ans. 78° 55′ W.

4. When it is noon at New Orleans, the time at Rio Janeiro is 3 h. 7 min. later; what is the longitude of Rio Janeiro? *Ans.* 43° 15′ W.

5. The time at Edinburgh is 2 h. 14 min. earlier than at St. Petersburg; what is the longitude of Edinburgh?

Ans. 3° 11′ W.

350. From Art. 349 we also derive the following

RULE. — To find the difference of time between two places when the difference of longitude is given: *Divide the numbers expressing the difference of longitude between the two places given in degrees, minutes, and seconds by 15. The resulting quotients will indicate the number of hours, minutes, and seconds of time between the two places.*

EXAMPLES.

What is the difference of time between

6. New York and Chicago? *Ans.* 54 min. $30\frac{4}{5}$ sec.

7. Boston and Mexico? *Ans.* 2 h. 10 min. 48 sec.

8. London and San Francisco?

9. London and Paris? *Ans.* 9 min. $44\frac{7}{10}$ sec.

10. Canton and Boston? *Ans.* 11 h. 42 min. 50 sec.

11. When it is 2 o'clock P. M. in Washington, what time is it in Boston? *Ans.* 2 h. 23 min. 47 sec. P. M.

12. When it is 8 o'clock A. M. in Philadelphia, what time is it in Quebec? *Ans.* 8 h. 15 min. $50\frac{2}{3}$ sec. A. M.

☞ For Dictation Exercises, see "Manual and Key," page 108.

351. Topical Review in Compound Numbers.

The pupil may illustrate the following topics to his class, using the weights and measures themselves when practicable and giving definitions and rules.

1. Quantity and unit of measure. (Arts. 282, 283.)

2. A denominate number; simple and compound numbers. (Arts. 284 – 286.)

3. Weight, — avoirdupois weight. Reduction. (Arts. 287 – 289.)

4. Changing numbers of higher denominations to units of a lower. (Art. 290.)

5. Changing a number to units of higher denominations. (Art. 291.)

6. Troy weight. (Art. 292.) Apothecaries' weight. (Art. 292, note.) 7. Comparison of weights. (Art. 293.)

8. Extension. (Arts. 294 – 298.) 9. Long measure. (Art. 299.) Surveyors' measure. (Art. 300.) Mariners' measure. (Art. 301.)

10. Measures of surface. (Arts. 303 – 308.) 11. Rectangles and their areas. (Arts. 309, 310, 312.) Measurement of land. (Art. 311.)

12. Measures of solids. (Arts. 313 – 316.) 13. Rectangular solids and their contents. (Arts. 317 – 319.) 14. Wood measure. (Arts. 320, 321.)

15. Liquid measure. (Art. 322.) 16. Dry measure. (Art. 323.)

17. Comparison of measures. (Art. 324.)

18. Circular measure. (Arts 325 – 331.) 19. Angles and their measures. (Art. 332.)

20. Measures of time. (Art. 333.) 21. Leap Year. (Art. 335.)

22. Divisions of the year. (Art. 336.)

23. Finding difference of time between two dates. (Arts. 337, 338.)

24. Miscellaneous tables. (Art. 339.)

25. Changing fractional numbers of a higher denomination to numbers of lower denominations. (Art. 340.)

26. Changing numbers of lower denominations to a fractional number of a higher. (Art. 342.)

27. Addition. (Art. 343.) 28. Subtraction. (Art. 344.)

29. Finding difference of latitude and longitude between two places. (Art. 345.)

30. Adding and subtracting denominate fractions. (Art. 346.)

31. Multiplication. (Art. 347.) 32. Division. (Art. 348.)

33. Longitude and time. (Arts. 349, 350.)

352. GENERAL REVIEW, No. 5.

1. Change 2 m. 120 rd. 5 yd. to feet.

2. Change 273261 seconds to numbers of higher denominations.

3. What part of a circle is $14° 7' 42'' + 22° 35' 12''$?

4. Change the difference between 124 lb. 2 oz. and 13 lb. 11 oz. to the decimal of a ton.

5. What is the cost of 5 loads of apples each containing 12 bu. 3 pk. 4 qt., at $ 1.20 a bushel?

6. What must I pay for 1 fourth of a cask of kerosene which contains 42 gal. 3 qt., at 15 cents a quart?

7. What is the price of $\frac{2}{3}$ of an acre of land at 27 cents per sq. ft.?

8. Change .4921875 of a bushel to numbers of lower denominations.

9. By the scales of a grocer, a quantity of silver weighs 1 lb. 2 oz., what does it weigh by the scales of a silversmith?

10. How many yards of carpeting, $\frac{3}{4}$ of a yard wide, will be required to cover a floor 25 feet long and 18 feet wide?

11. How many tons of ice may be put into a building measuring on the inside 42 feet in length, 32 in width, and 15 in height, a cubic foot of ice weighing 930 ounces?

12. What is the cost of sawing a pile of wood, 20 feet long, 4 feet wide, and 6 feet high, at $1.20 per cord?

13. When it is 2 o'clock P. M. in Boston, what is the time in Chicago?

14. If when it is 11 o'clock A. M. in New York it is 10 o'clock 35 min. $52\frac{1}{2}$ sec. A. M. in Pittsburg, what is the longitude of Pittsburg?

☞ For Dictation Exercises upon this Review, see "Manual and Key," page 109; for Miscellaneous Exercises, see pages 110, 111.

353. Miscellaneous Examples.

1. What cost 5 lb. 7 oz. of steak at 28 cents a pound?
Ans. \$ 1.52$\frac{1}{4}$.

2. What is the cost of 1 gallon of molasses, when 2 gal. 3 qt. and 5 gal. 1 pt. cost \$ 4.41? *Ans.* \$ .56.

3. What is the cost of 3 dozen bottles of cologne at \$ .75 a bottle, and $\frac{2}{3}$ of a gross of buttons at \$ .33 a dozen?

4. How many times will a wheel 6 ft. 2 in. in circumference turn in crossing a bridge that is 28 rd. 4 yd. long?

5. It is said that 2500 barrels of oil flowed from a certain oil well in Penn. in one day; if each barrel contained 32 gallons, how many gallons flowed in a minute? *Ans.* 55$\frac{5}{9}$ gal.

6. How many rails and how many posts are required to fence a rectangular piece of land 32 rods long and 16 rods wide, if the posts are set 8 feet apart, and the fence is 5 rails high?

7. If the land mentioned above was sold at \$ .12$\frac{1}{2}$ per sq. ft., and the proceeds invested in railroad stock at \$ 107 a share, how many shares were bought, and what money remained? *Ans.* 162 shares; \$ 90 remained.

8. How many yards of cloth, 30 inches wide, will be required to make a dress, the skirt to be 5 yards wide and 44 inches long, 3 yards being allowed for waist and sleeves? *Ans.* 10$\frac{1}{3}$ yds.

9. How many squares of patchwork, 9$\frac{1}{2}$ inches square, must be allowed for a quilt 8 ft. 3 in. long and 7 ft. 6 in. wide, $\frac{1}{4}$ of an inch being allowed on the sides of each square for seams? *Ans.* 110 squares.

10. How much cloth, $\frac{5}{8}$ of a yard wide, would line the above-mentioned quilt, no allowance being made for seams in the lining? *Ans* 8$\frac{1}{4}$ yds.

11. How many solid feet are there in a block of marble 2 ft. 3 in. long, 1 ft. 6 in. wide, and 2 ft. thick, and how many sq. ft. of surface does it contain?

12. How many square yards of plastering are there in the walls and ceiling of a room 13 feet long, $10\frac{1}{2}$ ft. wide, and 8 ft. high above the mop-boards, allowing 8 sq. yds. for windows and doors? *Ans.* $48\frac{17}{18}$ yards.

13. How many bushels can a bin contain that is on the inside 4 ft. long, 3 ft. wide, and 1 ft. 9 in. deep? (Art. 324.)

14. What must be the height of a bin 6 ft. long, and $2\frac{1}{2}$ ft. wide, inside, to contain 6 bushels of grain?

15. A man purchased a pile of wood 142 ft. 6 in. long, 8 ft. wide, and 4 ft. high, at $ 5 a cord, and sold 7 cd. 6 cu. ft. at $ 6 a cord, $12\frac{1}{2}$ cords at $ 8 a cord, and the remainder at $ $4\frac{1}{2}$ a cord; did he gain or lose, and how much? *Ans.* Gained $ $37.56\frac{1}{4}$.

16. How many gallons of water can a cistern contain that is, on the inside, 3 ft. long, 2 ft. wide, and 3 ft. deep?

17. If a cistern 4 feet deep is filled with water, what weight of water does 1 sq. in. of the bottom sustain? (See page 183, Ex. I.) *Ans.* $1\frac{53}{72}$ lb.

18. At 9 o'clock A. M. in Portland, what is the time in San Francisco? *Ans.* 5 o'clock, 31 m. $11\frac{1}{15}$ s. A. M.

19. A man sailing from Boston found his watch, which kept Boston time, was 1 hour and 10 minutes too slow; in what longitude was he? *Ans.* 53° 33′ 30″ W.

20. How many cu. in. are there in a stone which, immersed in a vessel full of water, displaces $1\frac{3}{4}$ qts.? (Art. 324.)

21. How far away is the lightning whose thunder is heard in 5 seconds after the flash is seen, if sound travels 1 mile in $4\frac{2}{3}$ seconds? *Ans.* 1 m. $22\frac{6}{7}$ rd.

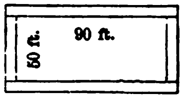
22. How far from the speaker is a hill which sends back an echo in $3\frac{1}{2}$ seconds from the time the sound is made?

354. EXAMPLES FOR ADVANCED PUPILS AND FOR REVIEW.

23. What is the cost of 130 pieces of imported muslin, averaging 42 yards in length and 45 inches in width, at \$.37 per yard, and \$.03½ per sq. yard for duties? *Ans.* \$2259.07½.

24. How many sq. yds of plastering are there in the walls and ceiling of a room 15 ft. long, 14¾ ft. wide, and 9 ft. high; the mop-boards being 8 in. high, the room having 3 windows, each 3 ft. wide and 6 ft. high, and 2 doors, each 3 ft. wide and 7 ft. high? *Ans.* $69\frac{49}{108}$ sq. yards.

25. How many sq. ft. in a walk around the outside of a rectangular garden which is 90 ft. long and 50 ft. wide, the walk being 3 ft. wide?



NOTE.—If we consider the walk at each end of the garden to equal in length the width of the garden, the length of the walk on each side of the garden must equal the length of the garden plus two widths of the walk, or 90 ft. + 6 ft. = 96 ft. The entire length of the walk must then be (96 ft. + 50 ft.) × 2 = 292. *Ans.* 876 sq. ft.

26. How many cubic yards of earth must be removed in digging a ditch 2½ ft. wide and 2 ft. deep, outside and next to the boundary of a garden 6 rods square? *Ans.* $75\frac{5}{27}$ cu. yards.

27. Suppose a ditch to be dug inside of the boundary of the above-mentioned garden, how many cubic yards of earth must be removed, the ditch being 2½ ft. wide, and 2 ft. deep? *Ans.* $71\frac{13}{27}$ cu. yards.

28. How many cords of wood can be put into a building which measures on the outside 42 ft. long, 26½ ft. wide, and 12 ft. high, the walls being 9 in. thick? *Ans.* $94\frac{59}{64}$ cd., or 94 cd., 7 cd. ft., 6 cu. ft.

29. What will be the cost, at \$.25 per sq. yd. of removing the earth for a cellar 10 ft. deep, and whose measurement inside of the wall, which is 2 ft. 6 in. thick, is 32 ft. long and 18 ft. wide? *Ans.* \$$78.79\frac{17}{27}$.

30. How many bricks 8 in. long, 4 in. wide, and 2 in. thick would be required to construct a wall 15 ft. high and 1½ ft. thick, to enclose an area 40 ft. long and 20 ft. wide, allowing 1 sixth for mortar?

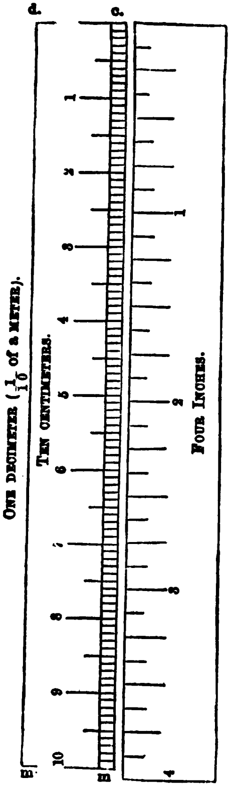
31. How many bricks will be required to construct the walls of a building 50 ft. long, 30 ft. wide, and 20 ft. high, outside measure, having 4 windows 6 ft. high, 3 ft. 4 in. wide, and 2 doors 7 ft. high, 4 ft. 6 in. wide, the walls to be 1 ft. thick, and 1 sixth of the wall to be allowed for mortar? *Ans.* 66,982½ bricks.

☞ For Dictation Exercises, see "Manual and Key," page 108.

THE METRIC SYSTEM.

MEASURES OF LENGTH.

TO THE TEACHER. — To introduce the metric system, bring into the presence of the pupils a METER, calling it "a new measure." (In the absence of a graduated meter, represent its length by a straight line upon the board.) The pupil, by measuring, will find the length of the new measure to be 3 feet $3\frac{3}{8}$ inches, or 39.37 inches.



355. A measure whose length is 39.37 inches is a unit of measure called a **meter** (meeter) from the French word *metre*, a measure.

356. The line d. m. in the margin is nearly 4 inches in length; it represents $\frac{1}{10}$ of a meter, and is called a **dec'imeter** (*deci* being from the Latin for 10.)

The line c. m. represents a decimeter divided into ten equal parts, with each of these parts again divided into 10 equal parts.

$\frac{1}{10}$ of a decimeter equals $\frac{1}{100}$ of a meter, and is called a **cen'timeter** (*centi* being from the Latin for 100).

$\frac{1}{10}$ of a centimeter equals $\frac{1}{1000}$ of a meter, and is called a **mil'limeter** (*milli* being from the Latin for 1000).

A unit equal to 10 meters is called a **dek'ameter** (*deka* being from the Greek for 10).

A unit equal to 10 dekameters is called a **hec'tometer** (*hecto* being from the Greek for 100).

A unit equal to 10 hectometers is called a **kil'ometer** (*kilo* being from the Greek for 1000).

A unit equal to 10 kilometers is called a **myr'iameter** (*myria* being from the Greek for 10,000).

357. The units named in Art. 356 are all parts or multiples of the meter; hence the meter is called the **primary unit of length.**

358. It will be observed that the names of the other units of length are formed by uniting with the name of the primary unit prefixes derived from the Latin and Greek numerals.

359. As a series of units of length forms a *scale of tens* (Art. 148), units of each order may be expressed as decimal numbers are expressed, thus: —

(myrm.)	(km.)	(hm.)	(dkm.)	(m.)		(dcm.)	(cm.)	(mm.)	Abbreviations.
Myriameter.	*Kilometers.*	Hectometers.	Dekameters.	METERS.	*Decimal Point.*	Decimeters.	Centimeters.	*Millimeters.*	Names of units.
1	7	3	4	5	.	8	4	2	Expression.

or in the denomination of the primary unit; thus:

1 7 3 4 5 . 8 4 2 meters.

NOTE I. — The primary unit is indicated in the table by capitals, and other principal units by italics.

NOTE II. — The five-cent piece, coined since 1866, is 2 centimeters in diameter.

NOTE III. — 5 meters are nearly equal to 1 rod.

NOTE IV. — 1 kilometer is a little less than $\frac{5}{8}$ of a mile. *Great distances are estimated in kilometers.*

EXERCISES.

1. Name the units of length in order from the lowest to the highest; from the highest to the lowest.

2. How many meters are there in 1 dekameter? 1 hectometer? 1 kilometer? 1 myriameter?

3. What part of a meter is 1 decimeter? 1 centimeter? 1 millimeter?

4. In which place from the decimal point and on which side of it are expressed centimeters? dekameters? kilometers? decimeters?

360. In reading a number expressed in figures, as 35.84^m, we read the part expressed at the left of the point in the denomination of the primary unit, and the part expressed at the right of the point either as a part of the primary unit or as units of the lowest order expressed; thus 35.84^m may be read "35 and 84 hundredths meters," or "35 meters 84 centimeters."

EXERCISES.

Read the numbers expressed by the following, in each of the ways indicated above: —

5. 75.85^m. | 6. 876.056^m. | 7. 7000.706^m.

361. Numbers of one order of units may be changed to those of another by processes already illustrated (see Arts. 121, 123).

NOTE. — If a number is expressed in figures, to indicate the change it is necessary simply to place the decimal point at the right of the place of the units in which the number is to be expressed, and annex to the expression the name indicating the order.

EXAMPLES.

1. Express 7.22^m as decimeters; as centimeters; as millimeters.
Ans. 72.2^{dcm}; 722^{cm}; $7,220^{mm}$.

2. Express 46.8^m as hectometers; as kilometers; as myriameters.
Ans. $.468^{hm}$; $.0468^{km}$; $.00468^{mym}$.

3. Express in meters and add 475^{dm}; 4.47^{hm}; 2.2568^{km}.
Ans. $4,743.18^m$.

4. Express in meters the difference between 985^{cm} and $.3^{km}$.
Ans. 290.15^m.

MEASURES OF SURFACE.

TO THE TEACHER. — To introduce measures of surface, represent the square meter on the board. Divide each side into 10 equal parts, and join the points of division by lines parallel to the sides.

The square meter will thus be divided into 10×10 or 100 smaller squares, each of which has for its side one decimeter, and which is consequently one square decimeter.

In the same manner show that the square decimeter equals 10×10 or 100 square centimeters, etc.

362. From the previous illustration, it will be seen that

1 sq. decimeter = .1 × .1 or .01 of a sq. meter.
1 sq. centimeter = .01 × .01 or .0001 of a sq. meter.
1 sq. millimeter = .001 × .001 or .000001 of a sq. meter.

363. The SQUARE METER is the primary unit for the measurement of small surfaces.

364. A square each of whose sides is 10 meters in length, called an **Are** (air), is the primary unit for land measure. The **Hec'tare** of 100 ares, and the **Cen'tare,** 100th of an are, are also used.

NOTE.—The centare and the square meter are like units.

365. From the preceding illustrations, it will be seen that a series of units of surface forms a *scale of hundreds.*

A unit of each order may then be expressed as follows:—

Hectare (ha).	ARE (ar).	Centare. (ca), or Sq. METER.	Sq. Decimeter.	Sq. Centimeter.	Sq. Millimeter.
1	0 1	0 1	0 1	0 1	0 1

NOTE.—The are equals 119.6 square yards, nearly 4 square rods, or about $\frac{1}{40}$ of an acre. The hectare equals about $2\frac{1}{2}$ acres.

EXAMPLES.

1. How many ares are there in 1 hectare?

2. What part of 1 are is 1 centare?

3. What part of 1 sq. meter is 1 sq. decimeter? 1 sq. centimeter? 1 sq. millimeter?

4. Express 14.25 ares as hectares; as centares.

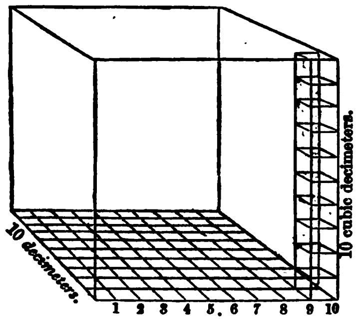
Ans. $.1425^{ha}$; 1425^{ca}.

5. Express as ares and add 47.53 hectares, 8.67 centares, 7.5 hectares.

Ans. $5{,}503.0867^{ar}$.

MEASURES OF VOLUME.

To THE TEACHER. — To introduce measures of solids or volume, represent upon the board, with the square meter as a base, 1 cubic meter. The figure in the margin represents a cubic meter and decimeters.



Show that if, upon the square meter as a base, the solid were 1 decimeter in height, it would contain 10×10 or 100 cubic decimeters; and that since it is 10 decimeters in height, it contains $10 \times 10 \times 10$ or 1000 cubic decimeters.

Show in the same manner that 1 cubic decimeter equals 1000 cubic centimeters, etc.

366. The CUBIC METER is the primary unit of volume.

367. From the illustration referred to above, it will be seen that a series of units of measure of volume forms a *scale of thousands*.

Hence, in expressing numbers denoting volume, three decimal places must be allowed for cubic decimeters, three for cubic centimeters, and three for cubic millimeters.

EXAMPLES.

1. What part of 1 cubic meter is 1 cubic decimeter? 1 cubic centimeter? 1 cubic millimeter.

Ans. $.001^{cu\ m}$; $.000001^{cu\ m}$; $.000000001^{cu\ m}$.

2. Express as cubic meters and add, $.974^{cu\ dcm}$; $49700^{cu\ cm}$.

Ans. $.050674^{cu\ m}$.

368. A cube, which equals 1 cubic meter, called a STERE (stair), is the primary unit of measure for firewood, stone, etc. **Dekasteres** and **decisteres** are also used.

A series of these units forms a *scale of tens*.

NOTE. — The stere equals 1.308 cubic yards, and is a little more than $\frac{1}{4}$ of a cord.

EXAMPLES.

3. How many steres of wood are there in a pile 15^m long, 1^m wide, and 2.4^m high? *Ans.* 36 steres.

4. How many steres of sand in a load 2.8^m long, 1^m wide, and 5^{dcm} deep? *Ans.* 1.4 steres.

MEASURES OF CAPACITY.

TO THE TEACHER. — To introduce the measures of capacity, bring into the presence of the pupil a cubic measure of tin or pasteboard whose capacity equals a liter. The pupil by measuring will find its edge to be 1 decimeter (Art. 356), and hence its capacity to be 1 cubic decimeter.

369. The LITER (leeter) is the primary unit of capacity, and is used for both liquid and dry measure.

370. A series of units of capacity forms a *scale of tens*, and hence the units are named and expressed in the same manner as units of length are named and expressed, thus: —

Kiloliters (kl.)	*Hectoliters* (hl.)	Dekaliters (dkl.)	LITERS (l.)		Deciliters (dcl.)	*Centiliter* (cl.)	Milliliters (ml.)
5	2	7	8	.	6	1	4

NOTE I. — The liter contains .908 of a quart, *dry measure*, or 1.0567 quarts, *liquid measure*.

NOTE II. — A kiloliter (1000 liters) has the same capacity as a cubic meter.

NOTE III. — A hectoliter equals about $2\frac{5}{6}$ bushels.

EXAMPLES.

1. Change 35.072 liters to deciliters; to milliliters; to kiloliters; to dekaliters. *Ans.* 350.72^{dcl}; $35{,}072^{ml}$; $.035072^{kl}$; 3.5072^{dkl}.

2. What cost 7 kiloliters of oil at 30 cents a liter? *Ans.* $ 2,100.

3. Change to liters and add 27.4 kiloliters, 87.62 deciliters, 469 centiliters. *Ans.* $27{,}413.452^l$.

MEASURES OF WEIGHT.

TO THE TEACHER. — To introduce the measures of weight, bring into the presence of the pupil a cubic measure of tin or other material, whose capacity is sufficient to contain 1 gram of water. The pupil, by measuring, will find its edge to be 1 centimeter, and its capacity 1 cubic centimeter.

371. The GRAM is the primary unit of weight.

372. A series of units of weight forms a *scale of tens*, and they are named and expressed as units of length are named and expressed, thus: —

Tonneaus (T.)	Quintals (q.)	Myriagram (myg.)	KILOGRAMS (kg.)	Hectograms (hg.)	Dekagrams (Dkg.)	GRAMS.		Decigrams (dcg.)	Centigrams (cg.)	Milligrams (mg.)
4	2	1	7	3	8	5	.	4	9	5

NOTE I. — The gram equals 15.432 grains.

NOTE II. — The five-cent piece, coined since 1866, weighs 5 grams.

NOTE III. — The kilogram, sometimes called the kilo, is considered the unit in weighing gross, heavy articles. The kilogram equals about $2\frac{1}{5}$ pounds avoirdupois, or, more nearly, 2.2046 pounds.

The tonneau equals a little more than 2204 pounds. 1 tonneau of water (1,000,000 grams) equals in size 1 cubic meter.

EXAMPLES.

1. What part of a tonneau is 1 kilogram? 1 gram?

2. At $ 15 per tonneau, what is the value of 1 kilogram of hay? 5 kilos? *Ans.* $ .015; $ .075.

373. MISCELLANEOUS EXAMPLES.

1. At $ 2 a stere, what is the value of 28 dekasteres of stone? 8 decisteres? *Ans.* $ 560; $ 1.60.

2. If $ 750 is paid for land at $ 140 per hectare, how many hectares can be bought?

3. How many cubic meters in a cistern 6.4^m long, 2.25^m wide, and 3.75^m deep? *Ans.* $54.^{cu\ m}$.

4. How many liters in the cistern mentioned in example 3?

NOTE. — The cubic meter equals 1000 liters; hence 54.$^{cu\ m}$ equal 54000 liters. *Ans.* 54,000^l.

Or, since the edge of the liter equals 1 decimeter, the product of the units of length, breadth, and depth in decimeters equals the number of liters; thus $64 \times 22.5 \times 37.5 = 54000$.

5. How many kilograms of water in a tank 5.6^m long, 5^m wide, and 3^m deep?

NOTE. — 1 cubic meter will contain 1000000 grams of water, or 1000 kilograms; $5.6 \times 5 \times 3 \times 1000 = 84000$. *Ans.* 84,000^{kg}.

6. What weight of water, in kilos, may be contained in a cistern 4^m deep, 1.5^m long, and 1.2^m wide? What weight of alcohol, — alcohol being .79 as heavy as water?

Ans. 7,200^{kg} of water; 5,688.^{kg} of alcohol.

7. How many tonneaux in a cube of granite whose edge is 2 meters, the weight of granite being equal to 2.7 of the weight of water?

Ans. 21.6 tonneaux.

374. The units of length, of surface, of volume, of capacity, and of weight constitute a system of measures; and because they are based upon the meter, the system is called the **Metric System.**

REMARKS. — 1. The metric system was first adopted in France in 1795.

To secure for a standard a unit which should be unchanging for all time, a length equal to one ten millionth of a quadrant, or one forty millionth of the circumference of the earth measured over the poles, was agreed upon as the base of the system; this is 39.3685 inches, or nearly 39.37 inches.

A platinum bar, on which was traced a line of this length, 39.37 inches, and declared to be the METER, also a platinum weight equal to 1 KILOGRAM, were deposited in the national archives of France for safe-keeping, as the standards of measures and weights.

2. The use of the metric system was legalized in Great Britain by Parliament in 1864, and in the United States by Congress in 1866; its use is extending among other nations of Europe and America.

This System is now employed in the United States Coast Survey and in books on Scientific subjects.

3. As no abbreviations for the names of the units of the metric system have been agreed upon in this country, it has been thought best to employ the method of abbreviating used by the French.

375. METRIC MEASURES LEGALIZED BY THE UNITED STATES WITH THEIR EQUIVALENTS NOW IN USE.

NOTE.—Although the equivalents here given are not entirely accurate, they are those which are established by Congress for use in legal proceedings, and in the interpretation of contracts, and are sufficiently accurate for all practical purposes.

Measures of Length.

METRIC DENOMINATIONS AND VALUES.		EQUIVALENTS IN DENOMINATIONS IN USE.
Myriameter. . . .	10,000 m. . .	6.2137 miles.
Kilometer	1,000 m. . .	0.62137 mile, or 3280 ft. 10 in.
Hectometer. . . .	100 m. . .	328 ft. 1 in.
Dekameter	10 m. . .	393.7 in.
Meter.	1 m. . .	39.37 in.
Decimeter	.1 m. . .	3.937 in.
Centimeter. . . .	.01 m. . .	0.3937 in.
Millimeter	.001 m. . .	0.0394 in.

Measures of Surface.

METRIC DENOMINATIONS AND VALUES.		EQUIVALENTS IN DENOMINATIONS IN USE.
Hectare	10,000 sq. m.	2.471 acres.
Are	100 sq. m.	119.6 sq. yards.
Centare	1 sq. m.	1550 sq. inches.

Measures of Capacity.

METRIC DENOMINATIONS AND VALUES.			EQUIVALENTS IN DENOMINATIONS IN USE.	
Names.	No. of Liters.	Cubic Measure.	Dry Measure.	Liquid or Wine Measure.
Kiloliter or Stere	1000	1 cu. m.	1.308 cu. yds.	264.17 gal.
Hectoliter. . .	100	.1 cu. m.	2 bu. 3.35 pks.	26.417 gal.
Dekaliter . . .	10	10 cu. dcm.	9.08 qts. . .	2.6417 gal.
Liter	1	1 cu. dcm.	0.908 qt. . .	1.0567 qts.
Deciliter . . .	.1	.1 cu. dcm.	6.1022 cu. in.	0.845 gill.
Centiliter . . .	.01	10 cu. cm.	0.6102 cu. in.	0.338 fld oz.
Milliliter . . .	.001	.1 cu. cm.	0.061 cu. in. .	0.27 fld dr.

Weights.

METRIC DENOMINATIONS AND VALUES.			EQUIVALENTS IN DENOMINATIONS IN USE.
Names.	No. of Grams.	Weight of what quantity of water at maximum density.	Avoirdupois Weight.
Millier or Tonneau .	1,000,000	1 cu. meter . .	2204.6 pounds.
Quintal	100,000	1 hectoliter. . .	220.46 "
Myriagram. . . .	10,000	10 liters	22.046 "
Kilogram or Kilo .	1,000	1 liter	2.2046 "
Hectogram. . . .	100	1 deciliter . . .	3.5274 ounces.
Dekagram	10	10 cu. centim. . .	0.3527 "
Gram	1	1 cu. centim. . .	15.432 gr. Troy.
Decigram	.1	.1 cu. centim. . .	1.5432 "
Centigram	.01	10 cu. millim. . .	0.1543 "
Milligram	.001	1 cu. millim. . .	0.0154 "

376. To change numbers in the metric system to equivalents now in use.

EXAMPLES.

1. Change 1000 kilometers to miles. *Ans.* 621.37 miles.

2. How many cubic yards in 72.5 steres? *Ans.* 94.83 cu. yds.

3. How many bushels in 7 hectoliters? *Ans.* 19 bu. 3.45 pk.

4. How many acres in 34.75 hectares? *Ans.* 85.86725 A.

5. The diameter of the 5-cent piece of 1866 is 2 centimeters; if 100 of these coins were placed in a line, how far would they extend in feet and inches? *Ans.* 6 ft. 6.74 in.

6. The 5-cent piece of 1866 weighs 5 grams; how many pounds avoirdupois are there in 1000 of these coins? *Ans.* 11,023 lb.

377. The following are some of the measures in common use with their equivalents in measures of the metric system:

An inch	= 2.54 centimeters.	A cu. yard	= .7646 cu. meter.
A foot	= .3048 meter.	A cord	= 3.624 steres.
A yard	= .9144 meter.	A liquid quart	= .9465 liter.
A rod	= 5.029 meters.	A gallon	= 3.786 liters.
A mile	= 1.6093 kilometers.	A dry quart	= 1.101 liters.
A sq. inch	= 6.452 sq. centimeters.	A peck	= 8.811 liters.
A sq. foot	= .0929 sq. meter.	A bushel	= 35.24 liters.
A sq. yard	= .8361 sq. meter.	An ounce av.	= 28.35 grams.
A sq. rod	= 25.29 sq. meters.	A pound av.	= .4536 kilogram.
An acre	= .4047 hectare.	A ton	= .9072 tonneau.
A sq. mile	= 259 hectares.	A grain Troy	= .0648 gram.
A cu. inch	= 16 39 cu. centimeters.	An ounce Troy	= 31.104 grams.
A cu. foot	= .02832 cu. meter.	A pound Troy	= .3732 kilogram.

378. To change measures now in use to equivalents in the metric system.

EXAMPLES.

1. Change 5 acres to hectares. *Ans.* .20235 acres.

2. In 48 ft. 6 in. how many meters? *Ans.* 14.7838 m.

3. In 5 tons 200 lbs. how many tonneaux? *Ans.* 4.62672 t.

4. In a pile of wood 40 feet long, 4 feet wide, and 6 feet high, how many steres of wood? *Ans.* 27.18 st.

PERCENTAGE.

379. ILL. EX. What is $\frac{7}{100}$ of $300. *Ans.* $21.

A number, as $21 in the ILL. EX., found by taking a number of hundredths of a given number, is **Percentage.**

Define percentage.

NOTE. — The word "percentage" is derived from the Latin "per centum," which means by the hundred.

380. The number of which a number of hundredths are taken for a percentage, as $300 in the ILL. EX., is the **basis** of percentage.

Define basis of percentage.

381. The number of hundredths of the basis which are taken to obtain a percentage, as $\frac{7}{100}$ in the ILL. EX., is the **rate per cent.**

Define rate per cent.

NOTE. — The sign % is used for the words "per cent."

382. 2 per cent may be expressed .02 or $\frac{2}{100}$ or 2%.
75 per cent " " .75 or $\frac{75}{100}$ or 75%.

Thus any rate per cent may be expressed either decimally, in the form of a common fraction, or by the sign %.

EXAMPLES.

Express the following decimally: —

1.	8%. *Ans.* .08.	3.	25%.	5.	$\frac{1}{2}$%.
2.	$7\frac{1}{2}$%. *Ans.* $.07\frac{1}{2}$.	4.	40%.	6.	$31\frac{1}{4}$%.

Express decimally the difference between 100% and

7.	75%. *Ans.* .25.	9.	$87\frac{1}{2}$%.	11.	$62\frac{1}{2}$%.
8.	$37\frac{1}{2}$%. *Ans.* $.62\frac{1}{2}$.	10.	$66\frac{2}{3}$%.	12.	$6\frac{1}{4}$%.

Express the following as common fractions and change them to their smallest terms: —

13. 100%.	*Ans.* $\frac{100}{100} = 1$.	21. 2%.	27. 12½%.
14. 90%.	*Ans.* $\frac{90}{100} = \frac{9}{10}$.	22. 50%.	28. 6¼%.
15. 80%.	*Ans.* $\frac{80}{100} = \frac{4}{5}$.	23. 30%.	29. 66⅔%.
16. 60%.	*Ans.* $\frac{60}{100} = \frac{3}{5}$.	24. 25%.	*Ans.* $\frac{66\frac{2}{3}}{100} = \frac{2}{3}$.
17. 40%.	*Ans.* $\frac{40}{100} = \frac{2}{5}$.	25. 75%.	30. 33⅓%.
18. 10%.	*Ans.* $\frac{10}{100} = \frac{1}{10}$.	26. 37½%.	31. 16⅔%.
19. 5%.	*Ans.* $\frac{5}{100} = \frac{1}{20}$.	*Ans.* $\frac{37\frac{1}{2}}{100} = \frac{3}{8}$.	32. 8⅓%.
20. 4%.	*Ans.* $\frac{4}{100} = \frac{1}{25}$.		

383. ILL. EX. What per cent of a number is ⅕ of the number?

OPERATION.

$$5\,)\,\underline{100\%}$$
$$20\%$$

Explanation. — Since any number equals 100% of itself, ⅕ of the number equals ⅕ of 100% or 20%. *Ans.* 20%.

EXAMPLES.

33. What % of a number is ¼ of the number? ⅕? ⅓? ⅗?

34. What % of a number is $\frac{1}{10}$ of the number? $\frac{1}{20}$? ⅝? ¾?

35. What % of a number is ⅔ of the number? $\frac{1}{16}$? ⅚? $\frac{3}{40}$?

384. TO FIND THE PERCENTAGE WHEN THE BASIS AND THE RATE PER CENT ARE GIVEN.

ILL. EX. What is 5% of \$20?

OPERATION.

$$\frac{\$20 \times 5}{100} = \$1.$$

Explanation. — 5% of \$20 is $\frac{5}{100}$ of \$20, which equals \$1. *Ans.* \$1.

From the above we derive the following

RULE. — To find the percentage when the basis and rate per cent are given: *Multiply the basis by the rate per cent.*

EXAMPLES.

36. What is 6% of \$75? *Ans.* \$4.50.

37. What is 10% of \$180? 7½% of 200 days?

38. What is the difference between 8% of \$450 and 6¼% of \$680? *Ans.* \$6½.

385. TO FIND THE AMOUNT OR REMAINDER, WHEN THE BASIS AND THE RATE PER CENT ARE GIVEN.

ILL. EX. At a certain time A and B each owed \$ 300; in one year A's debt was increased 20% and B's was diminished 20%, what was then the amount of A's debt? What remained of B's?

Explanation.—20% of \$ 300 is \$ 60, which is the percentage of increase of A's debt, and of decrease of B's; \$ 300 + \$ 60 = \$ 360, amount of A's debt. \$ 300 — \$ 60 = \$ 240, what remained of B's.

The basis plus the percentage is called the **amount**.

The basis minus the percentage is called the **remainder**.

The amount may be found by taking $\frac{120}{100}$ of the basis, and the remainder by taking $\frac{80}{100}$ of the basis. Hence the following rules,

RULE I. To find the amount when the basis and rate per cent are given: *Multiply the basis by* 1 *plus the rate per cent.*

RULE II. To find the remainder when the basis and rate per cent are given: *Multiply the basis by* 1 *minus the rate per cent.*

EXAMPLES.

39. A merchant had 7000 tons of coal, and shipped 65% of it; how much had he left? *Ans.* 2,450 tons.

40. At what price must a chair be sold which cost \$ 1.25 that 10% may be gained? *Ans.* \$ 1.37½.

41. A mechanic sold a shop which cost him \$ 800 at a loss of 12½%; what did he receive for it?

42. A man willed to his sons John and William \$ 5500 each; within one year John spent $7\frac{3}{10}$% of his money and William increased his 12½%; at the end of the year how much more money had William than John?

43. A trader sold 100 pairs of gloves at \$ .80 per pair; 5% of the price was deducted for prompt payment; what was the sum deducted? what was the balance paid?

Ans. \$ 4 deducted; \$ 76 paid.

NOTE.—That part of the nominal price of goods which is deducted, as \$ 4, in the above example, is called a **discount**.

44. A lot of crockery was sold for $128; if a discount of 6% was made for prompt payment, what was the balance paid? *Ans.* $120.32.

45. What is the balance of a bill of $270 for books after a discount of $33\frac{1}{3}$% has been made?

46. What is the balance of a bill of $64.50 after two discounts have been made; the first of 20% on the $64.50, the other of 5% upon what then remained? *Ans.* $49.02.

386. TO FIND THE BASIS WHEN THE RATE PER CENT AND THE PERCENTAGE ARE GIVEN.

ILL. EX. A man lost $63, which was 9% of all the money he had; how much money had he?

OPERATION.

$$\frac{\$\overset{7}{\cancel{63}} \times 100}{\cancel{9}} = \$700$$

Explanation.—If $63 is 9% or $\frac{9}{100}$ of all the money he had, $\frac{1}{100}$ of what he had must be $\frac{1}{9}$ of $63, and $\frac{100}{100}$ must be ($\frac{1}{9}$ of $63) × 100, which equals $700. *Ans.* $700.

From the above we derive the following

RULE.—To find the basis when the rate per cent and the percentage are given: *Divide the percentage by the rate per cent.*

EXAMPLES.

47. A man rents a house at $575 a year, which is 10% of its valuation; what is its valuation? *Ans.* $5,750.

48. A lawyer charged $7.50 for collecting money, which was 6% of what he collected; how much did he collect?

49. A man failing in business pays 25% of what he owes; if he pays one creditor $250.75, what did he owe him?

50. If the percentage is $18.75 and the rate 5%, what is the basis? *Ans.* $375.

51. If the percentage is $37\frac{1}{2}$ bushels and the rate 8%, what is the basis? *Ans.* $468\frac{3}{4}$ bushels.

☞ For Dictation Exercises upon these examples, see "Manual and Key," page 117.

387. To find the Basis, when the Amount, or the Remainder, and the Rate per cent are given.

Ill. Ex. I. After adding 5% to the weight of a lot of hay by salting, the weight was 1869 lbs.; what was the weight before salting?

OPERATION.

$$\frac{\overset{89}{\cancel{1869}} \times \overset{20}{\cancel{100}}}{\underset{21}{\cancel{105}}} = 1780$$

Explanation. — If 5% was added to the weight, the amount, 1869 lbs., must be 105% of the weight. If 1869 lbs. is $\frac{105}{100}$ of the weight, $\frac{1}{100}$ must be $\frac{1}{105}$ of 1869 lbs., and $\frac{100}{100}$ must be ($\frac{1}{105}$ of 1869 lbs.) $\times$ 100 = 1780 lbs. *Ans.* 1,780 lbs.

Ill. Ex. II. How many soldiers were there in a regiment at first, which, after losing 8% by sickness, has 644 remaining?

OPERATION.

$$\frac{\overset{7}{\cancel{644}} \times 100}{\cancel{92}} = 700$$

Explanation. — Since 8% are lost, 92% remain. If 644 soldiers are $\frac{92}{100}$ of the regiment, $\frac{1}{100}$ must be $\frac{1}{92}$ of 644 soldiers, and $\frac{100}{100}$ must be ($\frac{1}{92}$ of 644 soldiers) $\times$ 100 = 700 soldiers. *Ans.* 700 soldiers.

From the above we derive the following rules.

Rule I. To find the basis, amount and rate per cent being given: *Divide the amount by* 1 *plus the rate per cent.*

Rule II. To find the basis, remainder and rate per cent being given; *Divide the remainder by* 1 *minus the rate per cent.*

Examples.

52. What should be the weight of a loaf of bread before baking, that it may weigh 30 ounces afterwards, if in the process of baking it loses $6\frac{1}{4}$%? *Ans.* 32 oz.

53. A certain school has now 84 pupils, which is 50% more than it had last year; how many had it last year? 54.

54. After increasing the wages of his workmen $33\frac{1}{3}$%, a manufacturer paid his operatives $2 a day; what did he pay them before? *Ans.* $1.50.

55. By increasing the rate of speed in the machinery of a mill 7%, 1498 yards of cloth were woven in a day; how many yards could be woven by the former rate? *Ans.* 1,400 yds.

56. By diminishing the rate of speed of a locomotive 8%, it ran but 23 miles in an hour; how many miles did it run before? *Ans.* 25 miles.

57. When a discount of 25% had been made upon the published price of a book, it was sold for $ 2.25; what was the published price?

388. TO FIND THE RATE PER CENT, WHEN THE BASIS AND THE PERCENTAGE ARE GIVEN.

ILL. EX. If of 30 words 25 only are spelled correctly, what per cent of the words are spelled correctly?

OPERATION.	
30)25.00 $.83\frac{1}{3}$	*Explanation.* — If of 30 words 25 are spelled correctly, $\frac{25}{30}$ of the words are spelled correctly. $\frac{25}{30}$ changed to hundredths are $.83\frac{1}{3}$, or $83\frac{1}{3}$ per cent. *Ans.* $83\frac{1}{3}$%.

From the above we derive the following

RULE. — To find the rate per cent when the basis and percentage are given: *Divide the percentage by the basis, continuing the division to 100ths.*

EXAMPLES.

58. If the working hours of a day are changed from 10 hours to 8, what is the per cent of deduction? *Ans.* 20%.

59. If when wages are $ 3 a day, $ 1 be added, what is the per cent of increase? *Ans.* $33\frac{1}{3}$%.

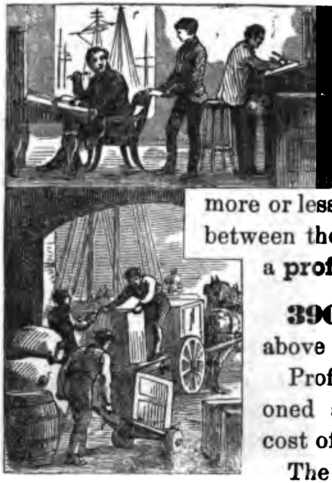
60. If wages are advanced from $ 20 to $ 24 a week, what is the per cent of increase? *Ans.* 20%.

61. If a coat, on which the charge is $ 36, is sold for $ 30, what is the per cent of discount? $16\frac{2}{3}$

62. What is the rate per cent when the basis is 40 bushels and the percentage 15 bushels?

☞ For Dictation Exercises upon these examples, see "Manual and Key," page 117.

PROFIT AND LOSS.



389. ILL. EX. I bought goods for $16; what shall I receive for them if I sell them for $4 more than they cost? for $4 less than they cost?

When goods are sold for more or less than they cost, the difference between the cost and the selling price is a **profit** or a **loss.**

390. The $4 of profit or loss above is 25% of the cost, $16.

Profit and loss may thus be reckoned at a certain per cent of the cost of the goods which are sold.

The COST is, then, the BASIS; the PROFIT OR LOSS the PERCENTAGE.

Hence the rules of percentage already illustrated apply to Profit and Loss.

EXAMPLES.

391. What is the profit or loss when goods which cost

1. 75 cents sell at a profit of 20%? *Ans.* 15 cents.
2. $2500 sell at a loss of 30%? *Ans.* $750.
3. $450 sell at a loss of 13%?
4. $4.88 sell at a profit of 37½%? *Ans.* $1.83.
5. $375.18 sell at a profit of 50%?

392. At what price must I sell goods which cost

6. $150 to make 75% profit? *Ans.* $262.50.
7. $720 to make 12½% profit? *Ans.* $810.
8. $17000 to lose 4%? *Ans.* $16,320.
9. $250 to lose .5%? *Ans.* $248.75.
10. $762 to gain 13½%? 11. $4.74 to gain 16⅔%?

393. What was the cost of goods when

12. A gain of $10 was 5% of the cost? *Ans.* $200.
13. A gain of $.06 was 20% of the cost? *Ans.* $.30.
14. A loss of $50 was $2\frac{1}{2}$% of the cost? *Ans.* $2000.
15. A loss of $.$33\frac{1}{3}$ was 4% of the cost? $8.33 $\frac{1}{3}$ ans
16. A gain of $315 was $2\frac{1}{4}$% of the cost? 14000. ans
17. A gardener sold a lot of plants on which he gained $84, which was $10\frac{1}{2}$% of the cost; what was the cost? $800.

394. What was the cost of goods which selling

18. At a loss of 25%, brought $30? *Ans.* $40.
19. At a gain of 50%, brought $.75? *Ans.* $.50.
20. At a loss of 8%, brought $1150?
21. At a gain of $12\frac{1}{2}$% brought $9.36? *Ans.* $8.32.
22. A horse becoming lame was sold for $150, which was at a loss of 70% of the price at which he had been valued; at what price had he been valued? *Ans.* $500.
23. I bought a store for a certain sum, and after paying a tax of $2\frac{1}{2}$% on the cost and $\frac{1}{2}$% for insurance, I sold it for $7828 which exactly covered the cost, tax, and insurance; what was the cost?

395. What is the % of profit or loss when goods which

24. Cost $20, sell for $22.50? *Ans.* $12\frac{1}{2}$% profit.
25. Cost $3200, sell for $2800? *Ans.* $12\frac{1}{2}$% loss.
26. Cost $15 a doz., sell at $2 each? *Ans.* 60% profit.
27. Cost 10 cents a yard, sell for $12\frac{1}{2}$ cents? 25%
28. Cost $6.25, sell at $8? at $$8.33\frac{1}{3}$? 28 33 $\frac{1}{3}$
29. Cost $10, sell at $12.50? at $$16\frac{2}{3}$?
30. When a half-ream of paper, which cost $2, is sold at 35 cents per quire? *Ans.* 75% profit.
31. When a hundred logs which cost $65 are sold at 78 cents each? 20

For Dictation Exercises in Profit and Loss, see "Manual and Key," page 118.

396. MISCELLANEOUS EXAMPLES.

1. If a quantity of gold weighed 5 ounces before refining, and 3 ounces afterwards, what was the per cent of loss? 40%

2. If a bird-fancier bought a pair of canaries for $ 5, and sold them for $ 7, what was the per cent of gain? 40%

3. A mother is 48 years of age and her daughter is 28; what per cent of the mother's age is the daughter's age? 58⅓%

4. If both mother and daughter live 12 years, what per cent of the mother's age will the daughter's age then be?

5. A merchant bought muslin at 24 cents a yard; at what price must he sell it to gain 25%? to lose 16⅔%?

6. If shell-bark hickory is sold at $ 10 per cord, and if the heating power of other kinds of wood in relation to hickory is as follows, poplar, 40%; white pine, 42%; pitch pine, 43%; white birch, 48%; hard maple, 60%; and white oak, 86%; what should be paid for 1 cord of each of these other kinds of wood? *Sum of Ans.* $ 31.90.

7. What should a bookseller charge for a book for which he paid at the rate of $ 54 a dozen, that he may make 20% upon the cost?

8. What per cent of profit is made by selling watch crystals at 30 cents each which cost $ 12½ per hundred?

9. What is the increase of a person's salary which, when raised 25%, amounts to $ 1500? *Ans.* $ 300.

10. What must I pay for 25 railroad tickets at 20% discount, when single tickets are sold at $ 3.35 apiece?

11. In a certain city a deduction of 1% a month is made on taxes paid before October 1, and an extra charge of 1% a month is made on those paid after October 1; how much less will A pay than B, the former paying August 1 and the latter December 1, the tax of each being $ 60?

12. What is the per cent of discount when a bill of \$375 is settled for \$337.50? *Ans.* 10%.

397. Examples for Advanced Pupils.

13. Prunes were sold at 15 cents per pound at a profit of 25%; for what should they have been sold to afford a profit of 40%?

NOTE.—First find the cost. *Ans.* \$.16$\frac{4}{5}$.

14. 25% was lost by selling oats at \$.45 per bushel; what per cent would have been lost or gained by selling them at \$.50?
Ans. 16$\frac{2}{3}$% lost.

15. Cloth was sold for \$3.30 per yard, at 10% above cost; what would it have sold for at 10% below cost? *Ans.* \$2.70.

16. For what should hay have been sold per ton to gain 16$\frac{2}{3}$%, if 33$\frac{1}{3}$% was lost by selling at \$16? *Ans.* \$28.

17. I sold a horse-rake for \$24 at a profit of 20%; what would have been the per cent of profit or loss if I had sold it for \$18? for \$12? *Ans.* 10% loss; loss.

18. By selling potatoes at a loss of 8 cents per bushel, I received but 80% of their cost; what per cent of their cost should I have received by selling them at a gain of 10 cents per bushel? *Ans.* 125%.

19. A lot of apples was sold at \$3.85 per barrel, which was 12$\frac{1}{2}$% less than they cost; but they cost 10% more than the present wholesale price; what is the present wholesale price? *Ans.* \$4.00.

20. If sugar at 6$\frac{3}{10}$ cents a pound is 12$\frac{1}{2}$% above the former wholesale price, but 55% below the present wholesale price, what per cent of advance has been made in the wholesale price? *Ans.* 150%.

21. $\frac{3}{4}$ of a lot of peaches bought at \$2 a basket were sold at a gain of 40%, the rest at a loss of 20%; what was the per cent of gain by the whole transaction? *Ans.* 25%.

22. $\frac{1}{2}$ of a lot of tea was sold at a profit of 50%; the remainder of the lot was sold at \$.65 per pound, which brought the average per cent of profit down to 40%, what was the cost of the tea per pound?
Ans. \$.50.

COMMISSION.

398. ILLUSTRATION.—One person, A, employs another person, B, to make purchases and sales of goods for him, to collect his bills or transact other business.

For the labor which B thus performs A pays B a percentage on the amount of money he uses in buying, or receives in selling, collecting, or in transacting other business. This percentage is called **commission.**

Define commission.

399. As B is employed to transact business for another person, B is called an **agent.**

400. Here the MONEY USED OR RECEIVED is the BASIS, and the COMMISSION is the PERCENTAGE.

Hence the processes already illustrated in percentage apply to operations in Commission.

401. EXAMPLES.

1. An agent bought for me 25 horses at $ 75 each; his commission being 3% of the price he paid, what was his commission, and what the entire cost of the horses?

NOTE.—By Art. 384, ($ 75 × 25) × $\frac{3}{100}$ = $ 56.25 Com.
By Art. 385, ($ 75 × 25) × $\frac{103}{100}$ = $ 1931.25 Cost.

2. What is the commission, at 2½%, on the purchase of 100 bales of cotton, at $ 200 per bale; and what is the entire cost? *Ans.* Com., $ 500; Cost, $ 20,500.

3. My agent sold 166 boxes of soap at $ 3 per box; if he is allowed a commission of 4½% on the amount of the sale, what is his commission, and what are the net proceeds?

NOTE.—The commission being deducted from the amount of sales, the remainder is the **net proceeds.**

Ans. Com., $ 22.41; Proceeds, $ 475.59.

4. What is the commission on the sale of 2750 lbs. of leather at \$.30 per lb., commission $3\frac{1}{2}\%$; and what are the net proceeds? *Ans.* Com., \$28.87½; Proceeds, \$796.12½.

5. If the rate of commission on the sale of a lot of land was 2%, and the percentage received by an agent for selling was \$28, what was the amount of the sale?

NOTE.—Here \$28 = $\frac{2}{100}$ of the amount of sales. *Ans.* \$1,400.

6. I bought, as an agent, a lot of cloth at \$5.20 a yard; my percentage of commission being \$22.10, and the rate 5%, what was the entire cost of the cloth, and what was the number of yards bought? *Ans.* \$442; 85 yards.

7. A farmer hired a man to cut and haul wood; the man was to receive 20% of the wood cut for his services; if the man delivered to the farmer 320 cords, how many cords were cut? and what percentage did the cutter receive?

NOTE.—Here 320 cords is the remainder after the percentage has been deducted. *Ans.* 400 cds. cut; 80 cds. percentage.

8. My agent sells for me a lot of cheese; after deducting his commission, which was $2\frac{1}{2}\%$ of the amount of sales, the net proceeds were \$405.60, what was the amount of sales? *Ans.* \$416.

9. A commission merchant received \$4160 with which to purchase carpeting, after deducting his commission of 4% on the money to be expended; what was the sum to be expended, and what was his commission?

NOTE.—Here \$4160 is an amount, the money to be expended being the basis, which may be considered 100% of itself, and the commission being the percentage, which equals 4% of the basis; therefore, \$4160 = $\frac{104}{100}$ of the sum to be expended. *Ans.* \$4,000; \$160.

10. I have sent to my agent at Paris \$2295, of which he is to expend what he can in silk after deducting his commission of 2% on the purchase; what sum can he expend, and what is his commission? *Ans.* \$2,250; \$45.

11. What part of a remittance of $ 464.43 will remain to be expended after $2\frac{3}{4}$% of the sum to be expended has been deducted? *Ans.* $ 452.

12. I send to my agent $ 4488.75, of which he is to lay out what he can in land at $ 15 per acre, after his commission of 5% has been deducted; how many acres can he buy, and what is his commission? *Ans.* 285 acres; $ 213.75.

402. MISCELLANEOUS EXAMPLES IN COMMISSION.

13. My agent writes me that he has sold, on my account, 10 sets American Encyclopædia, of 21 volumes each, at $ 3.90 per volume; what is his commission at $12\frac{1}{2}$% on the sale?

14. Find the net proceeds of the rent of a house for 9 months at $ 16 per month, after deducting the agent's commission of 8%, and $ 7.50 for repairs? *Ans.* $ 124.98.

15. My agent has sold for me 2000 yards of cloth at 24 cents a yard; he allowed a discount of 5% for cash and deducted his commission of $2\frac{1}{2}$% upon his cash receipts; what were my net proceeds? *Ans.* $ 444.60.

16. At 15 cents per yard, how many yards of cotton cloth can be bought with the net proceeds of $ 636.30, a commission of 1% upon the net proceeds being allowed for making the purchase? *Ans.* 4,200 yards.

17. The net proceeds from the sale of a lot of sheeting being $ 288, after a commission of 4% had been reserved by the agent, what was the number of yards sold at 25 cents a yard? *Ans.* 1,200 yards.

18. What sum must be sent to an agent that he may purchase 2000 pounds of sugar at $5\frac{3}{8}$ cents per pound, and retain his commission of $2\frac{2}{3}$% on the purchase? *Ans.* $ 110.37.

19. What is the rate per cent of commission when an agent reserves to himself $ 52.50 of $ 1552.50, sent him to purchase hay at $ 20 per ton? What number of tons can he purchase? *Ans.* $3\frac{1}{2}$%; 75 tons.

☞ **For Dictation Exercises in Commission, see "Manual and Key," page 120.**

STOCKS AND DIVIDENDS, AND BROKERAGE.

403. ILLUSTRATION. — Several persons, A, B, and C, associate themselves together for the purpose of manufacturing paper, or for other business purposes.

An association of persons formed for the purpose of transacting business is called a **company**.

404. A brings into the company $5000, B $7000, and C $10000, to be used in the business of manufacturing. These sums of money are thus invested for future production of other money.

Property invested for future production is **capital**.

405. A, B, and C employ their capital in manufacturing. Capital employed by a person or a company in business is **stock**.

406. The stock of the company A, B, and C amounts to $22000. This amount may be divided into 220 equal parts of $100 each.

Each of the equal parts into which stock is divided is a **share**.

NOTE. — The written statements which A, B, and C have of the number of shares each owns or possesses of the stock employed are called **certificates** of stock.

407. The owners of stock are called **stockholders**.

Define company; capital; stock; share or shares; stockholders.

408. The gain upon the capital of a company is divided among the stockholders; gain thus divided is called a **dividend**.

The DIVIDEND is reckoned as a PERCENTAGE, the CAPITAL being the BASIS: hence each stockholder's part of the dividend is the same per cent of his stock that the whole dividend is of the capital.

NOTE I. — Stocks may be bought and sold like other property.

NOTE II. — When a share of stock will sell at its original value, it is **at par**; when a share will sell for more than its original value, it is **above par**, or at a **premium**; when it sells for less than its original value, it is **below par**, or at a **discount**.

409. Persons who buy and sell stock as a business are called **stock-brokers**.

410. The commission paid to a broker is called **brokerage**.

The processes already illustrated in percentage apply to Stocks and Dividends, and Brokerage.

411. EXAMPLES.

1. What is the cost of 12 shares of bank stock, original value $ 100 each, at 6% above par?

NOTE. — $ 1200, the original value, is the basis of which the **amount** is required; 106% of $ 1200 = $ 1272. *Ans.* $ 1,272.

2. What cost 20 shares of railroad stock, original value $ 100 each, at $7\frac{1}{2}$% above par? *Ans.* $ 2,150.

3. What is the value of 5 shares of the stock of a company for working a silver-mine, the original value being $ 50 per share, at 8% below par?

NOTE. — $ 250, the original value, is the basis of which the remainder is required. *Ans.* $ 230.

4. What is the brokerage at $\frac{1}{4}$% on the sale of the above stock? *Ans.* $ .575.

5. When gold is at a premium of $26\frac{2}{3}$%, what must be paid, in currency, for $ 1200 in gold? *Ans.* $ 1,520.

NOTE. — Gold, at a premium, is bought and sold like other property.

6. What is the brokerage at $\frac{1}{4}$% upon the purchase of the above gold? 380

7. If I buy 4 shares of stock, originally worth $100 a share, at 10% below par, and sell it at 4% above par, what do I gain? *Ans.* $56.

8. What would be my loss if I should buy 5 shares of stock at 2% above par, and sell it at $10\frac{1}{2}$% below par, the par value being $100 each? *Ans.* $62.50.

9. What shall I lose by buying 2 shares of Pacific Mills stock at 90% above par, and selling it at $87\frac{1}{2}$% above par, par value being $1000 per share?

10. When stock, originally worth $300, sells for $325, at what per cent above par does it sell? *Ans.* $8\frac{1}{3}$%.

11. When 50 shares of stock, of which the par value is $100 per share, sell for $4000, at what per cent below par does the stock sell? *Ans.* 20%.

12. If by buying stock at par and selling it at 7% below par, $3.50 is lost on each share, what was the par value?

13. A dividend of $4\frac{1}{4}$% is paid upon the stock in a bank; the original value being $100 per share, what is paid on 75 shares? on 120 shares?

14. 3 semi-annual dividends of 6% each were paid on a certain railroad stock; what was the sum of the 3 dividends on stock, the par value of which was $1800?

15. Owning 27 shares of stock in a mining company, I received a dividend of $33.75, which was $2\frac{1}{2}$% of the par value of my stock; what was the par value of the stock per share? *Ans.* $50.

16. A person sold a lot of stock at $27.50 a share, which was 10% above par; what was the par value?

17. A broker sold 15 shares of stock for $780, which was 4% above par; what was the par value per share; and what was the brokerage at $\frac{1}{2}$% on the sale?

☞ For Dictation Exercises in Stocks, etc., see "Manual and Key," page 120.

INSURANCE.

412. ILLUSTRATION. — A, who owns a house, pays $100 a year, that $10000 of the value of his house may be secured to him, if it should be destroyed or damaged.

A also pays a sum of money that another sum may be secured to his heirs in case of his death, or that a certain sum may be secured to himself in case of sickness or accident.

The security one has against loss if his property is destroyed or damaged, or the security he has that a sum of money shall be paid to his heirs in case of his death, or to himself in case of sickness or accident, is **insurance.**

413. The security against loss of property is **property insurance.**

414. The security of a sum of money in case of death, sickness, or accident, is **life, health,** or **accident insurance.**

415. The parties that secure against loss are called **underwriters** or **insurers.**

416. Persons that are secured are called the **insured.**

417. The written contract that binds the underwriters is called the **policy.**

418. The sum, as $100 above, paid to the underwriters, is the **premium.**

Define insurance; property, life, health, and accident insurance; underwriters; policy; premium.

419. The premium, as $100 in the Ill. Ex., is a certain per cent of $10000, the sum insured.

The SUM INSURED is, then, the BASIS; the PREMIUM, the PERCENTAGE.

Hence the rules of percentage already illustrated apply to Insurance.

420. EXAMPLES.

1. What is the premium for insuring a brick house, the value of which is $4000, insurance being effected upon $\frac{3}{4}$ of the value, at 1%? *Ans.* $30.

NOTE. — Some insurance companies never insure for more than *three fourths* of the estimated value of the property.

2. What is the premium for insuring a steam-mill for $75000, at $1\frac{1}{2}$%? *Ans.* $1,125.

3. What is the cost of insuring a cargo of wheat for $6500, at $2\frac{1}{2}$%, including $1 for the policy? *Ans.* $163.50.

4. A premium of $\frac{1}{2}$% was paid for insuring property to the amount of $25000, and subsequently the property was damaged by fire to the amount of $8250; what was the net loss to the underwriters if they made good this loss?

NOTE. — $8250 minus the premium is the net loss. *Ans.* $8,125.

5. Insurance was effected on a store to the amount of $5600, at the rate of $10 on $1000, $1 being paid for the policy; what did the insurance cost? *Ans.* $57.

6. A merchant insured 8000 barrels of pork, worth $15 a barrel, for $\frac{3}{4}$ of its value, at $\frac{1}{4}$ of 1%; what was the premium for insurance?

7. If the whole property above was destroyed by fire, what was the net loss to the underwriters? *Ans.* $89,775.

8. Having goods valued at $2400, I effected an insurance on $\frac{5}{8}$ of their value at $\frac{2}{5}$%; what was the premium?

9. What will be my whole loss if the above-mentioned goods are destroyed by fire? *Ans.* $906.

10. The premium for insuring a house at $\frac{1}{2}$% was \$ 7.50; what was the sum insured? *Ans.* \$ 1,500.

11. The premium for insuring a lot of flour at \$ 9 a barrel was \$ 135; the rate of insurance was $1\frac{1}{4}$%; what was the number of barrels of flour?

12. I paid for insuring a stable \$ 2 on \$ 100; if the premium, with \$ 1 for the policy, was \$ 25, and the insurance upon $\frac{3}{4}$ of the value of the stable, what was the value of the stable? *Ans.* \$ 1,600.

13. I paid \$ 33.75 for insuring \$ 2700 on my house for 5 years; what was the per cent of the yearly premium?

Ans. $\frac{1}{4}$%.

14. An agent of the American Popular Life Insurance Company collected the premium on my life policy for \$ 5000 at the rate of \$ 19.37 on \$ 1000, and charged me \$ 1 for the policy; what was the sum collected? *Ans.* \$ 97.85.

NOTE.—Life insurance is estimated at a certain rate per \$ 100 or per \$ 1000 for which the person is insured. Life tables are constructed based upon the average length of life, by which the premiums are reckoned.

☞ For Dictation Exercises, see "Manual and Key," page 121.

TAXES.

421. When a number of persons, as the citizens of a state or town, or the members of a society, are associated, the expenses of the association are generally met by a sum assessed on the persons associated, on their property or on their income. This sum is called **a tax.**

422. When the tax is assessed on a person, it is called a **poll tax.**

NOTE.—In this country the male citizens of a city, county, or town, not exempted by law, pay an equal poll tax. These persons are sometimes called the *polls.*

423. When the tax is assessed on property, it is called a **property tax**, and is reckoned at a certain rate per cent on the estimated value of each person's property.

424. When the tax is assessed on the income, it is called an **income tax**, and is reckoned at a certain per cent of the income.

425. Movable property, such as money, stocks, cattle, ships, etc., is called **personal property.** Immovable property, as lands, houses, etc., is called **real estate.**

426. Persons appointed to assess taxes are called **assessors.** It is their duty to estimate the value of the taxable property, to ascertain the number of polls, and to apportion the sum to be raised by taxation among the individuals to be taxed.

427. ILL. EX. The tax of a certain town is \$ 15980. The town contains 980 polls, which are assessed \$ 2 each. The taxable property of the town is valued at \$ 1752500. What is the tax on \$ 1, and what will J. Brown's tax be, his property being estimated at \$ 13700, he paying 1 poll tax?

OPERATION.

Tax to be assessed on property,	$\$15980 - \$2 \times 980 = \$14020$
The rate on \$ 1,	$\$14020 \div \$1752500 = .008$
Brown's whole tax,	$\$13700 \times .008 + \$2 = \$111.60.$

Ans. \$ .008; \$ 111.60.

From the above we may derive the following

RULE. — For the assessment of taxes: 1. To find the rate of the property-tax, *take out of the whole tax the sum of the poll taxes, and divide the remainder by the number of dollars of taxable property.*

2. To find each person's tax, *multiply each person's taxable property by the rate, and to the product add his poll tax, if he has any.*

EXAMPLES.

1. The town of Chester is to raise $ 3971 by taxes; the taxable property of the town is $ 700000; there are 471 polls, each taxed $ 1. What will be the tax on $ 1, and what is the tax on $ 3500, with 1 poll tax? *Ans.* $ .005; $ 18.50.

2. The valuation of a certain city is $ 24000000; the amount of the state, county, and city taxes is $ 336000; of this $\frac{1}{7}$ is to be assessed upon the polls, of which there are 38400; what is the poll tax and the rate of property tax? *Ans.* Poll tax, $ 1.25; Rate, $ .012.

3. A tax of $ 13590 is to be assessed upon a certain town; the property is valued at $ 1500000; there are 3120 polls to be taxed 75 cents each; find the tax on $ 1. *Ans.* $7\frac{1}{2}$ mills, tax on $ 1.

NOTE. — Assessors commonly construct a table, showing the tax on $ 1, $ 2, $ 3, etc., from which they compute the individual taxes. From the above example we should have the following

TABLE,

Showing the tax on various sums at the rate of $7\frac{1}{2}$ mills on $ 1.

$ 1	pays	$ .007½	$ 10	pays	$ .075	$ 100	pays	$ .75
2	"	.015	20	"	.15	200	"	1.50
3	"	.022½	30	"	.225	300	"	2.25
4	"	.03	40	"	.30	400	"	3.00
5	"	.037½	50	"	.375	500	"	3.75
6	"	.045	60	"	.45	600	"	4.50
7	"	.052½	70	"	.525	700	"	5.25
8	"	.06	80	"	.60	800	"	6.00
9	"	.067½	90	"	.675	900	"	6.75
10	"	.075	100	"	.75	1000	"	7.50

ILL. EX. At the above rate, what is the tax of A, who is assessed on $ 8276, and who pays 2 poll taxes of $.75 each?

NOTE. — Find by the table the tax on $ 8000, $ 200, $ 70, and $ 6.

Ans. $ 62.07.

Find the tax of the following persons at the above rate: —

4. Of B, who is assessed for $ 1525 and 1 poll.
5. Of C, who is assessed for $ 7250 and 1 poll.
6. Of D, who is assessed for $ 863 and 2 polls.
7. Of E, who is assessed for $ 796 and 3 polls.

NATIONAL TAXES.

428. The expenses of the national government are met in part by taxes on profits from business, incomes, etc., and by a special tax on certain occupations, articles manufactured, or articles kept for use, as carriages, gold watches exceeding a certain valuation, on billiard-tables, gold plate, silver plate exceeding 40 oz. Troy, etc.

The money derived from taxes on profits, incomes, occupations, etc., is called **internal revenue.**

ILL. EX. The income of A. J. Ames for the year 1868 was $ 5940; take out of this sum the following items: $ 1000 exempted by law, $ 450 paid for rent, $ 175 for insurance, $ 1200 for clerk, $ 257 for repairs, $ 96.40 for town and county taxes; and reckon 5% on the balance. Add to this percentage, $ 2 tax on a gold watch, $ 2 on a carriage, also a tax of 5 cents per oz. on 28 oz. of silver (it being his excess of 40 oz.), and find the amount of Ames's income tax.

OPERATION.

Amount of Income, ...	$ 5940.00
Deductions. { Exempt, $ 1000; Rent, $ 450; Insurance, $ 175; Clerk, $ 1200; Repairs, $ 257; Taxes, $ 96.40; }	3178.40
Taxable income,	$ 2761.60
Income Tax, (5% of $ 2761.60)	138.08
Special tax: — Watch $ 2, Carriage $ 2, Silver plate $ 1.40	5.40
Amount of tax, *Ans.*	$ 143.48

EXAMPLE.

1. Find T. Hill's national tax for the year 1869, his income consisting of a salary of $ 4000, and proceeds of published works $ 1468.40, at 5% on all except the following deductions: $ 1000 exempted by law, $ 800 paid for rent, and $ 225.76 for taxes. Add to this percentage the special tax of 5 cents per oz. on 150 oz. silver plate, $ 2 each on two carriages, and $ 2 on a gold watch.

Ans. $ 185.63.

DUTIES OR CUSTOMS.

429. The expenses of the national government are met in part by a tax on imported goods and upon vessels.

The tax is laid upon a vessel according to the weight she is estimated to carry, which is called the vessel's **tonnage.**

The taxes upon imported goods and upon the tonnage of vessels are called **duties** or **customs.**

430. Duties are collected where goods are brought into the country; such places are called **ports of entry.**

431. At the ports of entry are one or more persons who collect the duties. These persons are called **custom-house officers.**

NOTE. — A list of imported goods showing the quantity, also the price and where purchased, is an **invoice.**

432. When a uniform tax is laid upon each imported article of a certain kind, the tax is called a **specific duty.**

433. When a tax of a certain per cent is laid upon the invoice price of goods, the tax is an **ad valorem duty.**

NOTE I. — The weight of goods, including whatever is used for packing, is called **gross weight.**

NOTE II. — The weight of the goods alone is called **net weight.**

NOTE III. — An allowance for a loss from the leaking and breaking of bottles, boxes, etc., is called **leakage** or **breakage.**

NOTE IV. — An allowance for the weight of boxes, etc., is called **tare.**

434. EXAMPLES.

1. What is the duty, at 2 cents per pound, on 8 boxes of sugar, averaging 275 pounds, tare being 5%?

OPERATION.

5% of (275 lb. × 8) = 110 lb. tare.

(275 lb. × 8) — 110 lb. = 2090 lb. net weight.

$.02 × 2090 = $41 80 duty. *Ans* $41.80.

2. What is the duty, at 5 cents per pound, on 450 boxes of raisins weighing 25 lbs. each, tare being 8%? *Ans.* $517.50.

3. What is the duty, at 8 cents per gallon, on 70 hogsheads of molasses, 63 gal. each, deducting 2% for leakage?

4. What is the duty, at 24% ad valorem, on 200 doz. bottles of porter invoiced at $1.40 per doz., the breakage being estimated at 3%? *Ans.* $65.184.

5. What is the duty, at 40% ad valorem, on 50 lbs. sewing silk, invoiced at $10 a pound?

☞ For Dictation Exercises on these examples, see "Manual and Key," page 121.

435. Topical Review in Percentage.

The pupil may present the following topics to his class, using common illustrations, giving definitions, and deriving rules from illustrative examples, which he will solve before the class.

1. Percentage, basis, rate per cent, manner of expressing per cent. (Arts. 379 – 383.)
2. Manner of finding percentage. (Art. 384) Amount and remainder. (Art. 385.)
3. Manner of finding the basis. (Arts. 386, 387.)
4. Manner of finding the rate per cent. (Art. 388.)
5. Profit and loss. (Arts. 389, 390.)
6. Commission. (Arts. 398 – 400.)
7. Stocks, Dividends, and Brokerage. (Arts. 403 – 410.)
8. Insurance. (Arts. 412 – 419.)
9. Taxes. (Arts. 421 – 427.)
10. National Taxes. (Art. 428.)
11. Duties and Customs. (Arts. 429 – 434.)

436. General Review, No. 6.

1. What is the premium on an insurance policy of $12600 at $1\frac{1}{2}$%?

2. What is the duty on 400 bags of coffee, 120 lbs. each, at $3\frac{1}{2}$ cents per lb., tare being 3%? *Ans.* $1,629.60.

3. What is 10% of 40% of 70% of $250? *Ans.* $7.

4. How much United States money at 12% discount will pay a bill of \$968 in Canada? *Ans.* \$1,100.

5. Bought a span of horses at \$180 each; I sold one for 5% less than cost, and the other for 25% more than cost; what did I gain by the whole transaction? *Ans.* \$36.

6. Paid duties \$325 in gold, gold being at a premium of 40%; what were the duties in current money? *Ans.* \$455.

7. I have marked goods which cost 25 cents a yard, 40% above cost; at what price are they marked?

8. I sold the above at a discount of 10% on the marked price, what per cent on the cost did I make? *Ans.* 26%.

9. A man whose property was valued at \$2450, and who paid a poll tax of \$1, was taxed \$20.60; what was the tax upon \$1? *Ans.* \$.008.

10. When gold is at a premium of 30%, how much gold will equal \$520 of current money? *Ans.* \$400.

11. How many \$100 shares of stock at 50% below par, must be paid for 10 at 25% above par? *Ans.* 25 shares.

12. Find the net proceeds of the following account of

SALES OF 42 BARRELS OF BEEF ON ACCOUNT OF H. M. BALL.

1869.		To whom sold.	Description.		Price.	
Nov.	1	Stone & Sons	12 bar.	Prime Beef	\$15.00	
"	2	D. Atwater	30 "	do. do.	14.80	
						\$624.00
		Charges.				
		Freight and cartage, ...			\$22.70	
		Storage and Commission at 3½% on sales,			\$	44.54
		Net proceeds, ...				\$579.46
				DAVIS & FURBER.		
		New York, Nov. 5, 1869.				

☞ Perform the above examples substituting 3 for 2 in each example. For other Dictation Exercises on this Review, see "Manual and Key," page 122.

SIMPLE INTEREST.

437. ILLUSTRATION. — A has the use of $ 100 of B's money for one year, and at the end of the year he pays B for its use $ 6, a sum equal to $\frac{6}{100}$ of $ 100.

Money, as $ 6 above, paid for the use of money for a definite period of time, is **interest.**

438. The sum for the use of which interest is paid, as $ 100 above, is the **principal.**

439. The principal, with the interest, as $ 100 + $ 6, or $ 106, is the **amount.**

Define interest, principal, amount.

440. Interest is found by taking a number of hundredths of the principal; it is therefore a PERCENTAGE, and the principal is the BASIS.

441. The number of hundredths of the principal taken to find the interest for 1 year is the **rate per cent per annum,** usually called the **rate.**

NOTE I. — Debts of all kinds draw interest from the time they become due, but not before, unless interest is specified. Interest on interest remaining unpaid, however, is not generally allowed.

NOTE II. — The rate of interest established by law is the **legal rate.**

NOTE III. — TABLE OF LEGAL RATES OF INTEREST FROM OFFICIAL SOURCES. 1869.

Rate when no rate is specified.	
6%	On debts due to U. S., in D. C., Me., N. H., Vt., Mass., R. I., Conn., Penn., W. Va., Fla., Iowa, Ill., Ind., Ohio, Mo., Ark., Tenn., Del., Ky., N. C., New Mexico, Md., Miss.
7%	N. Y., N. J., Ga., Minn., Wis., Mich., Kas., Cal.
8%	Ala., La., Va., Texas.
10%	Col., Ut., Neb., Ida., Or., Montana.

Rate allowed by special contract.	
As high as 8%	Ohio, N. C. (on borrowed money).
10%	Wis., Mich., Iowa, Ill., Ind., Mo., Miss.
12%	Minn., Kas., Neb., Or. ("Borrowed money," &c.) Tex.
Any rate.	Mass., R. I., Penn.,* S. C.,† Fla., Ark., Col., Ut., Montana, Cal.

* A debt of honor.
† No laws of usury.

NOTE IV.—When no rate is mentioned, the legal rate is understood. More than the legal interest is **usury**.

GENERAL METHOD OF COMPUTING INTEREST.

TO THE TEACHER.—If the teacher prefers, he may allow the pupil to omit the General Method of computing Interest, and pass directly to the 6% method. (Art. 444.)

After having learned the 6% method, for additional practice the pupil may perform the following examples by that method:—

442. ILL. EX. Find the interest of $309.60 for 1 y. 5 m. 18 d. at 10%.

OPERATION.

	$309.60	Principal.	
10% of $309.60 =	30.96	interest for 1 y.	
$\frac{1}{3}$ of $30.96 =	10.32	" " 4 m.	5 m.
$\frac{1}{4}$ of 10.32 =	2.58	" " 1 m.	
$\frac{1}{2}$ of 2.58 =	1.29	" " 15 d.	18 d.
$\frac{1}{5}$ of 1.29 =	.258	" " 3 d.	
	$45.408	Interest.	

Explanation.—The interest of $309.60 for 1 y. is 10%, or $\frac{1}{10}$ of the principal, for 4 months it is $\frac{1}{3}$ of the interest for 1 year, for 1 month it is $\frac{1}{4}$ of the interest for 4 months, for 15 days it is $\frac{1}{2}$ of the interest for 1 month, and for 3 days it is $\frac{1}{5}$ of the interest for 15 days. Adding these sums we have for the interest of $309.60 for 1 y. 5 m. 18 d.,

$45.41, *Ans.*

From the above operation we derive the following

RULE.—To compute interest for years, months, and days, at any rate per cent:—1. *Find the interest for* 1 *year by multiplying the principal by the given rate, and for a number of years, by multiplying the interest for* 1 *year by the number of the years.*

2. *Find the interest for months by taking aliquot parts of* 1 *year's interest, and for days by taking aliquot parts of* 1 *month's interest.*

NOTE I.—In the answers reject mills when less than 5, and call 5 or more 1 cent.

NOTE II.—In computing interest, 30 days are considered a month.

443. EXAMPLES.

Find the interest of

1.	\$ 720 for 1 y. 5 m. 20 d. at 6%.	*Ans.* \$ 63.60.
2.	\$ 234 for 1 y. 7 m. 12 d. at 8%.	*Ans.* \$ 30.26.
3.	\$ 428.75 for 1 y. 5 m. 21 d. at 7%.	*Ans.* \$ 44.27.
4.	\$ 1265.40 for 1 y. 9 m. 26 d. at 5%.	*Ans.* \$ 115.29.
5.	\$ 72.90 for 1 y. 10 m. 9 d. at 4%.	*Ans.* \$ 5.42.
6.	\$ 286.20 for 2 y. 1 m. 8 d. at 9%.	*Ans.* \$ 54.23.
7.	\$ 2500. for 2 y. 3 m. 7 d. at $7\frac{1}{2}$%.	*Ans.* \$ 425.52.
8.	\$ 752.20 for 1 y. 2 m. 25 d. at 10%.	*Ans.* \$ 92.98.
9.	\$ 3207 for 3 y. 5 m. 11 d. at $7\frac{3}{10}$%.	*Ans.* \$ 807.03.
10.	\$ 287.50 for 3 y. 11 m. 13 d. at 5%.	*Ans.* \$ 56.82.
11.	\$ 1200 for 4 y. 4 m. 4 d. at 12%.	*Ans.* \$ 625.60.
12.	\$ 212 for 6 m. 29 d. at 1% a month.	*Ans.* \$ 14.77.
13.	\$ 290 for 8 m. 27 d. at $\frac{1}{2}$% a month.	*Ans.* \$ 12.91.

14. \$ 629 from Jan. 1, 1868, to Jan. 16, 1870, at 7%.

NOTE.—Find the time by Art. 337, Note I. *Ans.* \$ 89.89.

15. \$ 422 from Jan. 1, 1869, to July 21, 1869, at 10%.

16. \$ 2000 from April 10, 1869, to June 20, 1869, at $7\frac{3}{10}$%.

Sum of last 2 *answers*, \$ 51.83.

17. If \$ 1200 is on interest from Oct. 4, 1868, to Jan. 11, 1870, at 10%, what sum will pay the principal with the interest at the latter date? *Ans.* \$ 1,352.33$\frac{1}{3}$.

18. If \$ 200 is loaned Oct. 1, 1869, what will pay the loan with the interest at 9%, on the 28th day of March following? *Ans.* \$ 208.85.

☞ For Dictation Exercises, see "Manual and Key," page 124.

SIX-PER-CENT METHODS OF COMPUTING INTEREST.

FIRST METHOD.

444. ILL. EX. At 6%, what part of the principal will the interest equal for 2 months?

Explanation.—Since the interest for 1 year or 12 months equals 6%, or .06 of the principal, the interest for 2 months, which is $\frac{1}{6}$ of a year, will equal $\frac{1}{6}$ of .06, or .01 of the principal.

INFERENCE.—If the interest for 2 months equals .01 of the principal, *the interest for any number of months will equal one half as many hundredths of the principal as there are months.*

EXERCISES.

What part of the principal does the interest equal for

	Months.	Ans.		Months.	Ans.		Months.	Ans.		Months.	Ans.
1.	2?	.01.	4.	1?	.00½.	7.	50?	$\frac{1}{4}$.	10.	66⅔?	$\frac{1}{3}$.
2.	20?	.1.	5.	10?	.0½.	8.	25?	$\frac{1}{8}$.	11.	33⅓?	$\frac{1}{6}$.
3.	200?	Prin.	6.	100?	$\frac{1}{2}$.	9.	12½?	$\frac{1}{16}$.	12.	16⅔?	$\frac{1}{12}$.

445. ILL. EX. Find the interest of $480 for 9 y. 3 m.

OPERATION.

Principal,	$480		
$\frac{1}{2}$ of $480 =	240	int. for	100 mo.
$\frac{1}{10}$ of $240 =	24	" "	10 mo.
$\frac{1}{10}$ of $24 =	2.40	" "	1 mo.
Interest,	$266.40	" "	111 mo.

Explanation.—We first change 9 years to months; 9 y. 3 m. equal 111 months.

We find the interest for 100 months by taking $\frac{50}{100}$, or $\frac{1}{2}$ of the principal, then for 10 months by taking $\frac{1}{10}$ of the interest for 100 months, and for 1 month by taking $\frac{1}{10}$ of the interest for 10 months.

We then take the sum of these items, and have $266.40 for the answer.

NOTE.—In the above operation the interest for 10 months and the interest for 1 month are expressed by simply moving the decimal point in previous expressions.

EXERCISES.

Find the interest of $400 for 22 months; for 11 months; for 55 months. *Ans.* $44; $22; $110.

446. ILL. EX. At 6%, what part of the principal will the interest equal for 6 days?

Explanation.—Since the interest for 2 months, or 60 days, equals .01 of the principal, the interest for 6 days, which is $\frac{1}{10}$ of 60 days, will equal $\frac{1}{10}$ of .01, or .001 of the principal.

INFERENCE.—If the interest for 6 days equals .001 of the principal, *the interest for any number of days will equal one sixth as many thousandths of the principal as there are days.*

EXERCISES.

What part of the principal does the interest equal for

	Days.	Ans.		Days.	Ans.		Days.	Ans.		Days.	Ans.
1.	60?	.01	3.	30?	$.00\frac{1}{2}$	5.	20?	$.00\frac{1}{3}$	7.	10?	$.00\frac{1}{6}$
2.	6?	.001	4.	3?	$.000\frac{1}{2}$	6.	2?	$.000\frac{1}{3}$	8.	1?	$.000\frac{1}{6}$

447. ILL. EX. Find the interest of $1200 for 93 days.

OPERATION.

Principal,		$1200	
$\frac{1}{100}$ of $1200 =		12	int. for 60 days.
$\frac{1}{2}$ of	$12 =	6	" " 30 "
$\frac{1}{10}$ of	$6 =	.60	" " 3 "
Interest,		$18.60	" " 93 "

Explanation.—We first find the interest for 60 days by taking $\frac{1}{100}$ of the principal, then for 30 days by taking $\frac{1}{2}$ of the interest for 60 days, and for 3 days by taking $\frac{1}{10}$ of the interest for 30 days.

We then find the sum of these items to be $18.60, *Ans.*

From the above illustrations and exercises we derive the following

RULE.—To compute interest at 6%: 1. *To find the interest for* 200 *months, take a sum equal to the principal; for* 20 *months, equal to* $\frac{1}{10}$ *of the principal; for* 2 *months, equal to* $\frac{1}{100}$ *of the principal; and for* 6 *days, equal to* $\frac{1}{1000}$ *of the principal.*

2. *For any other periods of time, take convenient multiples or aliquot parts of the interest for the times expressed above.*

448. EXAMPLES.

What is the interest of

$1200 for			$270 for		$927.90 for	
19.	33 days?	*Ans.* $6.60.	24.	3 m. 3 d.?	29.	3 m. 9 d.?
20.	63 days?	*Ans.* $12.60.	25.	1 y. 8 m.?	30.	4 m. 12 d.?
21.	22 days?	*Ans.* $4.40.	26.	3 y. 4 m.?	31.	5 m. 15 d.?
22.	11 days?	*Ans.* $2.20.	27.	2 y. 1 m.?	32.	7 m. 15 d.?
23.	2 m. 6 d.?	*Ans.* $13.20	28.	3 m. 6 d.?	33.	6 m. 18 d.?

SECOND METHOD.

449. ILL. EX. Find the interest of $720 for 1 y. 5 m. 23 d. at 6%.

OPERATION.

Principal,	$ 720
Rate,	.088$\frac{5}{6}$
Interest,	$ 63.96

Explanation. — The rate of interest for 1 y. 5 m., or 17 m., is .08½, or .085 (Art. 444); the rate for 23 days, is .003$\frac{5}{6}$ (Art. 446); the sum of these rates is .088$\frac{5}{6}$. Multiplying the principal by the rate, .088$\frac{5}{6}$, we have for the interest $63.96. *Ans.* $63.96.

From the above we derive the following

RULE. — To compute interest at 6%: *Take one half as many hundredths as there are months with one sixth as many thousandths as there are days, and multiply the principal by this number.*

450. EXAMPLES.

Find the interest or amount of the following at 6%, by either of the previous methods: —

34. Interest of $480 for 10 m. 24 d. *Ans.* $25.92.
35. Interest of $217.50 for 1 y. 3 m. *Ans.* $16.31.
36. Interest of $528 for 11 m. 18 d. *Ans.* $30.62.
37. Interest of $162.84 for 1 y. 6 d. *Ans.* $9.93.
38. Amount of $200 for 2 y. 7 m. *Ans.* $231.
39. Amount of $396 for 1 y. 5 m. 15 d. *Ans.* $430.65.
40. Interest of $1298 for 1 y. 8 m. 21 d. *Ans.* $134.34.
41. Interest of $129.72 for 6 m. 20 d. *Ans.* $4.32.
42. Amount of $3000 for 2 y. 1 m. 19 d. $384.50
43. Interest of $622.88 for 9 m. 29 d. 81.04
44. Amount of $440 for 3 y. 4 m. 9 d. 528.66
45. Amount of $1320 for 1 y. 1 m. 13 d.

Sum of last 4 answers, $5,352.86. 1408.66

451. To find the interest at any other rate than 6%.

ILL. EX. What is the interest of $96.30 for 2 y. 9 m. 10 d. at 7%?

Explanation. — The interest of $96.30 at 6% is $16.05. Since 7% is 1% more than 6%, by adding to the interest at 6% $\frac{1}{6}$ of itself, we shall obtain the interest at 7%. *Ans.* $18.73.

To find the interest at any rate per cent by the 6% methods: *First find the interest at 6%; then increase or diminish that interest by such a part of itself as will give the interest at the required rate.*

NOTE.—Observe that 1% = $\frac{1}{6}$ of 6%; 2% = $\frac{1}{3}$ of 6%; 3% = $\frac{1}{2}$ of 6%; 4% = 6% — ($\frac{1}{3}$ of 6%); 5% = 6% — ($\frac{1}{6}$ of 6%); 7% = 6% + ($\frac{1}{6}$ of 6%); $7\frac{1}{2}$% = 6% + ($\frac{1}{4}$ of 6%), etc.

452. EXAMPLES.

Find, by either of the preceding methods, the interest

46. Of $ 360 for 2 y. 1 m. 15 d. at 8%. *Ans.* $ 61.20.
47. Of $ 874 for 3 y. 4 m. at $7\frac{1}{2}$%. *Ans.* $ 218.50.
48. Of $ 250 for 93 days at 5%. *Ans.* $ 3.23.
49. Of $ 200 for 5 m. at 1% a month.

NOTE.—The interest for 5 months, at 1% a month, will be .05 of the principal.

50. Of $ 1000 for 60 days at 2% a month.

Find, by either of the preceding methods, the amount

51. Of $ 487.50 for 5 y. 8 m. 22 d. at 5%. *Ans.* $ 627.11.
52. Of $ 287.64 for 2 y. 5 m. 16 d. at 4%. *Ans.* $ 315.96.
53. Of $ 830 for 8 m. at $\frac{1}{2}$% a month. *Ans.* $ 863.20.

☞ For Dictation Exercices, see "Manual and Key," page 125.

453. ACCURATE INTEREST.

Interest for months and days is computed by the government of the United States and by Great Britain at 365 days to the year. This interest is called **accurate interest.**

ILL. EX. What is the accurate interest of $ 2190 at 5%, from December 28, 1869, to March 8, 1870?

OPERATION.

$$\begin{array}{r} \$\,2190 \\ .05 \\ \hline \$\,109.50 \end{array} \text{ int. for 1 year.}$$

$$\frac{\overset{.30}{\cancel{109.50}} \times 70}{\cancel{365}} = \$\,21.$$

Explanation.—The number of days between the two dates (by Art. 337, Note II., or Art. 338) is 70. The interest of $ 2190 for 1 year at 5% is $ 109.50; for 1 day it is $\frac{1}{365}$ of $ 109.50, and for 70 days it is $\frac{109.50}{365} \times 70$, which equals $ 21.

From the preceding example we derive the following

RULE. — To compute accurate interest for any time less than one year: *Find the exact number of days, and take as many 365ths of one year's interest as there are days.*

NOTE. — When interest is taken for a number of days by the ordinary methods, 1 day is reckoned as $\frac{1}{360}$ of a year. This is at the rate of a year's interest for $\frac{365}{360}$ of a year, and in leap year for $\frac{366}{360}$ of a year; thus, $\frac{5}{365}$ or $\frac{1}{73}$ and in leap year $\frac{1}{61}$ too much interest is taken; hence *accurate interest may be obtained by deducting $\frac{1}{73}$, and in leap year $\frac{1}{61}$, of the interest found for days by the ordinary processes.*

EXAMPLES.

Find the accurate interest of

54. \$8000 from March 15 to May 1, at 6%. *Ans.* \$61.81.
55. \$1000 from June 17 to Nov. 1, at 5%. *Ans.* \$18.77.
56. \$2500 from Sept. 25, 1868, to Jan. 1, 1869, at 6%.

☞ For Dictation Exercises, see "Manual and Key," page 125.

PROBLEMS IN INTEREST.

454. TO FIND THE TIME, WHEN THE INTEREST, PRINCIPAL, AND RATE PER CENT ARE GIVEN.

ILL. EX. In what time will \$500, on interest at 6%, gain \$75 of interest?

OPERATION.

Int. of \$500 for 1 y. = \$30.

$$30\,)\,75$$
$$2\tfrac{1}{2}$$

Explanation. — The interest of \$500 for 1 year at 6% is \$30. It will require as many years for \$500 to gain \$75 as there are 30's in 75, which is $2\frac{1}{2}$.

Ans. $2\frac{1}{2}$ years.

From the above we derive the following

RULE. — To find the time, when the interest, principal, and rate are given: *Divide the given interest by the interest of the principal at the given rate for one year.*

NOTE. — It will often be found more convenient to divide by the interest for 1 *month* or 1 *day*, in which case the answer will be in months or days.

Examples.

What time will be required

1. For $ 600 to gain $ 48 at 6%? *Ans.* $1\frac{1}{3}$ y.
2. For $ 125 to gain $ 6.25, at 10%? *Ans.* 6 m.
3. For $ 317.50 to gain $ 15.24, at 8%? *Ans.* 7 m. 6 d.
4. For $ 500 to gain $ 500, at 7%? at 12%? at $7\frac{1}{2}$%?
5. For $ 800 to amount to $ 898, at 7%?

Note. — The interest equals $ 898 less $ 800, or $ 98. *Ans.* 1 y. 9 m.

6. For $ 562.50 to amount to $ 630, at 9%?
7. I invested $ 3600 at 5% and received for principal and interest $ 4600; for what time was the money invested? *Ans.* 5 y. 6 m. 20 d.

455. To find the Rate, when the Interest, Principal, and Time are given.

Ill. Ex. At what rate must $ 80 be on interest 3 y. 6 m. to gain $ 14?

OPERATION.

$3\frac{1}{2}$% of $ 80 = $ 2.80

2.80) 14.00 (5
14.00

Explanation. — The interest of $ 80 for 3 y. 6 m. at 1% is $ 2.80.

If the gain is $ 2.80 when the rate is 1 per cent, to gain $ 14 the rate must be as many 1 per cents as there are 2.80's in 14, which is 5. *Ans.* 5%.

From the above we derive the following

Rule. — To find the rate, when the interest, principal, and time are given: *Divide the given interest by the interest of the principal for the given time at one per cent.*

Examples.

At what rate will

8. $ 1500 gain $ 315 in 3 y.? *Ans.* 7%.
9. $ 3750 gain $ 100 in 4 m.? *Ans.* 8%.
10. $ 750 gain $ 125 in 1 y. 8 m.?
11. $ $666\frac{2}{3}$ gain $ $33\frac{1}{3}$ in 8 m.? *Ans.* $7\frac{1}{2}$%.
12. $ 764.96 gain $ 95.62 in 2 y. 1 m.?
13. $ 2736 amount to $ 3116 in 2 y. 9 m. 10 d.? *Ans.* 5%.
14. $ 9000 amount to $ 9519 in 5 m. 23 d.? *Ans.* 12%.

456. To find the Principal, when the Interest or Amount, the Time, and Rate are given.

Ill. Ex. What principal on interest 2 y. 2 m. 12 d., at 10%, will gain $ 132 ? will amount to $ 732 ?

OPERATION I.

$$\begin{array}{r} .22\,)\,132.00\,(\,600 \\ \underline{132} \\ 000 \end{array}$$

Explanation I. — The interest of $ 1 at 10% for 2 y. 2 m. 12 d. is $ .22.

If the gain on $ 1 is $ .22, to gain $ 132, as many dollars must be on interest as there are .22's in 132, which is 600.

OPERATION II.

$$\begin{array}{r} 1.22\,)\,732.00\,(\,600 \\ \underline{732.} \\ 000 \end{array}$$

Explanation II. — The amount of $ 1 at 10% for 2 y. 2 m. 12 d. is $ 1.22. To amount to $ 732 as many dollars must be on interest as there are 1.22's in 732, which is 600. *Ans.* $ 600.

From the above we derive the following rules : —

I. To find the principal, the interest, time, and rate being given: *Divide the interest by the interest of* $ 1 *for the given time and rate.*

II. To find the principal, the amount, time, and rate being given: *Divide the amount by the amount of* $ 1 *for the given time and rate.*

Examples.

What principal on interest

15. Will gain $ 42 in 1 y. 6 m. at 7% ? *Ans.* $ 400.

16. Will gain $ 39 in 2 y. 6 m. at 1% a month ? *Ans.* $ 130.

17. Will amount to $ 260 in 8 m. at 6% ? *Ans.* $ 250.

18. What sum invested at 8% will yield $ 600 annually ? semi-annually ? *Ans.* $ 7,500 ; $ 15,000.

19. What sum invested at 5% will yield $ 30 monthly ?

20. What sum lent July 1 at 12% will amount to $ 783.30 Dec. 1, of the same year ? *Ans.* $ 746.

21. What sum of money put at interest Feb. 10, 1870, at 6%, will amount to $ 789.779 June 26, 1870 ? *Ans.* $ 772.27.

22. What principal on interest at 8% from Nov. 19, 1870, to March 9, 1871, will amount to $ 2201.60 ? *Ans.* $ 2,150.

☞ For Dictation Exercises upon Problems in Interest, see "Manual and Key," page 127.

DISCOUNT AND PRESENT WORTH.

457. ILL. EX. J. Waters sold to E. Bond goods to the amount of $214.20 on 4 months' credit without interest. What sum should Bond pay Waters to discharge the above debt at its date, the current rate of interest being 6% ?

Explanation. — Bond should pay such a sum as put at interest at 6%, for 4 months, will amount to $214.20; this, by Art. 456, is $210.

PROOF. — To prove the accuracy of the above operation, find the amount of $210 for 4 months; this equals $214.20.

458. A sum which will, without loss to either party, discharge a debt at a given time before it is due, is called the **present worth of the debt.**

459. The difference between a debt due at a future time and the present worth of it is called **true discount.**

460. It will be seen that the debt due at a future time is an AMOUNT, of which the PRINCIPAL is the present worth, and the INTEREST the discount. Hence the following

RULE. — 1. To find the present worth: *Divide the given sum by the amount of* $1 *for the given rate and time.* (Art. 456.)

2. To find the true discount: *Take the difference between the debt and its present worth.*

EXAMPLES.

1. What is the present worth of $185.43 due 6 months hence, without interest, when the current rate of interest is 10% ? What is the true discount ?

OPERATION. $\begin{cases} \$185.43 \div \$1.05 = 176.60. & \text{Present worth, } \$176.60. \\ \$185.43 - \$176.60 = \$8.83, & \text{Discount.} \end{cases}$

c note.

2. What is the present worth of $ 290.421 due in 6 months without interest, the rate of interest being 7%? What is the true discount? *Ans.* $ 280.60; $ 9.821.

3. Find the present worth and true discount of $ 916.37 due in 8 months at 9%. *Ans.* $ 864.50; $ 51.87.

4. Find the present worth and true discount of $ 700.73 due in 10 months at 4½%. *Ans.* $ 675.40; $ 25.33.

5. Find the present worth and true discount of $ 1865.30 due in 7 m. 15 d. at 2% a month. *Ans.* $ 1,622; $ 243.30.

6. Having bought goods to the amount of $ 500, Sept. 1, 1867, on 6 months' credit, what should I pay to discharge the debt, Jan. 1, 1868, interest being 6%?

Note. — By month is meant the time from any date in one month to the same date in another month.

Explanation. — 6 months from Sept. 1 is March 1, the time when the debt is due; the present worth of $ 500 for 2 months, the time between Jan. 1 and March 1, is $ 495.05; this sum paid Jan. 1 should discharge the debt.
Ans. $ 495.05.

7. A debt of $ 360 was contracted July 20, 1868, payable in one year without interest. What sum would discharge the above debt May 31, 1869, the rate of interest being 6%?
Ans. $ 357.02.

8. What sum would have discharged the above debt Feb. 20, 1869, interest being 10%? *Ans.* $ 345.60.

9. Money being worth 7%, what sum, paid Aug. 14, will discharge a bill of $ 630.90 for goods bought July 9 on 60 days' credit? *Ans.* $ 627.97.

10. Money being worth 9%, what sum, paid May 15, will discharge two bills, one of $ 742.75, for goods bought March 7 on 90 days' credit, the other of $ 342.36, for goods bought April 15 on 60 days' credit? *Ans.* $ 1078.68.

☞ For Dictation Exercises in Discount, see "Manual and Key," p. 128.

461. BANK DISCOUNT.

Stamp.	*$400*	*Boston, March 4, 1869.*

Four months after date I promise to pay to V. M. Rice, or order, four hundred dollars.
Value received. *J. S. Adams.*

462. The above is the written promise of one party, J. S. Adams, to pay to another party, V. M. Rice, or to any one to whom Rice may order it paid, a certain sum, $400, for value received. Such a promise is called a **promissory note,** or simply a **note.**

463. The sum named in the note, as $400 above, is the **face** of the note.

NOTE I. — The number of dollars of the face of the note should always be expressed in words.

NOTE II. — The words *value received* should always be written in the note, to avoid the necessity, in case of a suit at law, of proving that the note was given for a sufficient consideration.

NOTE III. — The signer of a note, as Adams of the above, is the **drawer** or **maker**; the person to whom the note is made payable, as Rice of the above, is the **payee**; the one who has the note in possession is the **holder.**

464. Suppose Rice, who holds the above note against Adams, to be in immediate want of money, by what process can he secure it upon this note? If both he and Adams are of undoubted credit, the officers of a bank may give him in exchange for the note their estimated value of it before it becomes due. The note is then said to be **discounted** at the bank.

465. Banks and business men, in discounting notes for short times, instead of taking true discount, which, we have seen (Art. 460), is interest upon the present worth, take

interest upon the face of the note from the date of discount. This interest is called **Bank discount.**

466. The face of a note, *minus* the discount, is the **proceeds, avails,** or **cash value** of the note.

467. Though the foregoing note is nominally due in 4 months from date, viz. July 4, in most of the States it would not be legally due till three days after it is nominally due. These three days are called **days of grace.**

468. The time at which a note is legally due is called its **maturity.**

NOTE I.—The time when the above note would be nominally and when legally due would be expressed thus: July $^4/_7$, '69.

NOTE II.—When a note is given for months, calendar months are understood, and the note is nominally due on the day corresponding with its date; if the month in which it falls due has no corresponding day it is due on the last day of that month.

NOTE III.—Notes maturing on the Sabbath or on a legal holiday must be paid on the business day next preceding.

NOTE IV.—In computing Bank Discount, the more general custom is to reckon the time in days; hence, in the examples in Bank Discount which follow, the time is thus reckoned.

469. Before discounting the above note the bank would require Rice to write his name upon the back of it. Writing one's name upon the back of the note is called **indorsing** the note.

NOTE V.—A note payable *to order,* as the above, and thus indorsed, is **negotiable,** i. e. salable; a note payable *to bearer* is negotiable without indorsing.

NOTE VI.—By indorsing this note, Rice binds himself to pay it at maturity in case Adams fails to pay it. An indorser is not responsible for the payment of a note, if *without recourse* is written above his name.

NOTE VII.—If a note is not paid at its maturity, a written notice of the fact is made out and sent to the indorsers by an officer called a notary. This notice is called a **protest.**

NOTE VIII.—In order that the indorsers may be held responsible, a protest must be mailed to them on the last day of grace if practicable.

470. To find the Bank Discount and the Proceeds of a Note, the Face of the Note being given.

Ill. Ex. If Rice has Adams's note (Art. 461) discounted at a bank June 1, at 6%, what is the bank discount; and what will he receive as the proceeds of the note?

Explanation. — The bank discount (Art. 465) of the note is the interest of it from the date of discount to maturity; the proceeds (Art. 466) are the face of it, *minus* the discount.

4 months from March 4 is July 4, which with 3 days of grace gives the maturity July 7. From June 1 to July 7 is 36 days. (Art. 468, IV.)

Interest on \$ 400 for 36 days = \$ 2.40, Discount.

\$ 400 — \$ 2.40 = \$ 397.60, Proceeds.

From the above we may derive the following

Rule. — 1. To find the bank discount of a note: *Compute interest on the face of the note from the date of discount to maturity.*

2. To find the proceeds of the note: *Take the interest out of the face of the note.*

Examples.

Find the date when the following notes become due, the time for which they are discounted, the bank discount, and the proceeds:

1. A 4 months' note for \$ 225, dated Dec. 10, 1868, and discounted Jan. 1, 1869, at 7%.

Ans. Due April 10/13; 102 d.; \$ 4.46 dis.; \$ 220.54 proceeds.

2. A 90 days' note for \$ 568 dated Sept. 20 and discounted Nov. 25 at 1% a month.

Ans. Due Dec. 19/22; 27 d.; \$ 5.11 dis.; \$ 562.89 proceeds.

3. A 6 months' note for \$ 862 dated Nov. 5, 1868, and discounted Dec. 26, 1868, at 8%.

Ans. Due May 5/8; 133 d.; \$ 25.48 dis.; \$ 836.52 proceeds.

4. What are the bank discount and the proceeds of a note for \$ 728, dated April 3, payable in 60 days, with interest at 6%, discounted at date at 7%?

Note. — When a note bearing interest is discounted, the *amount of the note at maturity*, instead of the face of it, is taken as the basis of discount.

Ans. \$ 9.01 dis.; \$ 726.63 proceeds.

Find the date when the following notes become due, the bank discount, and the proceeds:

5. $ \$ 960\frac{67}{100}$ *Harrisburg, Aug.* 10, 1869.

Three months after date, I promise to pay to the order of Geo. F. Phelps nine hundred sixty $\frac{67}{100}$ dollars, without defalcation, value received.

THOMAS J. FREEMAN.

The above was discounted August 26, at 7%.

Ans. Nov. 10/13; \$ 14.76 dis.; \$ 945.91 proceeds.

6. \$ 1000 *Chicago, Nov.* 18, 1868.

Sixty days after date, we promise to pay to the order of E. Staples & Co. one thousand dollars at the Fourth National Bank, Chicago, value received. WOODMAN & WADSWORTH.

The above was discounted Nov. 27, at 10%.

Ans. Jan. 17/20, 1869; \$ 15 dis.; \$ 985 proceeds.

7. I had three \$ 500 notes, dated severally September 1, October 1, and December 1, 1868, and payable in 6 months; these notes were discounted by a banker February 1, 1869, at 12%; what was the sum of the proceeds? *Ans.* \$ 1,464.

471. TO FIND THE FACE OF A NOTE THAT SHALL YIELD A GIVEN SUM WHEN DISCOUNTED AT A BANK FOR A CERTAIN TIME AND RATE.

ILL. EX. For what sum must a 60 days' note be drawn so that when it is discounted at a bank at 6% the proceeds shall be \$ 1400?

Explanation. — The bank discount of \$ 1 for 63 days at 6% is \$ .0105. The proceeds of \$ 1 for 63 days are \$ 1 — \$ .0105, which is \$ .9895. Since \$ .9895 is the proceeds of \$ 1, the note must be drawn for as many dollars as there are \$ .9895's in \$ 1400. *Ans.* \$ 1,414.86.

PROOF. — Interest on \$ 1414.86 for 63 days = \$ 14.86, Discount.
\$ 1414.86 — \$ 14.86 = \$ 1,400, Proceeds.

From the above may be derived the following

RULE. — To find the face of a note that shall yield a certain sum when discounted at a bank: *Divide the given sum by the proceeds of \$ 1 for the given time and rate.*

EXAMPLES.

8. For what sum must a 90 days' note be drawn so that when it is discounted at a bank at 6% the proceeds shall be $ 448.932?

Ans. $ 456.

9. What must be the face of a note given for 105 days, to obtain $ 244 from a bank, discount being at 8%? *Ans.* $ 250.

10. Two notes were discounted, each for 30 days and grace; the proceeds of the first were $ 1243.12½, the rate 6%; the proceeds of the second, $ 1688.78, the rate 7½%; what sum will discharge both notes at maturity? *Ans.* $ 2,950.

11. For what sums must three six-months' notes be drawn, dated severally April 1, May 1, and June 1, and discounted at a bank, September 1, at 6%, that the proceeds of each may be $ 1600?

Ans. $ 1,608.85; $ 1,617.25; $ 1,625.47.

☞ For Dictation Exercises in Banking, see "Manual and Key," page 129.

ANNUAL INTEREST.

NOTE. — Teachers of New Hampshire and Vermont will find a treatment of ANNUAL INTEREST on page 321.

COMPOUND INTEREST.

472. ILLUSTRATION. — January 1, 1860, I had a deposit of $ 1000 in a savings bank; at the end of 6 months the deposit was increased by the interest, which was 3% of the $ 1000; at the end of another six months the amount thus formed was increased by the interest, which was 3% of the amount.

Interest was here employed semi-annually to increase the principal, and each successive amount became a new principal upon which interest was computed.

Interest employed to increase the principal at stated intervals is compound interest.

NOTE I. — Interest which is computed upon the principal only is **simple interest**.

NOTE II. — When the kind of interest is not specified, simple interest is meant.

NOTE III. — Interest may be compounded annually, semi-annually, quarterly, or at any specified time.

473. ILL. EX. In the illustration above, what was the amount due at the end of the third six months? What interest had accrued?

OPERATION.

Principal, ...	$ 1000.
	1.03
Amount for new principal, .	1030.
	1.03
Amount for new principal, .	1060.90
	1.03
Amount due, . . .	1092.727
Deducting principal, .	1000.
Accrued interest, . . .	$ 92.727

From the operation in the margin we derive the following

RULE. — To compute compound interest:

1. *Find the amount of the given sum for the first interval of time. With this as a new principal, find the amount for the second interval of time, and thus continue for the entire time. The last amount will be the amount at compound interest.*

2. *The last amount minus the given principal will be the compound interest.*

EXAMPLES.

1. What is the amount of $ 261 for 3 years at 6%, interest compounding annually? semi-annually? *Ans.* $ 310.86; $ 311.65.

2. What are the amount and interest of $ 2000 at 7% for 1 y. 6 m., interest compounding semi-annually? *Ans.* $ 2,217.44; $ 217.44.

3. What is the amount of $ 2000 for 2 y. 6 m. at 7%, interest compounding annually?

NOTE. — $\$ 2000 \times 1.07 \times 1.07 \times 1.035 = \$ 2369.94$. *Ans.* $ 2,369.94.

4. What is the amount of $ 340 for 1 y. 3 m. at 8%, interest compounding semi-annually? *Ans.* 375.10.

5. What is the amount, the interest compounding quarterly? *Ans.* $ 375.39.

6. What is the interest of $ 789 at 6% from Nov. 17, 1866, to July 3, 1869, interest compounding annually? (Art. 387, Note I.) *Ans.* $ 130.76.

474. The process of computing compound interest may be shortened by the use of the following

TABLE,

Showing the amount of $1 at compound interest from 1 year to 15 years, at 3, 4, 4½, 5, 6, and 7 per cent.

Years.	3 per cent.	4 per cent.	4½ per cent.	5 per cent.	6 per cent.	7 per cent.
1	1.030000	1.040000	1.045000	1.050000	1.060000	1.070000
2	1.060900	1.081600	1.092025	1.102500	1.123600	1.144900
3	1.092727	1.124864	1.141166	1.157625	1.191016	1.225043
4	1.125509	1.169859	1.192519	1.215506	1.262477	1.310796
5	1.159274	1.216653	1.246182	1.276282	1.338226	1.402552
6	1.194052	1.265319	1.302260	1.340096	1.418519	1.500730
7	1.229874	1.315932	1.360862	1.407100	1.503630	1.605781
8	1.266770	1.368569	1.422101	1.477455	1.593848	1.718186
9	1.304773	1.423312	1.486095	1.551328	1.689479	1.838459
10	1.343916	1.480244	1.552969	1.628895	1.790848	1.967151
11	1.384234	1.539454	1.622853	1.710339	1.898299	2.104852
12	1.425761	1.601032	1.695881	1.795856	2.012197	2.252192
13	1.468534	1.665073	1.772196	1.885649	2.132928	2.409845
14	1.512590	1.731676	1.851945	1.979932	2.260904	2.578534
15	1.557967	1.800943	1.935282	2.078928	2.396558	2.759031

ILL. EX. At 6% compound interest for 3 y. 5 m., what is the amount of $200? What is the compound interest of $200?

OPERATION.

$1.191016 × 1.025 = $1.220791
$1.220791 × 200 = $244.158 amount.
.220791 × 200 = $44.158 interest.

Explanation.—We find by the table the amount of $1 for 3 years to be $1.191016. We next find the amount of $1.191016 for 5 months, which is $1.220791, which equals the amount of $1 for 3 y. 5 m.

Multiplying $1.220791 by 200 we obtain for the amount of $200, $244.158, 1*st Ans.*

Having found the amount, we can obtain the interest by taking the difference between the amount and principal, or it can be obtained by multiplying .220791, the interest of $1 for 3 y. 5 m., by 200.

2d Ans. $44.158.

EXAMPLES.

Interest compounding annually, find, by the use of the above table, the interest

7. Of \$ 860 for 5 y. at 6%. *Ans.* \$ 290.87.

8. Of \$ 300 for 13 y. 3 m. at 5%. *Ans.* \$ 272.77.

9. Of \$ 400 for 25 y. at 6%.

NOTE.—Find by the table the amount of \$ 400 for 15 years, then find the amount of this amount for 10 years. *Ans.* \$ 1,316.75.

10. Interest compounding semi-annually, what is the amount of \$ 3500 for 7 years at 6%?

NOTE.—The interest on the above compounds 13 times at 3%; hence we take from the table the amount of \$ 1 for 14 years at 3% and multiply it by \$500. *Ans.* \$ 5,294.07.

Interest compounding semi-annually, what is the amount

11. Of \$ 480 for 6 y. at 9%? at 8%? 1*st Ans.* \$ 814.02.

12. Of \$ 357.60 for 4 y. 6 m. at 10%? at 6%? 1*st Ans.* \$ 554.75.

PARTIAL PAYMENTS.

475. ILLUSTRATION.—A note was given for \$ 1500 Sept. 1, 1865, payable on demand with interest at 6%; to discharge the interest and in part pay the note, payments were made as follows:—

September 1, 1866, \$ 500. September 1, 1867, \$ 25.
March 1, 1867, \$ 122.70. March 1, 1868, \$ 500.

476. Payments in part of a note or other debt are called **partial payments.**

477. A record of the amount of each payment, with its date, is made upon the back of the note; such a record is an **indorsement.**

478. ILL. EX. If the above note was settled September 1, 1868, what was the balance due?

The method prescribed by the Supreme Court of the United States, and adopted by most of the States, for finding the balance due on notes at the time of settlement when partial payments are made, is illustrated in the following

OPERATION OF THE ILLUSTRATIVE EXAMPLE.

Principal on interest from Sept. 1, 1865, ...	$ 1500.00
Interest on $ 1500 from Sept. 1, '65, to Sept. 1, '66 (1 y.),	90.00
Amount, ...	1590.00
First payment, ...	500.00
Remainder for a new principal, ...	1090.00
Interest on $ 1090 from Sept. 1, '66, to March 1, '67 (6 m.),	32.70
Amount, ...	1122.70
Second payment, ...	122.70
Remainder for new principal, ...	1000.00
Interest on $ 1000 from March 1 to Sept. 1, 1867 (6 m.),	30.00
Third payment, $ 25 (will not discharge interest).	
Interest on $ 1000 from Sept. 1, '67, to March 1, '68 (6 m.),	30.00
Amount, ...	1060 00
Third and fourth payments $ 25 + $ 500 = . .	525.00
Remainder for new principal, ...	535.00
Interest on $ 535 from March 1 to September 1, 1868 (6 m.),	16.05
Amount, balance due September 1, 1868, . . .	$ 551.05

NOTE. — By the above method the balance due is found whenever payments are made that equal or exceed the interest due. But that interest may not be taken on interest, no balance is found when payments are made that are less than the interest due. The method is stated in the following, called the

UNITED STATES RULE.

1. *Find the amount of the principal to the time when the payment or the sum of the payments equals or exceeds the interest; take out of this amount a sum equal to the payment or payments.*

2. *With the remainder as a new principal, proceed as before to the time of settlement.*

NOTE. — In partial payments find the time by Art. 337, Note I.

EXAMPLES.

1. **$350.** *Cleveland, April* 5, 1867.

On demand, with interest at 6%, I promise to pay James Shaw three hundred fifty dollars. Value received.

ISAAC NORTON.

INDORSEMENTS.—Received April 5, 1868, $75; Jan. 5, 1869, $150.
What balance was due April 5, 1869? *Ans.* $161.71.

2. **108\frac{60}{100}$.** *Albany, N. Y., Dec.* 28, 1865.

Four months after date I promise to pay to the order of Van Duzen & Co. one hundred eight $\frac{60}{100}$ dollars.

FRANCIS CONNOR.

INDORSEMENTS.—Received January 10, 1867, $26; December 3, 1867, $48; February 24, 1869, $34.

What was due on the above July 8, 1869?

NOTE.—Interest commences May 1, 1866 (Art. 441, Note I., and Art. 467). The rate of interest is 7% (Art. 441, Note III.). *Ans.* $15.67.

3. **$625.** *Boston, October* 4, 1864.

On demand, with interest at 6%, we promise Hickling & Carter to pay them or order six hundred twenty-five dollars. Value received.

BARKER & BOND.

INDORSEMENTS.—Received August 10, 1867, $75; December 15, 1867, $225; November 18, 1868, $150.

What balance was due October 4, 1869? *Ans.* $336.42.

4. **$600.** *San Francisco, February* 1, 1865.

On demand, I promise to pay Sargent, Bright, & Co. six hundred dollars with interest at 12%. Value received.

JAMES R. GIBBS.

INDORSEMENTS.—Received August 4, 1865, $100; May 1, 1866, $45; May 1, 1867, $200.83.

What balance was due August 31, 1867? *Ans.* $419.04.

5. A note for $3784.25, dated July 10, 1866, was given for 5 years with interest at 6%; the note was indorsed as follows:—

Jan. 16, 1867, $148.21; July 11, 1867, $50; Dec. 24, 1868, $2789.25; Feb. 12, 1869, $1000. What was due Dec. 14, 1869? *Ans.* $379.98.

☞ For Dictation Exercises in Partial Payments, see "Manual and Key," page 131.

479. ILL. EX. A note of $225 given March 1, 1868, is indorsed as follows: —

Received July 1, 1868, $84.50.
Received September 1, 1868, $75.
Received January 1, 1869, $40.50.

What is due March 1, 1869, interest being 6%?

When partial payments are made upon notes on interest for short periods of time, as the above, it is customary to compute interest by the following, called

THE MERCHANTS' RULE.

Compute interest on the principal from the time it begins to draw interest to the time of settlement, also upon each payment from the time it is paid to the time of settlement.

Take the difference between the amount of the principal and the sum of the payments with their interests; this difference is the balance due at the time of settlement.

OPERATION.

Principal on interest from Mar. 1, '68,		$ 225.00
Interest to March 1, '69 (1 year),		13.50
Amount of note, ...		238.50
Payment July 1, '68,	$ 84.50	
Interest to Mar. 1, '69 (8 m.),	3.38	
Payment Sept. 1, '68,	75.00	
Interest to Mar. 1, '69 (6 m.),	2.25	
Payment Jan. 1, '69,	40.50	
Interest to Mar. 1, '69 (2 m.),	.41	
Sum of payments and their interest,		206.04
Balance due March 1, 1869,		*Ans.* $ 32.46

EXAMPLES.

6. A note of $1000, given April 16, 1868, was indorsed as follows: —

Received August 16, 1868, $400.
Received October 20, 1868, $360.

What was due Feb. 25, 1869, interest at 6%? *Ans.* $271.40.

7. A note of $420, given Jan. 1, 1868, interest at 8%, was indorsed as follows: —

May 1, 1868, $100. July 1, 1868, $110.
June 1, 1868, $90. Oct. 27, 1868, $87.

What was due Dec. 1, 1868? *Ans.* $51.21.

480. ILL. EX. A note of $964, given July 7, 1867, with interest annually, was indorsed as follows: —

Oct. 7, 1867, $65. July 22, 1868, $150.
Jan. 7, 1868, $400.

What was due October 7, 1868, interest at 6%?

OPERATION.

Principal on interest from July 7, 1867, ...		$964.00
Interest to July 7, 1868 (1 year), ...		57.84
Amount of note for 1 year, ...		1021.84
Amount of $65 to July 7, 1868 (9 m.),	$67.925	
Amount of $400 to July 7, 1868 (6 m.),	412.00	
Payments with their interests, ...		479.925
Balance for new principal, July 7, 1868, ...		541.915
Interest to October 7, 1868 (3 m.), ...		8.129
Amount of note for balance of time, ...		550.044
Amount of $150 to October 7, 1868 ($2\frac{1}{2}$ m.), . . .		151.875
Balance due October 7, 1868, ...		398.169

Ans. $398.17.

Explanation. — When the interest is payable annually, interest is computed upon the principal, and upon the payments made within one year to the end of the year.

The balance due at that time forms the principal for the next year, or for the remaining time, if less than a year.

NOTE. — The above is but a slight modification of the Merchants' Rule.

EXAMPLES.

8. Perform example 1, Art. 478, by Merchants' Rule, allowing interest annually. *Ans.* $161.51.

9. Perform example 4, Art. 478, by Merchants' Rule, allowing interest annually. *Ans.* $417.03.

NOTE. — The Connecticut Rule for Partial Payments is given on page 324; the Rule for Partial Payments with annual interest, on page 322.

☞ For Dictation Exercises, see "Manual and Key," page 182.

481. INTEREST ACCOUNTS.

Dr.		Edgar G. French.					Cr.
1869.				1869.			
Mar. 1.	To Mdse.	300	00	April 5.	By Cash,	180	00
May 10.	" do.	675	00	May 20.	" do.	330	00
				June 19.	" do.	270	00

The above account with Edgar G. French is taken from the ledger of William Odin & Co.

The left-hand or debit side shows what goods French bought of Odin & Co., with the dates of the purchases.

The right-hand or credit side shows what cash French paid Odin & Co., with the dates of the payments.

482. ILL. EX. Suppose the above account to be settled July 1, 1869, interest at 6% being allowed on each item from its date to the time of settlement, what balance will be due?

The above account, with the interest on the several items and the balance due, is as follows: —

EDGAR G. FRENCH in account current and interest account with WILLIAM ODIN & Co. to July 1, 1869.

1869.		Items.		Days.	Int.		1869.		Items.		Days.	Int.	
Mar. 1.	To Mdse.	300	00	122	6	10	April 5.	By Cash,	180	00	87	2	61
May 10.	" do.	675	00	52	5	85	May 20.	" do.	330	00	42	2	31
							June 19.	" do.	270	00	12		54
								" *Dr. Bal. Int.*				6	49
	Add Bal. Int.	6	49				*July* 1.	*Bal. due.*	201	49			
		981	49		11	95			981	49		11	95

Explanation. — Having found the interest on the debit side to be $ 6.49 more than that on the credit side, we add that sum with the debits of merchandise, and out of the amount take the total of cash paid, and have for the balance due Odin & Co. $ 201.49. *Ans.* $ 201.49.

A statement of the mercantile transactions of one person or party with another, as the above, is an **account current.**

NOTE I.—In accounts current the details of each transaction should be drawn from the book of original entry and presented with the several items. To save space, some details are here omitted.

NOTE II.—In the settlement of mercantile accounts, interest is calculated or not, according to custom or the agreement of the parties, and 3 days of grace are generally allowed on items of merchandise on credit.

NOTE TO THE TEACHER.—In interest accounts and in average, accountants generally, though not universally, reckon the time in days. To avoid confusing the pupil with various methods, time is here reckoned exclusively in days.

EXAMPLES.

What is the balance due on the following accounts Jan. 1, 1870, reckoning interest at 6% on each item from the time when due?

(1.)

Dr. EASTERN RAILROAD CO. Cr.

Date.				Due.	Date.				Due.
1869.				1869.	1869.				1869.
July 15.	To Mdse. 3 m.	794	82	Oct. 18	Sept. 25.	By Mdse. 3 m.	200	00	Dec. 28.
Aug. 27.	" do. 3 m.	408	33	Nov. 30.	Nov. 8.	" Cash,	287	00	Nov. 8.
Sept. 25.	" do. 3 m.	217	90	Dec. 28.					

NOTE.—To find the time when due, add 3 m. + 3 d. to the date of each item of merchandise. *Ans.* Bal. due from E. R. R. Co., $ 943.61.

(2.)

Dr. GEORGE C. RAYMOND. Cr.

1869.				1869.			
April 7.	To Mdse. 3 m.	250	00	July 25.	By Mdse. 2 m.	375	00
June 1.	" do. 2 m.	300	00	Aug. 7.	" Cash,	500	00
June 29.	" do. 2 m.	321	40	Nov. 1.	" do.	600	00

Ans. Balance due to G. C. R., $ 606.56.

AVERAGE, OR EQUATION OF PAYMENTS.

INTEREST METHOD.

483. ILL. EX. A owes B several sums, as follows: $ 60 due Jan. 21, 1869, $ 80 due Feb. 2, $ 100 due Mar. 10. At what date may A pay all these sums at once without loss of interest to either party?

Explanation. — If all the sums are to be paid at once, it should be at such a time after or before some selected date as will be required for the total, \$ 240, to gain as much interest as the several sums would gain between the selected date and the times they severally become due. For convenience, the selected date may be *the earliest date at which any sum becomes due*, which, in this example, is Jan. 21.

OPERATION.

Due.	Items.	Days.	Interest.
Jan. 21.	\$ 60	0	\$ .00
Feb. 2.	80	12	.16
Mar. 10.	100	48	.80

$\frac{1}{6}$ of $\frac{1}{1000}$ of \$ 240 = \$ .04) \$ .96 (24

Jan. 21 + 24 d. = Feb. 14.

We then find the interest of each item at 6% from Jan. 21 till it becomes due.

The total of the interests thus found is \$ .96, and the sum of the items is \$ 240. The interest of \$ 240 for 1 day is $\frac{1}{6}$ of $\frac{1}{1000}$ of \$ 240, which equals \$ .04.

It will require as many days for \$ 240 to gain \$ .96 as there are .04's in .96, which is 24. A date 24 days after Jan. 21 is Feb. 14, the time when the whole may be paid in one sum. *Ans.* Feb. 14.

484. The process of finding the time when the payment of several items, due at different times, may be made at once without loss of interest to either party is **average,** or **equation of payments.**

485. The date at which several sums due at different times may be paid at once, as Feb. 14 above, is the **average date** or **equated time of payment.**

486. From the operation above may be derived the following

Rule. — To find the average date for the payment of several sums due at different dates: — 1. *Select some convenient date as the earliest date at which any item matures.*

2. *Compute the interest on each item from the selected date to the date of its maturity.*

3. *Divide the sum of the interests thus found by the interest of the sum*

of the items for 1 day. The quotient will express the number of days from the selected date to the average date of payment.

4. *Add this number to the selected date; the result will be the average date required.*

NOTE I. — If any item contains a number of cents, disregard them if less than 50, call them $ 1 if 50 or more.

NOTE II. — If a result contains a fraction of a day, disregard it if less than $\frac{1}{2}$, call it 1 day if $\frac{1}{2}$ or more.

NOTE III. — The interest may be reckoned at any rate per cent.

NOTE IV. — Instead of dividing by the interest for 1 day, we may divide by the interest for 1 month (at 6%, $\frac{1}{2}$ of $\frac{1}{100}$ of principal), in which case the time to elapse before the average date will be in months. Though this method is sometimes more convenient, it will not always give an accurate result.

487. PRODUCT METHOD.

The following is the operation of ILL. EX. Art. 483 by the product method: —

OPERATION.

Days.				Products.
0	×	60	=	0
12	×	80	=	960
48	×	100	=	4800
		240	)	5760
				24

Jan. 21 + 24 = Feb. 14.

Explanation. — The interest of $ 80 for 12 days equals the interest of $ 1 for 960 days; the interest of $ 100 for 48 days equals the interest of $ 1 for 4800 days.

Adding the several products, we find that the total of interest of the several sums for their respective times equals the interest of $ 1 for 5760 days. But the interest of $ 1 for 5760 days equals the interest of $ 240 for $\frac{1}{240}$ of 5760 days, or 24 days. Jan. 21 + 24 days = Feb. 14, *Ans.* Hence the following

RULE. — To find the average date for the payment of several sums due at different dates: 1. *Select some convenient date, as the earliest date at which any item matures.*

2. *Multiply the time each item has to run by the number of units in the item.*

3. *Divide the sum of the products thus obtained by the number of units in the sum of the items; the quotient will express the time from the selected date to the average date of payment.*

4. *Add this time to the selected date; the result will be the average date required.*

NOTE. — The examples in this book are performed by the interest method, which, by the use of Interest Tables, has the advantage of brevity.

488. PROOF. — *Find the sum of the interests on all items due before the average date, from the date at which they are due to the average date; also find the sum of the interests on all items due after the average date from that date to the date at which they are due. If these sums are equal, the work is correct.*

489. EXAMPLES.

1. What is the average date for paying $ 162 due Oct. 1, $ 120 due Oct. 10, and $ 150 due Nov. 6? *Ans.* Oct. 16.

2. I owe three notes as follows: a $ 300 note, payable March 31; a $ 200 note, payable April 30; a $ 100 note, payable May 30: what is the equated time for paying all at once? *Ans.* April 20.

3. A purchased goods August 5, amounting to $ 2100; $ 700 to be paid Nov. 5, $ 700 Dec. 5, and $ 700 Feb. 5; when may the whole be paid without loss to either party? *Ans.* Dec. 16.

4. I purchased goods of Eben Sutton to the amount of $ 3000; $ 800 to be paid Sept. 10, $ 1250 to be paid Oct. 1, and $ 950 to be paid Oct. 28; what is the average date of payment, and what must be the date of a note payable in 2 months, with grace, that it may become due at the average date of payment? *Ans.* Oct. 4; Aug. 1.

5. Bought goods of George Peirce & Co. as follows: —

Jan.	5,	1868,	a	bill of	$ 180.20	on 2	months'	credit.*
"	8,	"	"	"	216.48	"	"	"
"	9,	"	"	"	84.18	"	"	"
"	12,	"	"	"	522.00	"	"	"
"	21,	"	"	"	221.74	"	"	"

What is the average date for paying the whole? *Ans.* Mar. 15, '68.

6. July 10, 1869, my ledger contains items of account against John Cahill, as follows: —

1869.	Jan. 25.	To	Mdse.	on 6	mo.	credit,	$ 199.84.
"	April 3.	"	"	" 4	"	"	600.27.
"	June 16.	"	"	" 60	days'	"	500.00.

What is the average date for paying the whole? *Ans.* Aug. 9, '69.

* Add 3 days of grace to times of credit. (Art. 482. Note II.)

7. When shall a note to settle the following account be made payable?

E. E. WHITE

To WICKERSHAM & Co., Dr.

1869.	April 10.	To Mdse. on 30 days' cr.		$ 100
"	May 16.	" " " 60 " "		200
"	June 3.	" " " 30 " "		420
"	July 18.	" Cash,		150

Ans. July 5, 1869.

8. Find the average date of payment of the following account: —

J. B. MOORE

To E. DENNISON, Dr.

1868.	Dec.	5	To Mdse. on 4 m. . . .	$ 600	00
"	"	24	" " " 3 m. . . .	624	00
"	"	25	" " " 3 m. . . .	210	00
1869.	Jan.	3	" " " 60 days . .	242	00
"	"	21	" " " 60 " . . .	204	00

Ans. March 28, 1869.

AVERAGE OF ACCOUNTS.

490. TO FIND THE AVERAGE TIME FOR THE SETTLEMENT OF AN ACCOUNT WHEN THERE ARE BOTH DEBIT AND CREDIT ITEMS.

ILLUSTRATIVE EXAMPLE I.

My ledger contains the following account: —

Dr. HENRY B. DYER. Cr.

1869.					1869.				
Dec.	7	To Merchandise.	1750	00	Nov.	7	By charges	250	00

The above is an account of merchandise sold on my account by Henry B. Dyer, commission merchant, with his charges for the sale.

At what date should Dyer pay the balance of the account?

OPERATION.

Dr.

Due.	Item.	Days.	Interest.
Dec. 7.	$1750	30	$8.75

Cr.

Nov. 7.	250	0	0

$\frac{1}{6}$ of $\frac{1}{1000}$ of 1500 = .25) 8.75
35 d.

Nov. 7 + 35 d. = Dec. 12.

Explanation. — By paying me the balance of the account, $1500, Nov. 7, Dyer would virtually receive the $250 due him at that date; but as $1750 was not due till 30 days after that date, he would incur a loss equal to the interest on $1750 for 30 days, which, at 6%, is $8.75.

Instead, then, of paying me the balance Nov. 7, he may defer the payment as many days after Nov. 7 as will be required for $1500 on interest at 6% to gain $8.75, which is 35 days. A date 35 days after Nov. 7 is Dec. 12. *Ans.* Dec. 12, 1869.

ILLUSTRATIVE EXAMPLE II.

Dr. RICE & FISKE. Cr.

1868.					1868.				
Feb.	3	To Mdse. 30 d. cr.	840	00	Jan.	28	By Mdse. 90 d. cr.	700	00
Mar.	4	" do. 60 d. cr.	620	00	Apr.	15	" Cash,	400	00

The above account shows that

Rice & Fiske owe me
$840 due 33 d.* after Feb. 3.
620 " 63 " Mar. 4.

I owe Rice & Fiske
$700 due 93 d.* after Jan. 28.
400 " April 15.

When should R. & F. pay the balance of the account?

OPERATION.

Dr. Due. 1868.	Item.	Days.	Int. from Mar. 7.	Cr. Due. 1868.	Item.	Days.	Int. from Mar. 7.
Mar. 7.	$840	0	0	Apr. 30.	$700	54	$6.30
May 6.	620	60	$6.20	Apr. 15.	400	39	2.60
	$1460		$6.20		$1100		$8.90
	1100						6.20
Bal. of acct.	$360					Bal. of int.	$2.70

$\frac{1}{6}$ of $\frac{1}{1000}$ of $360 = $.06; $2.70 ÷ $.06 = 45.
45 days, counted back from Mar. 7, gives Jan. 22, 1868, *Ans.*

* Time of credit with grace.

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Explanation. — The earliest date at which any item on either side of the account becomes due is March 7, 1868.

By computing the interest on the several items from this date to the dates of the several maturities, and finding the total of interest also of the items of the account, we find that at the above date, March 7, 1868,

R. & F. owe me	$1460,	which	they	may	retain	till it	yields	$6.20 int.,
I owe R. & F.	1100,	"	I	"	"	"	"	8.90 "

The bal. of acct., $360, and the balance of interest, $2.70, are therefore due me March 7, 1868.

If the balance only is paid me, it should be at a date as many days before March 7 as will be required for $360, on interest, to gain $2.70, which is 45 days. 45 days before March 7, 1868, is Jan. 22, 1868.

Ans. Jan. 22, 1868.

NOTE. — It will be noticed that in ILL. EX. I. more interest is due on the larger side of the account, and that in ILL. EX. II. more interest is due on the smaller side of the account.

From the above illustrations we derive the following

RULE. — To find the average or equated time for the settlement of an account when there are both debit and credit items: —

1. *Find the interest on the several items of the account from the earliest date at which any item becomes due to their several maturities.*

2. *Find the balance of interest of the debit and credit sides of the account, also the balance of the items.*

3. *Divide the balance of interest by the interest of the balance of the items for one day. The quotient will be the time in days between the selected date and the average time of settlement.*

4. *Count this time* FORWARD *from the selected date, when the larger side of the account has more interest than the smaller, and* BACK *when it has less. The result will be the date of settlement.*

NOTE. — When the interest on the smaller side of the account exceeds the interest on the larger side, as in ILL. EX. II., the balance may be payable earlier than the date of any of the transactions. Settlement must, in such case, take place after the average date.

When settlement takes place after the average date, the balance of the account draws interest from the average date to the date of settlement.

When settlement takes place before the average date, discount is allowed on the balance from the date of settlement to the average date.

491. PROOF. — If an account has been averaged correctly, the difference between the totals of interest on the two sides of the account for the time between the average date and the several maturities will be less than the interest on the balance of the account for a half-day. The following is the

PROOF OF ILL. EX. II. (Art. 490.)

Dr. Due.	Item.		Int. from Jan. 22.	Cr. Due.	Item.		Int. from Jan. 22.
Mar. 7.	840	45 ds.	$ 6.30	Apr. 30.	700	99 ds.	$ 11.55
May 6.	620	105 ds.	10.85	" 15.	400	84 ds.	5.60
		Dr. Int.	$ 17.15			*Cr. Int.*	$ 17.15

492. EXAMPLES.

1. When should the balance of the following account be paid?

Dr. DAVID DANFORTH. Cr.

1869.					1869.				
Mar.	6	To Cash,	$ 200	00	Mar.	2	By Mdse. 60 d. cr.	$ 500	00
Mar.	8	" Real Estate 30 d. cr.	400	00	Mar.	5	" do. 60 d. cr.	400	00

Ans. July 25, 1869.

2. When should the balance of the following be paid?

Dr. WINTHROP & Co. Cr.

1869.					1869.				
Nov.	12	To Cash,	$ 950	00	Oct.	7	By Mdse. 60 d. cr.	$ 500	00

Ans. Oct. 13, 1869.

3. Average the following account: —

Dr. RICE, INGALLS, & Co. Cr.

1869.					1869.				
Mar.	18	To Mdse.	$ 150	00	Apr.	1	By Mdse.	$ 600	00
Mar.	30	" do.	500	00	Apr.	20	" do.	200	00

Ans. May 17, 1869.

☞ For Dictation Exercises in Interest Accounts and Average, see "Manual and Key," pages 135 – 144.

4. In my ledger is the following account; when is the balance due?

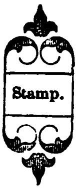
Dr. CHARLES DUDLEY. Cr.

1869.					1869.				
Jan.	6	To Mdse. 3 m. cr.	$239	92	Jan.	1	By bal. of acct.	$261	20
"	25	" do. 30 d. cr.	482	20	"	15	" Real Estate, 3 m. cr.	3000	00
Feb.	21	" do. 3 m. cr.	75	12	Feb.	7	" Mdse. 2 m. cr.	480	00
May	29	" do. 2 m. cr.	1200	00					

Ans. Jan. 31, 1869.

493. EXCHANGE.

$2000 *Philadelphia, April 8, 1869.*



At ten days' sight, pay to the order of John Drew,
—— *TWO THOUSAND DOLLARS,* ——
Value received, and charge to account of
To William J. Ives, *Alfred Conant.*
Pittsburg.

The above is a written order directing one person to pay money to another. Such an order is a **draft.**

ILLUSTRATION.—In explanation of the above draft, we may suppose that John Drew, residing in Philadelphia, owes a person, Calvin S. Pennell, for example, in Pittsburg, two thousand dollars; to make the payment, Drew buys of Alfred Conant, banker, of Philadelphia, this draft, in which Conant directs his correspondent, William J. Ives, of Pittsburg, to pay two thousand dollars to the order of John Drew. Drew indorses the draft by writing on the back of it,

"Pay to the order of Calvin S. Pennell,
John Drew,"

and remits it to Pennell.

Upon receiving the draft, Pennell immediately presents it to Ives. If Ives acknowledges the obligation, he writes across the face of the draft,

"Accepted; William J. Ives,"

also the date of presentation, and thus becomes responsible for its payment at its maturity.

NOTE. — If Pennell desires to secure the money on the draft before it matures, he indorses it, and thus renders it *negotiable*.

494. The method of making payments by drafts is **exchange**; drafts are sometimes called **bills of exchange**.

NOTE I. — Art. 463, with Notes, applies to drafts.

NOTE II. — In the payment of a draft, three days of grace are allowed or not, according to the custom of the place upon which the draft is made.

NOTE III. — Drafts may be at *par*, at a *premium*, or at a *discount*.

495. The draft given in Art. 493 is addressed to a person residing in the same country as the drawer. Such a draft is an **Inland** or **Domestic Bill of Exchange**.

A draft addressed to a person residing in a foreign country is a **Foreign Bill of Exchange**.

The rules already illustrated in percentage apply to Exchange.

DOMESTIC EXCHANGE.

EXAMPLES.

496. 1. What must be paid for a draft on Cincinnati for $ 350 at 1½% premium? at ¾% discount? *Ans.* $ 355.25; $ 347.37½.

2. Bought goods for $ 600 and sold them at a gain of 16⅔%, and remitted the amount to Portland in a draft payable at 60 days' sight, which I bought at 1¼% discount; what did I pay for the draft?

NOTE. — In selling a draft upon time, bankers usually include in their discount or premium for exchange the discount for the given time; hence, in solving examples like the above, unless otherwise directed, the pupil may proceed as if no time were specified. *Ans.* $ 691.25.

3. What must be paid for two drafts on Eldredge and Brother of Philadelphia for $ 540 each, one at sixty days' sight at a discount of 2%, and the other at sight at a premium of ½%? *Ans.* $ 1,071.90.

497. ILL. EX. What is the face of a draft on Cleveland, which may be bought for $ 275 at ½% premium?

Explanation. — The exchange value of $ 1, at ½% premium, is $ 1.005. With $ 275 a draft can be purchased for as many dollars as there are $ 1.005's in $ 275. *Ans.* $ 273.63.

4. What is the face of a draft that may be bought for $ 500 at a discount of 1½%? *Ans.* $ 507.61.

5. The net proceeds of a lot of cheese sold for my correspondent in New York are $ 1872; with this sum I buy for remittance a draft on J. Watson & Co. of New York at a premium of $\frac{1}{10}$%; what is the face of the draft? *Ans.* $ 1,870.13.

FOREIGN EXCHANGE.

498. The method of computing foreign exchange is the same as that of computing inland exchange, except that the currency of one country is changed to that of the other.

NOTE. — If a person wishes to remit a given sum to any foreign country, a bill on London or Paris is almost always convenient alike to creditor and debtor; hence *exchange between the United States and foreign countries is generally effected through London or Paris.*

499. ENGLISH CURRENCY.

4 farthings (qr. or far.)	= 1 penny,	marked	d.
12 d.	= 1 shilling,	"	s.
20 s.	= 1 pound,	"	£.

NOTE I. — The *guinea* of 21s., and the *crown* of 5 s., are also used. The coin which represents the £ value is called a *sovereign.*

NOTE II. — The value of £1 sterling, which is the English sovereign, is $ 4.44$\frac{4}{9}$ in the old coin of the United States. But Congress has from time to time reduced the weight and purity of United States coins, making their value as metals less than their value as coins, that they might not be used for transportation or the arts, and has established the legal value of the pound sterling at $ 4.84. The commercial value of the pound varies as it is in greater or less demand.

The basis of Exchange, however, is the old or nominal value of the pound ($ 4.44$\frac{4}{9}$), compared with which the pound at $ 4.84 is at 8$\frac{1}{9}$% premium. Hence, for example, when exchange on England is quoted at 10% premium, it is about 1% premium upon the legal value.

500. FRENCH CURRENCY.

100 centimes = 1 franc, marked fr.

NOTE. — 1 franc = $18\frac{3}{5}$ cents.

501. EXAMPLES IN REDUCTION OF CURRENCY AND EXCHANGE.

NOTE I. — In the following examples, by United States money is meant United States gold coin, unless it is otherwise specified.

6. What is the nominal value of £30 English money expressed in United States money?

OPERATION. — £1 = $\$4.44\frac{4}{9} = \$\frac{40}{9}$; $\$\frac{40}{9} \times 30 = \$133\frac{1}{3}$, *Ans.*

7. What is the legal value of the above? (Art. 499, Note II.)

8. What is the value of the above at $9\frac{1}{2}\%$ advance upon the nominal value? $\frac{\$40 \times 30 \times 1.095}{9} = \$146$, *Ans.*

9. What is the value of £40 at 9% advance?

10. What is the cost in Philadelphia of a bill on London for £50 5 s. 9 d. at 10% premium?

NOTE II. — £50 5 s. 9 d. equals £50.2875 (Art. 342), which may be changed to United States money as above. *Ans.* \$245.85.

ABBREVIATED METHOD OF REDUCTION. — £1 = $\$\frac{40}{9}$, and £1 equals 40 sixpences; then 40 sixpences equal $\$\frac{40}{9}$, and 1 sixpence equals $\$\frac{1}{9}$; hence 9 sixpences equal \$1. Therefore,

To change English money to United States money, *Change the given sum to sixpences, and divide the number of sixpences by 9. The quotient will express the nominal value of the given sum in dollars of United States money.*

To obtain the real value add the premium.

NOTE III. — To change pounds, shillings, and pence to sixpences, *multiply the number of the pounds by* 40, *the number of the shillings by* 2, *and divide the number of the pence by* 6. This can generally be done without the aid of the slate. Example 10 would be performed by the above method as follows: —

£50 5 s. 9 d. = 2011.5 sixpences. $\frac{2011.5 \times 1.10}{9} = \$245.85$. *Ans.*

EXAMPLES.

11. What is the cost of £40 15 s. at 9% premium? *Ans.* \$197.41.

12. What is the cost of £1000 at $9\frac{3}{4}\%$ premium? *Ans.* \$4,877.78.

13. Bought goods in Liverpool to the amount of £167 7 s.; what must I pay in U. S. paper currency for a draft of the same at 10% advance, gold being at a premium of 35%?

£167 7 s. = 6694 sixpences. $\frac{6694 \times 1.10 \times 1.35}{9} = \$\,1{,}104.51$, *Ans.*

14. Bought goods in London to the amount of £725 12 s.; what must I pay in U. S. paper currency for a draft of the same at 9½% advance, gold being at a premium of 33%? *Ans.* $ 4,696.57.

15. What shall I pay in U. S. paper money for the following draft at 9¾% advance, gold being at a premium of 30%?

Boston, May 17, 1869.

Exchange. *£ 100 Stg.*

No. 10.

Sixty days after Sight of this first of Exchange, second and third unpaid, pay to Brown, Smith, & Co. or Order, One Hundred Pounds Sterling, Value received, and charge the same to account of Your obdt. servts.,

Dupee, Beck & Sayles.

To Messrs. Baring Brothers & Co.,

London.

NOTE. — To provide against accident in the transmission of a draft, it is customary to send two, at least, of the same tenor and date, by different modes of conveyance or at different times, the payment of either one of which cancels the others.

16. What amount of English money at 10% advance can be bought for $ 675?

OPERATION.

$\frac{\$\,675 \times 9}{40 \times 1.10} = £138\ 1\text{ s. } 4\frac{4}{11}\text{ d.}$

Explanation. — $ 675 changed to pounds at par equals £$\frac{675 \times 9}{40}$.

As many pounds at 10% advance can be bought for this sum as there are 1.10's in $\frac{675 \times 9}{40}$, or £138 1 s. $4\frac{4}{11}$ d. *Ans.* £138 1 s. $4\frac{4}{11}$ d.

17. What amount of English money at 9½% advance can be bought for $ 328.50? *Ans.* £67 10 s.

18. There was shipped to Liverpool from New York in one week $ 6870205 in specie; what amount of English currency, at 9½% premium, could be bought with it? *Ans.* £1,411,685 19 s. 2 d.+

19. For $ 500 in U. S. paper money, what amount of English money can be bought at 9% advance, gold being at a premium of 28%?
Ans. £80 12 s. 8 d.+

20. What is the cost of a draft on Paris for 500 francs, exchange being 5.2 francs per dollar? *Ans.* $ 96.15.

21. What is the cost in paper currency of a set of exchange on Paris for 200 francs, at 5.21 fr. per dollar in coin, gold being at a premium of 25%? *Ans.* $ 47.98.

22. The shares in the Panama Railroad were originally 500 francs each; what was their value in United States paper money, exchange being 4.25 fr. per dollar? 117.65

23. What is the amount of a draft on Paris that I can purchase for $ 200 U. S. paper money, exchange being 5.22 fr. per gold dollar, and gold being at a premium of 20%? *Ans.* 870 fr.

BONDS.

502. Governments and corporations sometimes borrow money, giving, as evidence of the loans, certificates payable at or within some definite time, with interest at stated periods. Such certificates and notes are called **bonds.**

NOTE I. — Bonds sometimes have *interest tickets* attached, promising to the bearer certain sums of interest as they become due upon the bonds. The interest tickets are called **coupons.**

NOTE II. — When the interest specified on a coupon is paid, the coupon is cut off and retained as a receipt.

The principles already illustrated in percentage apply to operations upon bonds.

UNITED STATES BONDS.

503. The extraordinary expenses of the government of the United States for the past few years have been met in part by the sale of bonds.

NOTE. — U. S. bonds are issued in denominations of $ 50, $ 100, $ 500, $ 1000, $ 5000, and $ 10000.

The following is a list of the principal

UNITED STATES BONDS NOT REDEEMED IN 1869.

Bonds called	Payable in	Bearing interest at
"5 per cents" of '71 and '74.	1871 and 1874 respectively.	5%
"6 per cents" of '81, '92, '94.	1881, '92, '94 respectively.	6%
"5-20's," issued '61, '62, '64, '65.	20 yrs., redeemable in 5 yrs.	6%
"10-40's," issued 1864.	40 yrs., redeemable in 10 yrs.	5%

per annum, payable semi-annually in gold.

EXAMPLES.

1. When gold is at 35% premium, what is my semi-annual income in currency from 12 U. S. 6% bonds of $1000 each?

OPERATION. — $\$12000 \times .03 \times 1.35 = \$486.$ *Ans.* $486.

2. U. S. 6%'s are quoted to-day at 119⅛; what must I pay for 6 bonds of $100 each, brokerage being ¼% on the amount invested?

NOTE. — By 119⅛ is meant 119⅛% or 19⅛% premium. *Ans.* $716.54.

3. What is the value in currency of the semi-annual interest on the above bonds, gold being quoted at 128? *Ans.* $23.04.

4. What per cent upon my investment do I make semi-annually by buying a U. S. 5-20 bond at 110, gold being at a premium of 32%? *Ans.* 3⅗%.

5. The premium on gold being 32%, which will yield the greater per cent of income, money invested in U. S. 10-40's at 108, or in R. R. stock purchased at 12% advance, which pays a semi-annual dividend of 4%?

Ans. R. R. stock; R. R. paying semi-annually $3\frac{4}{7}$%; bonds $3\frac{1}{18}$%.

6. Which is the better investment, U. S. 5%'s at 112, or U. S. 6%'s at 115? *Ans.* 6%'s. 6%'s yield $2\frac{14}{23}$%, 5%'s yield $2\frac{13}{56}$% semi-annually in gold.

7. How many $100 U. S. bonds at 115 can I purchase with $1978, and what money will remain? *Ans.* 17 bonds; $23 remain.

8. How much money must be invested in U. S. 5-20's at 114, to yield a semi-annual income of $400 in gold? *Ans.* $15,200.

504. Topical Review in Percentage.

The pupil may present the following topics to the class, using common illustrations, giving definitions, and deriving rules from illustrative examples, which he will solve before the class: —

1. Interest, — principal, amount, rate, legal rate. (Arts. 437 – 441.)
2. General method of computing interest. (Art. 442.)
3. To compute interest at 6%, 1st method. (Arts. 444 – 447.)
4. To compute interest at 6%, 2d method. (Art. 449.)
5. To compute interest at any rate by the 6% methods. (Art. 451.)
6. To compute accurate interest. (Art. 453.)
7. To find the time, when the interest, the principal, and the rate per cent are given. (Art. 454.)
8. To find the rate, when the interest, the principal, and the time are given. (Art. 455.)
9. To find the principal, when the interest or amount, the time, and the rate are given. (Art. 456.)
10. Discount and present worth. (Arts. 457 – 460.)
11. Promissory notes. (Arts. 461 – 463, with notes.)
12. Bank discount, proceeds or avails, days of grace, maturity of notes. (Arts. 464 – 468, with notes.)
13. Indorsing notes. (Art. 469, with notes.)
14. To find the bank discount and proceeds of a note. (Art. 470.)
15. To find the face of a note. (Art. 471.)
16. Compound Interest. (Arts. 472 – 474.)

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17. Partial payments, United States rule. (Arts. 475 – 478.)
18. Partial payments, merchants' rule. (Art. 479.)
19. To balance accounts current with interest. (Arts. 481, 482.)
20. Average or equation of payments. — Interest method. (Arts. 483 – 486.) Product method. (Art. 487.) Proof. (Art. 488.)
21. Average of accounts. (Arts. 490.) Proof. (Art. 491.)
22. Drafts and exchange. (Arts. 493, 494, with notes.)
23. Domestic exchange, manner of computing. (Arts. 495 – 497.)
24. Foreign exchange; English currency; French currency; manner of computing foreign exchange. (Arts. 498 – 501.)
25. Bonds. (Arts. 502, 503.)

505. General Review, No. 7.

1. What is the amount at simple interest of $ 227 for 1 y. 2 m. 16 d at 9%? *Ans.* $ 251.74.

2. What is the amount of the above at compound interest, the interest compounding semi-annually? *Ans.* $ 252.60.

3. Compute accurate interest (365 days to the year) on $ 1000 at 6%, from May 2, 1869, to Nov. 1, 1869. *Ans.* $ 30.08.

4. At what rate per cent will $ 200 on interest from Dec. 18, 1869, to July 12, 1870, gain $ 13.60? *Ans.* 12%.

5. At what date must a $ 1200 note have begun to draw interest which, at 6%, yielded $ 280 Oct. 15, 1868? *Ans.* Nov. 25, 1864.

6. By true discount at 6%, what sum would discharge a debt of $ 720, April 4, which is payable May 7, without interest? *Ans.* $ 716.06.

7. What is the bank discount at 12% on a note for $ 60, given Oct. 17, for 3 months, and discounted Jan. 1 following? *Ans.* $ .38.

8. For what sum must a note be drawn that, when discounted at a bank for 2 months, at 6%, the proceeds shall be $ 240? *Ans.* $ 242.55.

9. On a note for $ 2000, dated Sept. 4, 1867, interest from date at 10%, there was paid $ 40 at the expiration of 3 months, and $ 200 at the expiration of 6 and of 9 months; what balance was due Sept. 4, 1868, by United States rule? *Ans.* $ 1,749.16.

10. What was due on the above by merchants' rule? *Ans.* $ 1,742.

11. What is the cost in U. S. currency (paper), of a 60 days' draft on London for £425, gold being at a premium of 41%, exchange being 9½% premium? *Ans.* $ 2,916.35.

12. What amount of French currency can I buy for $ 315, exchange being at 5.12 fr. per dollar? *Ans.* 1,612.8 fr.

13. What is the face of a draft that costs $ 720, exchange being at ½% discount? *Ans.* $ 723.62.

☞ Perform the above examples, substituting 3 for 2 in each example. For other dictations on this Review, see "Manual and Key," page 144.

506. Miscellaneous Examples.

1. What is the brokerage, at $\frac{1}{4}$ of 1%, on the purchase of 28 shares of stock in the Globe Bank, at $22\frac{1}{2}$% premium, par value $ 100?
Ans. $ 8.58.

2. A house valued at $ 3000 rents at $ $20.83\frac{1}{3}$ a month; what per cent per annum does the property pay? *Ans.* $8\frac{1}{3}$%.

3. How much money, on interest at $12\frac{1}{2}$%, will yield an annual income of $ 750?

4. What sum of money, borrowed April 1, 1869, at 10%, will amount to $ 288, Dec. 1, 1869? *Ans.* $ 270.

5. What is the difference between a discount of 35% on $ 1600, and a discount of 30% on $ 1600 with 5% on the balance? *Ans.* $ 24.

6. What per cent on his investment was a person's gain, who bought 20 shares of Panama Railroad stock at 20% below par, the par value being 500 francs each, and which he afterwards sold for 31000 francs? *Ans.* $287\frac{1}{2}$%.

7. What per cent is saved by using for basting, common skein cotton at $ .10 per doz. skeins of 54 yards each, instead of using Coat's spool cotton at $ .10 a spool of 200 yards? *Ans.* 224%.

8. What is the value in currency of the semi-annual interest of 3 U. S. 10-40 bonds of $ 1000 each, when gold is at 135%?
Ans. $ 101.25.

9. When gold is at a premium of 30%, what is the cost, in gold, of a passage to Liverpool for which $ 162.50 is paid in currency?
Ans. $ 125.

10. What per cent of profit is made on 1 acre of land that yields 200 barrels of onions which sell at $ 1.45 per barrel, the cost of cultivating and marketing being $ 135? *Ans.* $114\frac{22}{27}$%.

11. I imported from Paris 3 cases of shawls, their total weight being 674 lbs. invoiced at 8532 francs. What was the amount of the duties at $ .22 per lb. and 35% ad valorem? *Ans.* $ 703.71.

12. On his 25th birthday, a gentlemen insured his life for $ 5000, by paying a single premium of $ 262.23 per $ 1000. If he died on his 40th birthday, how much more did his family receive than was expended for the premium, calculating compound interest on the premium at 6% per annum? *Ans.* $ 1,857.75.

13. Find the amount due on the following note April 5, 1870, at 8%.

$620. *St. Louis, Dec.* 5, 1869.

Three months after date, I promise to pay James T. Brooks, or order, six hundred twenty dollars, value received. Jos. BOYNTON.

NOTE. — See Art. 467. *Ans.* $623.86.

14. What are the proceeds of the above note, by bank discount, Jan. 20, 1870, at 7% discount? *Ans.* $614.33.

15. What is the true present worth of $620 due in 3 months, interest being at 6%? *Ans.* $610.84.

16. 5% discount was allowed me for paying $250 of a bill of $500 before the bill became due; what balance was due upon the bill after this payment was made?

NOTE. — If 5% was allowed me on the payment, $250 must be 95% of the part of the debt cancelled. *Ans.* $236.84.

17. Sept. 10, 1867, a note was given for $800, with interest at 9%. On this note was paid $100 July 1, 1868, and $175.26 Oct. 1, 1868; what balance was due Jan. 1, 1869? *Ans.* $613.50.

18. Bought a U. S. 6% bond of $100, at 7% premium; I gave this bond in exchange for 2 sewing-machines which I rented for 1 year, at $3 each per month. The value of the machines having depreciated $10 each, the bonds having advanced to 113, and gold being 134, what was my gain or loss by the exchange? *Ans.* Gained $37.96.

19. Compute interest at 6% on each item of the following account, from the time it becomes due to the date of settlement, Nov. 13, 1869, and find the balance then due: —

Dr. E. P. PEABODY & Co. Cr.

Date.		When due.	Item.	Days.	Int.	Date.		When due.	Item.	Days.	Int.
1869.		1869.				1869.		1869.			
Aug. 6	To balance old account, . .	Aug. 6	720 00			Aug. 8	By note at 3 m. .	Nov. 11	300 00		
Sept. 4	To Mdse. 30 d. as per bill rend'd.	Oct. 7	820 00			Oct. 15	By Draft on Lee & Co.	Oct. 15	120 00		
						Oct. 17	By Cash,	Oct. 17	400 00		

Ans. Balance due from Peabody, $734.46.

20. Average the above account, to find at what date a note for the balance should begin to draw interest. *Ans.* July 15, 1869.

☞ For Miscellaneous Exercises, see "Manual," pages 146, 147.

RATIO.

507. By comparing the numbers 6 and 2, we find that 6 is 4 more than 2, or that their difference is 4; also that 6 equals three 2's, or that their quotient is 3.

508. The difference or quotient of two numbers of the same kind is **ratio.**

509. The difference of two numbers of the same kind is **arithmetical ratio.**

510. The quotient of two numbers of the same kind is **geometrical ratio.**

NOTE. — When the kind of ratio is not named, geometrical ratio is understood. Geometrical ratio only will be considered in this connection.

511. The ratio of 2 to 4 is indicated thus, 2 : 4. The expression is read, "The ratio of 2 to 4."

512. 1. The ratio of 3 to 6 is $\frac{1}{2}$, that is, 3 is $\frac{1}{2}$ of 6.

2. The ratio of 6 to 3 is 2, that is, 6 is two 3's.

Because the numbers 3 and 6 limit or determine the ratio in each of the examples above, 3 and 6 are called **terms** of the ratio. The two terms of a ratio taken together are called a **couplet.**

The first term of a couplet is the **antecedent.**

The second term of a couplet is the **consequent.**

EXERCISES.

Name the antecedents of the ratios indicated below: name the consequents.

1.	3 : 12.	3.	3 : $1\frac{1}{2}$.
2.	5 : 2.	4.	$2\frac{1}{3}$: $\frac{2}{3}$.

513. By Art. 512 we see that the ratio is found *by dividing the antecedent by the consequent.*

EXAMPLES.

What is the ratio of

1.	3 to 12?	*Ans.* $\frac{1}{4}$.	4.	3 to $1\frac{1}{2}$?	2
2.	5 to 2?	*Ans.* $2\frac{1}{2}$.	5.	$2\frac{1}{3}$ to $\frac{2}{3}$?	
3.	5 to 12?	*Ans.* $\frac{5}{12}$.	6.	10 to $3\frac{1}{3}$?	

NOTE.— Some arithmeticians determine the ratio by dividing the consequent by the antecedent.

514. If having two or more ratios as \$3 : \$4 $= \frac{3}{4}$, and 3 days : 5 days $= \frac{3}{5}$, we multiply one ratio by the other, thus, $\frac{3 \times 3}{4 \times 5}$, the product is a **compound ratio.**

This compound ratio may be indicated thus :

$$\left.\begin{array}{r} \$3 \ : \ \$4 \\ 3 \text{ days} \ : \ 5 \text{ days} \end{array}\right\} \text{or } 3 \times 3 : 4 \times 5 = \frac{9}{20}.$$

From the above, we see that a compound ratio is found *by dividing the product of the number of units in each of the antecedents by the product of the number of units in each of the consequents.*

EXAMPLES.

Find the ratio in each of the following examples :—

7. $\left.\begin{array}{r} 2 : 3 \\ 7 : 8 \end{array}\right\} = ?$ *Ans.* $\frac{\cancel{2} \times 7}{3 \times \cancel{8}_{4}} = \frac{7}{12}$.

8. $\left.\begin{array}{r} \$6 : \$8 \\ 5 \text{ lb.} : 3 \text{ lb.} \end{array}\right\} = ?$

PROPORTION.

515. The two ratios, 4 days : 8 days, and 6 hours : 12 hours, are equal to each other.

An equality of ratios is a **proportion.**

The above proportion may be expressed thus :—

4 days : 8 days = 6 hours : 12 hours.

The expression is read : "The ratio of 4 days to 8 days equals the ratio of 6 hours to 12 hours," or "4 days is to 8 days as 6 hours is to 12 hours."

516. The first and fourth terms of a proportion are called **extremes,** the second and third are called **means.**

EXERCISES.

Read the following, and name the means and the extremes of each proportion:—

1. 5 men : 8 men = $ 15 : $ 24
2. 3 : 6 = 6 : 12.

NOTE.—In the second proportion, as 6 is the consequent of the first couplet and the antecedent of the second, it is called a **mean proportional** between the other two terms.

517. TO FIND A MISSING TERM OF A PROPORTION WHEN THE OTHER THREE TERMS ARE GIVEN.

In the proportion 8 days : 2 days = $ 12 : $ 3, the ratio 4 is found by dividing the antecedent of either couplet by its consequent. (Art. 513.) Since an antecedent divided by its consequent equals the ratio, an antecedent must be a product of which its consequent is one factor and the ratio the other factor. Hence

1. *A missing antecedent in a proportion may be found by multiplying its consequent by the ratio of the given couplet.*

2. *A missing consequent in a proportion may be found by dividing its antecedent by the ratio of the given couplet.*

518. ILL. EX. I. Find the missing antecedent in the proportion 4 pk. : 5 pk. = ? : $ 12.

OPERATION.

(*a*) $\frac{\$12 \times 4}{5} = \$9\frac{3}{5}$.

(*b*) 4 pk. : 5 pk. = $ $9\frac{3}{5}$: $ 12.

Explanation. — As every antecedent equals its consequent multiplied by the ratio (Art. 517), the missing antecedent equals the consequent, $ 12 multiplied by $\frac{4}{5}$, which equals $ $9\frac{3}{5}$, and the proportion is 4 pk. : 5 pk. = $ $9\frac{3}{5}$: $ 12.

ILL. EX. II. Find the missing consequent in the proportion 4 pk. : 5 pk. = $ 12 : ?

OPERATION.

(*c*) $\frac{\$12 \times 5}{4} = \$15$.

(*d*) 4 pk. : 5 pk. = $ 12 : $ 15.

Explanation. — As every consequent equals its antecedent divided by the ratio (Art. 517), the missing consequent equals the antecedent $ 12, divided by the ratio, $\frac{4}{5}$, which equals $ 15, and the proportion is 4 pk. : 5 pk. = $ 12 : $ 15.

EXAMPLES.

Find the missing terms of the following proportions: —

1. $8° : 7° = ? : 28.$ *Ans.* 32.
2. $? : 3\frac{1}{2} = 2 : 14.$ *Ans.* $\frac{1}{2}$.
3. $3 : 7 = 9 : ?$ *Ans.* 21.
4. $8 : ? = 16\frac{1}{2} : 33.$
5. $? : 9 = 17 : 3.$
6. $4\frac{1}{2} : 6 = 16 : ?$

519. DEDUCTIONS.

(See Note II. on opposite page.)

Examining operation (*c*), Art. 518, we see that 1 fourth of \$12 × 5 equals \$15; therefore the whole of \$12 × 5 must equal \$15 × 4. Comparing the factors of the products

$$\$12 \times 5 = \$15 \times 4$$

with the terms of proportion (*d*), 4 pk. : 5 pk. = \$12 : \$15, we learn that

(1.) *If we multiply the third term of a proportion by the number of units in the second term, and the fourth term by the number of units in the first term, the two products will be equal.*

From the preceding deduction, also from operation (*c*), it follows that

(2.) The fourth term of any proportion may be found *by multiplying the third term by the number of units in the second term, and dividing the product by the number of units in the first term.*

NOTE TO THE TEACHER. — The principle given in Deduction 1 is sometimes enunciated thus: "*In any proportion the product of the means equals the product of the extremes,*" which is not true when the terms are *denominate* numbers; thus in the proportion 4 pk. : 5 pk. = \$12 : \$15, dollars multiplied by pecks would be neither dollars nor pecks nor anything else; but \$15 multiplied by *the number of units* in the first term, 4, would be \$60, and \$12 multiplied by *the number of units* in the second term, 5, would be \$60. These products are equal.

520. ILL. EX. If 5 pounds of sugar cost 65 cents, what will 14 pounds cost?

OPERATION BY ANALYSIS.

$$\frac{\$\overset{.13}{\cancel{.65}} \times 14}{\cancel{5}} = \$1.82.$$

Explanation. — If 5 pounds cost 65 cents, 1 pound will cost 1 fifth of 65 cents, and 14 pounds will cost 14 fifths of 65 cents; cancelling, we have \$.13 × 14 = \$1.82, *Ans.*

OPERATION BY PROPORTION.

5 lb. : 14 lb. = \$ .65 : \$ 1.82.

$$\frac{\$\,\overset{.13}{\cancel{.65}} \times 14}{\cancel{5}} = \$\,1.82$$

Explanation. — The ratio of 5 pounds to 14 pounds must equal the ratio of \$ .65, the price of 5 pounds, to the price of 14 pounds. We have, then, three terms of a proportion (5 lb. : 14 lb. = \$ .65 : ?), from which we find the 4th term to be \$ 1.82. (Art. 519, (2).)

Note I. — In practice the denomination of the first and second terms of a proportion may be omitted.

Rule for solving Examples by Proportion.

1. *Make the number that is of the same kind as the required answer, the third term.*

2. *Determine, from the conditions of the question, whether the answer should be greater or less than the third term; if greater, make the larger of the other two numbers the second term, and the smaller the first; if less, make the smaller number the second term, and the larger the first.*

3. *Multiply the third term by the number of units in the second term, and divide that product by the number of units in the first term.*

Note II. — If the teacher prefers, Art. 519 may be omitted, and for clause 3 of the rule above the following may be substituted: —

3. *Divide the third term by the ratio of the first term to the second.* (Art. 517, 2.)

Note III. — It is recommended that the pupils perform the following examples both by analysis and by proportion.

Examples.

1. If 5 men can do a piece of work in 6 days, in how many days can 7 men do it? *Ans.* $4\frac{2}{7}$ d.

2. If 15 men can do a piece of work in 12 days, how many men can do it in 20 days? *Ans.* 9 men.

3. If 72 cents is paid for 6 hours' washing, how much should be paid for $8\frac{1}{2}$ hours' washing? \$1.62

4. What is the cost of 3 cord feet of wood at \$ 7 a cord? *Ans.* \$ $2.62\frac{1}{2}$.

5. If it takes 3 days of 7 hours each to do a piece of work, how many days of 9 hours each will it take to do the same work?

6. If 150 soldiers can dig a trench in $6\frac{1}{2}$ hours, how many will be required to dig the trench in $4\frac{1}{3}$ hours? *Ans.* 225 soldiers.

7. If a barrel of beef lasts a family of 9 persons 24 weeks, how long will it last the rest of the family at the same rate, when 2 of the members are absent? *Ans.* **$30\frac{6}{7}$ weeks.**

8. If a hind wheel, $11\frac{2}{3}$ feet in circumference, turns 323 times in going a certain distance, how many times will the forward wheel, which is $8\frac{1}{2}$ feet in circumference, turn in going the same distance?

9. How many yards of cambric, 24 inches wide, will be required to line a table-cover 10 ft. 3 in. long and 6 ft. wide? *Ans.* $10\frac{1}{4}$ yards.

10. If the width of a croquet-ground is to its length as 2 to 3, what must be the width of a ground that is 40 feet in length? 26 2/3

11. If 6 lbs. of sugar can be bought for \$ 1, what will $16\frac{1}{2}$ lbs. cost at the same rate?

12. A man failing in trade owed \$ 13725, and his property was valued at \$ 5270; how much could he pay a creditor to whom he owed \$ 368? *Ans.* \$ 141.30.

13. What is the price of a gross of buttons when $7\frac{1}{2}$ dozens sell for \$ 3.88?

14. If my semi-annual income on U. S. 6%'s is \$ 157.50 when gold is at $131\frac{1}{4}$, what would it be if gold were at $137\frac{1}{2}$? *Ans.* \$ 165.00.

Perform the following examples by analysis alone:—

15. Charles has 5 minutes' start of Samuel in a foot race, and runs at the rate of 18 rods in a minute; in how many minutes can Samuel, who runs at the rate of 22 rods in a minute, overtake Charles?

Ans. $22\frac{1}{2}$ minutes.

16. A, B, and C are husking corn together; A can husk the corn in 12 hours, B in 8 hours, and C in 14 hours; in how many hours can they together husk it? *Ans.* $3\frac{27}{47}$ h.

17. A and B can, together, copy a manuscript in 12 hours, and B can copy it alone in 20 hours; in what time can A copy it alone? 30 h.

18. A merchant gave a chest of tea containing $48\frac{3}{8}$ lbs. worth \$ 1.12 per lb. in exchange for sugars at \$ .11 and \$ .13 per lb., taking the same quantity of each; how many pounds of sugar did he receive?

Ans. $451\frac{1}{2}$ lbs.

19. A man bought several sets of cutlery, each containing 60 pieces, at the rate of \$ .40 per piece, and sold them at the rate of \$ .55 per piece, clearing \$ 54; how many sets did he buy? *Ans.* 6 sets.

COMPOUND PROPORTION.

521. In the proportion $\left.\begin{matrix}3\text{ lb.} : 10\text{ lb.}\\ 5\text{ lb.} : 3\text{ lb.}\end{matrix}\right\} = \$3 : \$15$, an equality is expressed between a compound and a simple ratio.

A proportion in which either or both ratios are compound is a **compound proportion.**

ILL. EX. If 2 boys weed 3 rows of strawberries in 5 hours, how many rows can 4 boys weed in 8 hours?

OPERATION BY ANALYSIS.

$$\frac{\overset{\text{rows.}}{3} \times \overset{2}{\cancel{4}} \times 8}{\cancel{2} \times 5} = 9\tfrac{3}{5}\text{ rows.}$$

Explanation. — If 2 boys weed 3 rows in 5 hours, 1 boy can weed 1 half of 3 rows in the same time, and 4 boys can weed 4 halves of 3 rows.

If this number of rows can be weeded by 4 boys in 5 hours, 1 fifth of this number can be weeded in 1 hour, and 8 fifths of this number can be weeded in 8 hours. Cancelling, we have $\frac{3 \times 2 \times 8}{5}$ rows $= 9\frac{3}{5}$ rows.

OPERATION BY PROPORTION.

(1) $\quad 2\text{ (boys)} : 4\text{ (boys)} = 3\text{ (rows)} : \frac{3 \times 4}{2}\text{ (rows)}.$

(2) $\quad 5\text{ (hours)} : 8\text{ (hours)} = \frac{3 \times 4}{2}\text{ (rows)} : \frac{3 \times 4 \times 8}{2 \times 5}\text{ (rows)}.$

Explanation. — Here the number of rows weeded depends upon the ratio of 2 boys to 4 boys and upon the ratio of 5 hours to 8 hours. Considering the first ratio only, we have 2 boys is to 4 boys as 3 rows is to the number of rows weeded, which we find to be $\frac{3 \times 4}{2}$ rows. (Art. 519, (2).)

To find how many rows can be weeded in 8 hours when $\frac{3 \times 4}{2}$ rows can be weeded in 5 hours, we state the proportion thus: 5 hours is to 8 hours as $\frac{3 \times 4}{2}$ rows is to the number of rows that can be weeded in 8 hours, which we find to be $\frac{3 \times \overset{2}{\cancel{4}} \times 8}{\cancel{2} \times 5}$ rows, or $9\frac{3}{5}$ rows.

Instead of solving the example by using the proportions separately, we may employ the two ratios at once in a compound proportion, thus,

$$\left.\begin{array}{l}2 \text{ boys} : 4 \text{ boys} \\ 5 \text{ hours} : 8 \text{ hours}\end{array}\right\} = 3 \text{ rows} : ?$$

and find the missing term, as before, to be $\frac{3 \times \overset{2}{\cancel{4}} \times 8}{\cancel{2} \times 5}$ rows $= 9\frac{3}{5}$ rows.

Rule for solving Examples by Compound Proportion.

1. *Make the number that is of the same kind as the answer the third term.*

2. *Take each two terms that are of the same kind, and consider whether, depending upon them alone, the answer will be greater or less than the third term. Arrange them as in simple proportion.*

3. *Multiply the third term by the product of the number of units in each of the second terms, and divide the result by the product of the number of units in each of the first terms.**

Examples.

20. If 3 teams draw 50 tons of coal in 2 days, how many tons can 4 teams draw the same distance in 5 days? *Ans.* $166\frac{2}{3}$ tons.

21. If \$2160 is paid for 4 lots of land when the price is 15 cents a foot, what should be paid for 2 lots of the same size, when land is worth 50 cents a foot? *Ans.* \$3,600.

22. If a carpenter receives \$18 for 6 days' work of 10 hours each, what should he receive for 5 days' work of 8 hours each? *Ans.* \$12.

23. If 12 yards of silk, $\frac{3}{8}$ of a yard wide, costs \$29.04, what should 10 yards of silk, $1\frac{1}{4}$ yards wide and of the same quality, cost?

24. If \$100 gains \$7 interest at 9% for a given time, what will \$150 gain at 8% for the same time? *Ans.* $\$9.33\frac{1}{3}$.

25. If a quarter's gas bill was \$12.50, when gas was $2\frac{1}{2}$ mills per thousand ft., and 3 burners were kept lighted for 4 hours each evening, what should be my gas bill for a quarter when gas is 4 mills per thousand ft., and 5 burners are kept lighted for $3\frac{1}{2}$ hours each evening?

26. If 5 women can make 12 garments in $2\frac{1}{2}$ days, of 12 hours each, how many like garments can 2 women, with one sewing-machine, make in 1 day of 10 hours; 1 woman with the sewing-machine doing the work of 7 women without? *Ans.* $6\frac{2}{5}$ garments.

* Or, *Divide the third term by the compound ratio of the first terms to the second.*

☞ For Dictation Exercises, see "Manual and Key," page 149.

PARTNERSHIP.

522. ILL. EX. A and B associated themselves together in business for one year with a capital of \$ 5000, of which A furnished \$ 2000 and B \$ 3000, agreeing to share their profits or losses in proportion to their stock in trade; their net gains were \$ 1750; what ought each to receive?

OPERATION.

$$\frac{\$1750 \times 2}{5} = \$700, \text{ A's share.}$$

$$\frac{\$1750 \times 3}{5} = \$1050, \text{ B's share.}$$

Explanation. — Their stock in trade was \$ 5000, of which A's share is $\frac{2}{5}$ and B's $\frac{3}{5}$. As their profits were to be shared in the same proportion, A should receive $\frac{2}{5}$ of \$ 1750, or \$ 700, and B $\frac{3}{5}$ of \$ 1750, or \$ 1050.

523. Persons associated together, as A and B, for the transaction of business, are called **partners**; the partners thus associated are called a **firm, house,** or **company**; the association is called **partnership.**

524. The gains or losses of a firm are shared according to the agreement or contract of the partners.

EXAMPLES.

525. WHEN THE GAINS OR LOSSES ARE SHARED IN PROPORTION TO THE CAPITAL INVESTED.

1. Black owns $\frac{1}{2}$ of a ferry-boat, White $\frac{1}{3}$, and Brown the remainder; what share of a profit of \$ 800 should each receive?
Ans. Black, \$ 400; White, \$ 266.66$\frac{2}{3}$; Brown, \$ 133.33$\frac{1}{3}$.

2. N bought $\frac{3}{8}$ of a boat for \$ 150; if the boat was afterwards sold for \$ 475, what was N's gain? *Ans.* \$ 28.12$\frac{1}{2}$.

3. A, B, and C formed a company; A put in \$ 2500, B \$ 3800, and C \$ 2000; they hired D to conduct the business, for which he was to receive half the profits; they gained \$ 3500; what was each partner's share? *Ans.* A, \$ 527$\frac{9}{83}$; B, \$ 801$\frac{17}{83}$; C, \$ 421$\frac{57}{83}$.

4. Smith and Stiles hired a pasture together for \$ 200; Smith put in 15 cows, and Styles 18 cows and 16 sheep; what should each pay if the feed for 8 sheep is equal to that for 1 cow?

Ans. Smith, \$ $85\frac{5}{7}$; Styles, \$ $114\frac{2}{7}$.

5. C and D bought a house and land for \$ 10500, C paying \$ 6000 and D the remainder; they paid \$ 160 for repairs, \$ 95 for taxes and insurance, and received \$ 1000 for a year's rent; how much of the net proceeds of the year's rent should each receive?

Ans. C, \$ $425\frac{5}{7}$; D, \$ $319\frac{2}{7}$.

6. Suppose the house was insured for $\frac{3}{4}$ of the value of the house and land, and was entirely consumed by fire after being rented 6 months; what would be the loss to C and D, the land being worth \$ 700? C, \$ 960; D, \$ 720.

7. M, N, and O bought land for \$ 15000, M paying \$ 4000, N \$ 5000, and O the remainder; after paying \$ 150 for taxes, and \$ 315 for surveying and fencing, they sold the land for 75% advance upon the first cost, \$ 15000; what was each one's share of the gain?

Ans. M, \$ 2,876; N, \$ 3,595; O, \$ 4,314.

8. A person failing in business owed D \$ 750, E \$ 550, F \$ 25, and G \$ 1175; his property sold for \$ 1860; the expense of the sale being \$ 80, what should each creditor receive?

NOTE. — It is evident that a bankrupt's property should be shared by the creditors in proportion to the sums due them, hence the above and similar examples should be solved by principles developed in this article.

Ans. D, \$ 534; E, \$ 391.60; F, \$ 17.80; G, \$ 836.60.

9. Divide \$ 25 into three parts that shall be to one another as 5, 2, and 3. *Ans.* \$ $12\frac{1}{2}$; \$ 5; \$ $7\frac{1}{2}$.

10. For services received, Smith promised to A $\frac{1}{2}$ of a lot of apples, to B $\frac{1}{3}$ of the lot, and to C $\frac{1}{4}$. As Smith had nothing more with which to pay them, they shared the apples in the proportion of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$; what part of the whole lot did each receive?

Ans. A received $\frac{6}{13}$; B, $\frac{4}{13}$; C, $\frac{3}{13}$.

11. A man left by will \$ 18000 to Harvard College, \$ 12000 to the Blind Asylum, \$ 5000 to the Children's Hospital, and \$ 5000 to an orphan asylum; but on settling his estate it was found that his property was worth but \$ 25000; how should it be divided?

526. WHEN THE GAIN OR LOSS IS SHARED IN PROPORTION TO THE CAPITAL AND THE TIME FOR WHICH THE CAPITAL IS INVESTED.

ILL. EX. Ames and Bowles received $1265 for cutting timber; Ames furnished 5 men for 20 days, and Bowles furnished 4 men for 30 days; what was the share of each?

OPERATION.

$5 \times 20 = 100$ | A's share, $\frac{100}{220} = \frac{5}{11}$;

$4 \times 30 = \frac{120}{220}$ | B's share, $\frac{120}{220} = \frac{6}{11}$.

$\frac{\$1265 \times 5}{11} = \$575$; $\frac{1265 \times 6}{11} = \$690$

Explanation. — The work of 5 men for 20 days is equal to the work of 100 men for 1 day; the work of 4 men for 30 days is equal to the work of 120 men for 1 day. They together furnish work equal to that of 220 men for 1 day. A, then, should have $\frac{100}{220}$, or $\frac{5}{11}$ of $1265, which is $575; and B should have $\frac{120}{220}$, or $\frac{6}{11}$ of $1265, which is $690.

12. X and Y hired a pasture together for $87.50. X put in 3 cows for 5 months, and Y 5 cows for 4 months; what ought each to pay?

527. WHEN THE GAIN OR LOSS IS SHARED ACCORDING TO SPECIAL CONTRACT OF THE PARTNERS.

13. A and B form a partnership. A furnishes $3000 and B $5000. A's share of gain or loss is to be $\frac{1}{3}$ and B's $\frac{2}{3}$; if they gain 30% on their investment, what is each one's share of the gain?

14. If A and B share in the proportion of 4 to 5, and they lose 20% of their investment, what is each one's share of the loss?

15. M and N commence business on the 1st of Jan.; M furnishes $6550 in cash, and N $3720 in merchandise; each is to receive interest at 7% on his investment, and the partners are to share their gains and losses equally. On the 1st of July they have a settlement, when it is found that their resources amount to $18216.30, and their liabilities to $4628; what is the net gain of the firm and the net capital of each partner?

NOTE. — By **resources** is meant property, whether in money, merchandise, notes, or debts, due from others; and by **liabilities**, debts which are due to others. The resources of a firm, less its liabilities, is the **net capital**. The difference between the net capital at the end of a year and the capital at the beginning of the year with interest, is the **net gain** or **loss** for the year. *Ans.* $2,958.85; M's, $8,258.68; N's, $5,329.62.

INVOLUTION.

528. Name some products made by using 2's only as factors.

Products.		Factors.
4	=	2×2
8	=	$2 \times 2 \times 2$
16	=	$2 \times 2 \times 2 \times 2$

A product made by using equal factors only is a **power**.

529. A power, as 4, made by using two equal factors, is a **second power**. A power made by using three equal factors is a **third power**. A power made by using four equal factors is a **fourth power**, etc.

530. The process of forming powers is **involution**.

NOTE.—The process of forming any power of a number is sometimes called **raising** the number to that power; the process of forming the second power is called **squaring** the number; the process of forming the third power is called **cubing** the number.

531. The second power of 2 is indicated thus, 2^2; the third power thus, 2^3; the fourth power thus, 2^4, etc.

The expressions 2^2, 2^3, 2^4, are read, "the second power, or square, of 2," "the third power, or cube, of 2," "the fourth power of 2," respectively.

532. The number expressed by the small figure at the right is called the **index** or **exponent** of the power.

533. TO FIND ANY POWER OF A NUMBER.

ILL. EX. Find the second power or square of 100.

OPERATION. $100^2 = 100 \times 100 = 10000$. *Ans.* 10,000.

EXAMPLES.

1. Find and commit to memory the second power of each of the integral numbers from 1 to 12 inclusive. *Ans.* 1, 4, 9, 16, etc.

2. Find and commit to memory the third power of each of the integral numbers from 1 to 12 inclusive. *Ans.* 1, 8, 27, 64, etc.

Find the powers of the following numbers, as indicated by the exponents: —

3.	25^2.	*Ans.* 625.	9.	$(6\frac{2}{3})^4$.
4.	$(\frac{3}{8})^2$.	*Ans.* $\frac{9}{64}$.	10.	$(\frac{1}{2})^5$.
5.	$.15^2$.	*Ans.* .0225.	11.	16^2.
6.	$(4\frac{5}{7})^2$	*Ans.* $22\frac{11}{49}$.	12.	Square 80.
7.	2.1^3.	*Ans.* 9.261.	13.	Cube $30\frac{1}{4}$.
8.	$.5^4$.	*Ans.* .0625.	14.	Raise 2.2 to the 3d power.

15. What is the difference between the square and the cube of 64?

16. What is the amount, at compound interest, of $ 1.06 for 5 years, at 6%? (Raise 1.06 to the 6th power.) *Ans.* $ 1.418519.

EVOLUTION.

534. Name one of the two equal factors of 9; one of the three equal factors of 8.

One of the equal factors of which a power is composed is a **root.**

535. One of the two equal factors of a second power is called a **second** or **square root.** One of the three equal factors of a third power is a **third** or **cube root.** One of the four equal factors of a fourth power is a **fourth root,** etc.

536. The process of finding the root of a power is **evolution.**

NOTE. — The process of finding the root of a power is sometimes called **extracting the root.**

537. The second or square root of 64 is indicated thus, $\sqrt{64}$; the third or cube root thus, $\sqrt[3]{64}$, etc.

In the expressions $\sqrt{4} = 2$, $\sqrt[3]{8} = 2$, $\sqrt[4]{16} = 2$, the character $\sqrt{\ }$ denotes a root, and is called the **radical sign.** The radical sign, when used alone, denotes the second or square root; $\sqrt[3]{\ }$ denotes the third or cube root; $\sqrt[4]{\ }$ denotes the fourth root.

The number expressed by the small figure at the left of the radical sign is called the **index** of the root.

SECOND OR SQUARE ROOT.

538. ILL. EX. Find the square root of 1156.

NOTE.—From the definition of the square root (Art. 535), it follows, that to find the square root of 1156 is to find one of its two equal factors.

539. Before attempting to find the square root of a number we will ascertain in what part of the power the square of the terms, or different orders of units of the root, may be found.

The second power or square of

1,	a unit of the lowest order of integers,	1^2, is	1.
10,	" " next higher order of integers,	10^2, "	100.
100,	" " " " " " "	100^2, "	10000.
1000,	" " " " " " "	1000^2, "	1000000.

From the above it will be seen that the second power of a unit of any order equals 100 of the second power of a unit of the next lower order, and hence that it must be expressed two places at the left of the expression for the power of a unit of the next lower order.

540. From the above illustrations it will be seen, that the first two figures at the right of the expression for the second power of an integral number will express no part of the second power of the root above units; that the next two figures will express no part of the

second power of the root above tens; that the next two figures will express no part of the second power of the root above hundreds, and so on.

Hence if we place a dot over the units' figure of the expression for any second power, and a dot over every alternate figure from the place of units, we shall indicate the part of the number in which the square of the units of the different orders of the root are expressed.

Then to find the number of terms or orders of units in the square root of 1156, we place a dot over each alternate figure in the expression, beginning at the units' place, thus; $1\dot{1}5\dot{6}$, and having two dots, we know that there will be two terms in the root, and that the number expressed by the left-hand group contains the square of the tens of the root.

541. We will now raise a number consisting of two terms, 34 for example, to its second power, that we may learn of what parts the second power is composed.

OPERATION.

$$
\begin{array}{lllr}
 & & & \text{tens. units.} \\
34 \times 34 = & \left\{\begin{array}{l} 30 + 4 = \\ 30 + 4 = \end{array}\right. & & \begin{array}{r} 34 \\ 34 \end{array} \\
\hline
 & 4^2 = & 16 = & 16 \\
30 \times 4 & = & 120 = & 12 \text{ tens.} \\
30 \times 4 & = & 120 = & 12 \text{ tens.} \\
30^2 & = & 900 = & 9 \text{ hun'ds.} \\
\hline
30^2 + \left\{\begin{array}{l} 30 \times 4 \\ 30 \times 4 \end{array}\right\} + 4^2 & = & 1156 = & 1156
\end{array}
$$

Explanation. — The square of 34 = $(30+4) \times (30+4.)$ Multiplying, as in the operation at the left, and preserving each partial product, we see that the second power, 1156, is made up of (1) 30^2, or *the square of the tens;* (2) $30 \times 4 + 30 \times 4$, or *two products of the tens $\times$ the units;* (3) 4^2, or *the square of the units.*

In a similar manner it may be shown that the second power of any root that consists of tens with units contains

(1.) *The square of the tens;* (2.) *two products of the tens $\times$ the units;* (3.) *the square of the units.*

Which may be expressed thus: —

$$tens^2 + 2\ (tens \times units) + units^2.$$

542. By the operation, Art. 541, we see that the square of the tens of the root can be expressed in no place lower than the hundreds' of the power; that the 2 products of the tens × the units can be expressed in no place lower than the tens', and that the square of the units can be expressed in no place lower than the units'.

543. We are now prepared to extract the square root of 1156, which we do by taking out of the power the same partial products which were used to form it.

OPERATION.

*tens*² + 2 (*tens* × *units*) + *units*²	
	$1\dot{1}5\dot{6}$ (34
(3 tens)² =	9 hundreds.
3 tens × 2 = 6 tens)	25 Dividend.
6 tens × 4 =	24 tens.
	16
4² =	16 units.
	0

Explanation. — By pointing the expression into groups of two figures each, we see that the square root of 1156 consists of two terms, tens and units, the second power of which must contain the tens² + 2 (tens × units) + units². (Art. 541.)

As the square of the tens is expressed in no place lower than the hundreds', the 11 (hunds.) must contain the square of the tens.

The greatest square contained in 11 (hunds.) is 9 (hunds.), the square root of which is 3 (tens). This we express as the first term or tens of the root.

Taking the square of 3 (tens) = 9 (hunds.) out of 11 (hunds.), there remain 2 (hunds.), which we unite with 5 (tens) of the power, making 25 (tens).

Now as the second part of the power, "2 (tens × units)," can be expressed in no place lower than the tens', the 25 (tens) must contain a product of which the tens × 2 is one factor, and the units of the root the other factor. (3 tens) × 2 = 6 (tens); dividing 25 (tens) by 6 (tens), we find 4 units to be the other factor, and hence the units of the root.

Taking out of 25 (tens) 6 (tens) × 4 = 24 (tens), we have 1 (ten) left, which united with 6 (units) equals 16 units, which must contain the square of 4 units. Taking out of 16 the square of 4, = 16, nothing remains; therefore the square root of 1156 is 34.

544. As 1 unit of any order is 1 ten of the next lower order, the principle explained above may be applied to extracting the square root of powers whose roots consist of more than two terms; for after finding as above the first two terms, the terms found may be considered as tens, and the term sought as units; and after having found three terms, the three terms found may be considered as tens and the term sought as units, and so on.

545. From the foregoing may be derived the following

RULE. — To extract the square root of a number: — 1. *Separate the expression into groups of two figures each by placing a dot over the units' figure and over every alternate figure from the units.*

2. *Take the largest square contained in the number expressed by the left-hand group out of that number, and express its square root as the highest term of the required root; with the remainder unite the next term of the power for a dividend.*

3. *Multiply the term of the root already found by 2 for a divisor, by which divide the dividend and express the quotient as the next term of the root.*

4. *Multiply the divisor by this term, take the product out of the dividend, and with the remainder unite the next term of the power.*

5. *Take out of the number thus formed the square of the second term of the root.*

[Having thus obtained the first two terms of the root, if there are other terms to be found,]

6. *Unite with the remainder the next term of the power for a new dividend.*

7. *Multiply the terms of the root already found by 2, and apply the rule as in paragraph 3 and onward.*

EXAMPLES.

Find the square root of

1.	784.	*Ans.* 28.	3.	120409.	*Ans.* 347.
2.	2401.	*Ans.* 49.	4.	2856100.	*Ans.* 1,690.

546. ILL. EX. I. **Find the square root of 927.2025.**

OPERATION.

$$tens^2 + 2\,(tens \times units) + units^2.$$

$$\begin{array}{rrl}
 & & \dot{9}\dot{2}\dot{7}.\dot{2}0\dot{2}\dot{5}(30.45 \\
3^2 = & & 9 \\
\left.\begin{array}{r} 3 \times 2 = 6 \\ 30 \times 2 = 60 \end{array}\right\} & 60\,) & 272 \text{ Dividend.} \\
 & 60 \times 4 = & \underline{240} \\
 & & 320 \\
4^2 = & & \underline{16} \\
304 \times 2 = & 608\,) & 3042 \text{ Dividend.} \\
 & 608 \times 5 = & \underline{3040} \\
 & & 25 \\
5^2 = & & \underline{25} \\
 & & 0
\end{array}$$

NOTE I.—By squaring .1, .01, .001, etc., it will be seen that the second power of a unit of any order is expressed two places at the right of the expression of the second power of a unit of the next higher order. And, hence that the manner of pointing the expression and of extracting the root of a second power of integers applies to decimals.

NOTE II.—*When, as in the operation in* ILL. EX. I., *the divisor is not contained in the dividend, place a zero as the next figure in the expression for the root, also at the right of the expression for the divisor, and for a new dividend unite the next two terms of the power with the previous dividend.*

ILL. EX. II. Find the square root of $\frac{7}{8}$.

OPERATION.

$$tens^2 + 2\,(tens \times units) + units^2.$$

$$\begin{array}{rrl}
\frac{7}{8} = & & .\dot{8}7\dot{5}\dot{0}(.935+ \\
9^2 = & & \underline{81} \\
9 \times 2 = & 18\,) & 65 \text{ Dividend.} \\
 & 18 \times 3 = & \underline{54} \\
 & & 110 \\
3^2 = & & \underline{9} \\
93 \times 2 = & 186\,) & 1010 \text{ Dividend.} \\
 & 186 \times 5 = & \underline{930} \\
 & & 800 \\
5^2 = & & \underline{25} \\
 & & 775
\end{array}$$

NOTE III.—The square root of a fractional number whose numerator and denominator are both square numbers, as $\frac{9}{16}$, may be found by taking the square root of the numerator and of the denominator for the square root.

But when, as in ILL. EX. II., the numerator and denominator of the fractional number are not square numbers, the fractional number must first be changed to a decimal.

NOTE IV.—When there is a remainder after all the terms of the power have been used, annex zeros to the expression for the remainder, and continue the operation as far as desired.

547. EXAMPLES.

Find the square root of

5. 58.3696. *Ans.* 7.64.
6. 251001. *Ans.* 501.
7. .041209. *Ans.* .203.
8. 125741.16. *Ans.* 354.6.
9. 4016016. *Ans.* 2,004.
10. .000961. *Ans.* .031.

Find the square root of

11. $\frac{81}{144}$. *Ans.* $\frac{9}{12}$.
12. $\frac{15}{16}$. *Ans.* .9682+
13. .284. *Ans.* .5329+
14. .0763. *Ans.* .2762+
15. $9\frac{1}{8}$. *Ans.* 3.0207+
16. $27\frac{5}{6}$. *Ans.* 5.2757+
17. $\frac{5}{7}$ of $\frac{9}{12}$. *Ans.* .7319+
18. .9 *Ans.* .9486+

548. OPTIONAL EXAMPLES.

NOTE.—Roots of imperfect powers may be found to ten thousandths.

19. $\sqrt{3} = ?$
20. $\sqrt{30} = ?$
21. $\sqrt{6.4} = ?$
22. $\sqrt{1\frac{24}{25}} = ?$
23. $\sqrt{1656.49} = ?$
24. $\sqrt{300.3289} = ?$
25. $\sqrt{.0581} = ?$
26. $\sqrt{.00434} = ?$
27. $\sqrt{341.1409} = ?$
28. $\sqrt{144\frac{4}{9}} = ?$
29. $\sqrt{9 \times (3\frac{1}{2})^2} = ?$
30. $\sqrt{6\frac{19}{25}} = ?$
31. $\sqrt{10 \times (\frac{5}{2})^3} = ?$
32. $\sqrt{272\frac{1}{4}} = ?$

APPLICATIONS OF SQUARE ROOT.

549. 33. A gardener having 1024 strawberry plants set them in rows, putting as many plants in a row as he had rows; how many rows had he? *Ans.* 32 rows.

34. How many men must be placed in rank and in file that the largest square may be formed of 2000 men, and how many men will be left out? *Ans.* 44 in rank and file each; 64 left out.

35. I have a rectangular lot of land, 64 rods long and 36 rods wide, and a square lot of equal area; how many more rods of fencing will be required to fence the former than the latter lot? *Ans.* 8 rods.

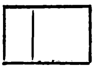
36. How many rods of fencing will be required to enclose a square lot of land containing 1 acre? 10 acres?

Sum of answers, 210.5964 rods+

37. How many to enclose a quarter section of government land? (See note, page 160.) *Ans.* 640 rods.

38. I have two square gardens whose areas are to each other as 3 to 5, the smaller of which contains ⅔ of an acre; what is the length of one side of the larger? *Ans.* 13⅓ rods.

39. It requires 28⅙ sq. yds. of carpeting to cover a floor whose width is to its length as 2 to 3; what are the dimensions of the room?



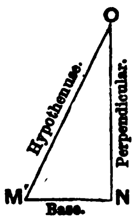
NOTE.—⅔ of the room will form a square whose side is the width of the room. *Ans.* width, 13 ft.; length, 19½ ft.

40. A rectangular court, of which the length is to the width as 4 to 3, can be paved with 7500 blocks of granite 1 foot square, what are the dimensions of the court? *Ans.* 100 ft. long, 75 ft. wide.

41. I have a cistern 8 feet deep, whose capacity is 1500 gallons; what are the other dimensions of the cistern inside, the length being equal to the width? *Ans.* 5.0065 ft.+

TO FIND EITHER SIDE OF A RIGHT-ANGLED TRIANGLE, WHEN THE OTHER TWO SIDES ARE GIVEN.

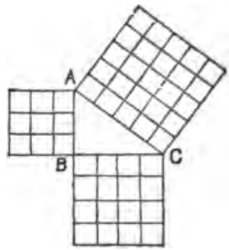
550. A plane surface, as M N O, bounded by three straight lines and having three angles (Art. 303), is a **triangle.**



551. A triangle, as M N O, which contains a right angle (Art. 332, Note I.), is a **right-angled triangle.**

552. In a right-angled triangle, the side opposite the right angle is the **hypothenuse**, one of the other two sides is the **base** and the other the **perpendicular.**

553. Suppose the figure A B C to be a right-angled triangle, whose hypothenuse is 5 ft., base 4 ft., and perpendicular 3 ft. A square formed upon the hypothenuse, A C, will contain 25 square feet; one formed upon the base, B C, will contain 16 square feet; and one formed upon the perpendicular, A B, will contain 9 square feet. Adding the squares A B and B C, we have for the sum 25 square feet; therefore the square upon the hypothenuse, A C, is equal to the sum of the two squares upon A B and B C.



It can be shown generally that *the square upon the hypothenuse of any right-angled triangle equals the sum of the squares upon the other two sides. Hence*

RULE I. — To find the hypothenuse, when the other two sides are given: *Square the number of units in each of the two given sides and extract the square root of the sum of these squares.*

RULE II. — To find either of the two sides which form the right angle, when the hypothenuse and one other side are given: *Square the number of units in the hypothenuse and the given side, and extract the square root of the difference of these squares.*

EXAMPLES.

42. What is the length of the hypothenuse of a right-angled triangle whose base is 12 rods and whose perpendicular is 9 rods?

Ans. 15 rods.

43. If one side of a right-angled triangle is 6 rods long, and the hypothenuse $6\frac{1}{2}$ rods long, what is the length of the other side?

Ans. $2\frac{1}{2}$ rods.

44. What is the length of the base of a right-angled triangle of which the hypothenuse is 195 ft. and the perpendicular 180 ft.?

45. What is the height of a tree, the top of which may be reached by a kite-line 200 feet long held at a point 50 feet in a horizontal line from the foot of the tree? *Ans.* 193.6 ft.+

46. How far from a wall should the foot of a ladder, 33½ ft. long, be placed that it may just reach a window in the wall 30 feet from the ground? *Ans.* 14.90 ft.+

47. How far from the wall should the foot of the above-mentioned ladder be placed that it may reach 2 feet higher than before?
Ans. 9.912 ft.+

48. A steamship and a sloop start from the same place at the same time, the former going east at the rate of 18½ miles an hour, and the latter going south at the rate of 4 miles an hour; how far apart are they at the end of 2 hours? *Ans.* 37.85 m.+

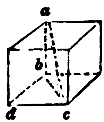
49. My house is 48 feet wide, and the ridge-pole is 10 feet above the beams which connect the top of the walls; what is the length of the rafters, which extend from the ridge-pole to the outer edge of the wall? *Ans.* 26 ft.

50. What length, in feet, of hand-rail will be required for a flight of 19 stairs, each 7½ in. high and 10 in. wide? *Ans.* 18.75 ft.*

51. What is the length, in rods, of a line from corner to corner diagonally of a square lot containing 1 acre of land?
Ans. 17.888 rods.+

52. How much shorter is the walk from corner to corner diagonally across a square common containing 16 acres, than the walk upon two of its sides? *Ans.* 29.638 rods.+

53. What must be the length of a festoon of evergreen to reach from the centre to one corner of a ceiling 20 ft. square, allowing 3 ft. for the drooping of the festoon? *Ans.* 17.14 ft.+



54. What is the length of a line from one corner through the centre to the opposite corner of a cubical block whose edge is 15 inches?
Ans. 25.98 in.+

NOTE. — The sum of the squares of the units in the length, in the breadth, and in the thickness of a cube or any rectangular solid, equals the square of the units in the diagonal. Let the pupil show this by the diagram.

55. What is the distance from the lower corner to the opposite upper corner of a room which is 25 feet long, 20 feet wide, 10 feet high? *Ans.* 33.54 ft.+

* For explanation, also for Dictation Exercises, see "Manual and Key," page 154.

THIRD OR CUBE ROOT.

554. ILL. Ex. Find the cube root of 39304.

NOTE.—From the definition of cube root (Art. 535), it follows that to find the cube root of 39304 is to find one of its three equal factors.

555. Before attempting to find the cube root of 39304, we will ascertain in what part of the power the cube of the terms or different orders of units of the root may be found.

The third power of

1,	a unit of the lowest order of integers,	1^3 is	1.
10	" " next higher order of integers,	10^3 is	1000.
100	" " " " " "	100^3 is	1000 000.
1000	" " " " " "	1000^3 is	1000 000 000.

From the above it will be seen that the third power of a unit of any order equals 1000 of the third power of a unit of the next lower order, and hence that it must be expressed three places at the left of the expression for the power of the unit of the next lower order.

556. From the above illustrations it will also be seen, that the first three figures at the right of the expression of a third power of an integral number will express no part of the third power of the root above units; that the next three figures will express no part of the third power of the root above tens; that the next three figures will express no part of the third power of the root above hundreds, and so on.

Hence, if we place a dot over the units' figure in the expression for any third power, and a dot over every third figure from the place of units, we shall indicate the part of the number in which the third power of the units of the different orders of the root are expressed.

Then, to find the number of terms or orders of units in the cube root of 39304, we place a dot over every third figure in the expression beginning at the units' place; thus, $3\dot{9}30\dot{4}$; and having two dots we know that there will be two terms in the root, and from Art. 555 we know that the number expressed by the left-hand group contains the cube of the tens of the root.

557. We will now raise a number consisting of two terms, 34 for example, to its third power, that we may learn of what parts the third power is composed.

OPERATION.

$$
\begin{array}{lcr}
34^2 & = & 30^2 + 2\,(30 \times 4) + 4^2 \\
\text{Multiplying again by} & & 30 + 4 \\
\hline
34^2 \times 4 & = & (30^2 \times 4) + 2\,(30 \times 4^2) + 4^3 \\
34^2 \times 30 & = & 30^3 + 2\,(30^2 \times 4) + (30 \times 4^2) \\
\hline
34^3 & = & 30^3 + 3\,(30^2 \times 4) + 3\,(30 \times 4^2) + 4^3
\end{array}
$$

Explanation. — Squaring 34, as in Art. 541, we have $30^2 + 2\,(30 \times 4) + 4^2$; we first multiply this product by 4, then by 30, and finally take the sum of these products, and thus find the parts composing the cube of 34 to be

(1.)	$30^3 =$	the cube of the tens	= 27 thousands.
(2.)	$3\,(30^2 \times 4) =$	3 products of the square of the tens × the units	= 108 hundreds.
(3.)	$3\,(30 \times 4^2) =$	3 products of the tens × the square of the units	= 144 tens.
(4.)	$4^3 =$	the cube of the units	= 64 units.
	$34^3 =$		39304

In a similar manner it may be shown that the third power of any root that consists of tens with units, contains

(1.) The cube of the tens;
(2.) 3 products of the square of the tens × the units;
(3.) 3 products of the tens × the square of the units;
(4.) the cube of the units.

Which may be expressed thus: —

$$tens^3 + 3\,(tens^2 \times units) + 3\,(tens \times units^2) + units^3.$$

558. By the operation, Art. 557, we see that the cube of the tens can be expressed in no place lower than the thousands'; that the 3 products of the tens² × the units can be expressed in no place lower than the hundreds'; that the 3 products of the tens × the square of the units can be expressed in no place lower than the tens'; and that the cube of the units can be expressed in no place lower than the units'

559. We are now prepared to extract the cube root of 39304. This we do by taking out of the power the same partial products that were used to form it.

OPERATION.

tens^3 + 3 (*tens*^{2} × *units*) + 3 (*tens* × *units*^{2}) + *units*^{3}.

	$\dot{3}9\,30\dot{4}$(34
(3 tens)^3 =	27 thousands.
(3 tens)^2 × 3 = 27 hundreds)	123 Dividend.
27 hundreds × 4 =	108 hundreds.
	150
3 tens × 4^2 × 3 =	144 tens.
	64
4^3 =	64 units.

Explanation. — By pointing the expression into groups of three figures each, we see that the cube root of 39304 consists of two terms, tens and units, the third power of which must contain the tens^3 + 3 (tens^2 × units) + 3 (tens × units^2) + units^3. (Art. 557.)

As the cube of the tens is expressed in no place lower than the thousands', the 39 (thous.) must contain the cube of the tens.

The greatest cube contained in 39 (thous.) is 27 (thous.), the cube root of which is 3 (tens). This we express as the first term, or tens of the root.

Taking the cube of 3 (tens) = 27 (thous.) out of 39 (thous.) there remain 12 (thous.) which we unite with 3 (hunds.) of the power, making 123 (hunds.).

Now, as the "3 products of the tens^2 × the units" can be expressed in no place lower than the hundreds', 123 (hunds.) must contain a product of which the tens^2 × 3 is one factor and the units of the root the other factor; (3 tens)^2 × 3 = 27 (hunds.); dividing 123 (hunds.) by 27 (hunds.), we find 4 units to be the other factor, and hence the units of the root.

Taking out of 123 (hunds.) 27 (hunds.) × 4 = 108 (hunds.), we have 15 (hunds.) left, which united with 0 (tens), equals 150 (tens). This must contain the "3 products of the tens × units^2," = 144 (tens); taking out of 150 (tens) 144 (tens), we have 6 (tens) left, which united with 4 (units) equals 64 (units). This must contain the cube of the units, = 64 (units); taking out of 64 (units) 64 (units), nothing remains; therefore the cube root of 39304 is 34.

560. As 1 unit of any order is 1 ten of the next lower order, the principle explained above may be applied to extracting the cube root of powers whose roots consist of more than two terms; for after finding, as above, the first two terms of the root, the terms found can be considered as tens and the term sought as units; and after having found three terms, the three terms found may be considered as tens and the term sought as units, and so on.

561. From the foregoing may be derived the following

RULE. — To extract the cube root of a number: 1. *Separate the expression into groups of three figures each by placing a dot over the units' figure and over every third figure from the units'.*

2. *Take the largest cube contained in the number expressed by the left-hand group out of that number, and express its cube root as the highest term of the required root; with the remainder unite the next term of the power for a dividend.*

3. *Multiply the square of the term of the root already found by 3 for a divisor; by which divide the dividend, and express the quotient as the next term of the root.*

4. *Multiply the divisor by this term, take the product out of the dividend, and with the remainder unite the next term of the power.*

5. *Take out of the number thus formed the product of the tens multiplied by the square of the units multiplied by 3, and with the remainder unite the next term of the power.*

6. *Take out of the number thus formed the cube of the second term of the root.*

[Having thus obtained the first two terms of the root, if there are other terms to be found,]

7. *Unite with the remainder the next term of the power for a net dividend.*

8. *Multiply the square of the terms of the root already found by 3, and apply the rule as in paragraph 3 and onward.*

EXAMPLES.

Find the cube root of the following: —

1.	4913.	*Ans.* 17.	4.	39651821.	*Ans.* 341.
2.	12167.	*Ans.* 23.	5.	230346397.	*Ans.* 613.
3.	551368.	*Ans.* 82.	6.	3609741304.	*Ans.* 1,534.

562. ILL. EX. **Find the cube root of 65207.515625.**

OPERATION.

$tens^3 + 3\,(tens^2 \times units) + 3\,(tens \times units^2) + units^3.$

		$\dot{6}5\dot{2}07.51\dot{5}62\dot{5}$ (40.25
$4^3 =$		64
$4^2 \times 3 = 48$ $40^2 \times 3 = 4800$	} 4800)	12075 Dividend.
	$4800 \times 2 =$	9600
		24751
$40 \times 2^2 \times 3 =$		480
		242715
$2^3 =$		8
$402^2 \times 3 =$	484812)	2427076 Dividend.
	$484812 \times 5 =$	2424060
		30162
$402 \times 5^2 \times 3 =$		30150
		125
$5^3 =$		125
		0

NOTE I. — By cubing .1, .01, .001, etc., it will be seen that the third power of a unit of any order of decimals is expressed three places at the right of the expression of the next higher order. And hence that the manner of pointing the expression and of extracting the root of a third power of integers applies to decimals.

NOTE II. — *When*, as in the operation of ILL. EX., *the divisor is not contained in the dividend, place a zero as the next figure in the expression for the root; place also two zeros at the right of the expression for the divisor, and for a new dividend unite the next three terms of the power with the previous dividend.*

NOTE III. — The cube root of a fractional number whose numerator and denominator are both cubes, as $\frac{8}{27}$, may be found by taking the cube root of the numerator and of the denominator for the cube root.

But when the numerator and denominator of the fractional number are not cubes, the fractional number must first be changed to a decimal.

NOTE IV. — When there is a remainder after all the terms of the power have been used, annex zeros, and continue the operation as far as desired.

In the examples which follow the roots are found to the fourth term.

EXAMPLES.

Find the cube root of

7.	117649000.	*Ans.* 490.	12.	$\frac{216}{2197}$.	*Ans.* $\frac{6}{13}$.
8.	.042875.	*Ans.* .35.	13.	$5.0\frac{1}{2}$.	*Ans.* 1.715+.
9.	128.024064.	*Ans.* 5.04.	14.	$1\frac{27}{125}$.	*Ans.* 1.067+
10.	.000017576.	*Ans.* .026.	15.	3.	*Ans.* 1.442+
11.	$\frac{64}{125}$.	*Ans.* $\frac{4}{5}$.	16.	.015.	*Ans.* .2466+

563. 17. If a cubic meter contains 61023.377953 cubic inches, what is the length of a meter? *Ans.* 39.37 in.

18. What must be the inside dimensions of a cubical cistern that will hold 1000 gallons? (Art. 324.) *Ans.* 61.35 in.+

19. What must be the inside dimensions of a cubical cistern that will hold 5000 gallons? *Ans.* 104.9 in.+

20. I have a cubical bin that holds 72 bushels of grain; what is its depth? *Ans.* 53.69+ in.

21. What must be the depth of a cubical bin to hold $\frac{1}{2}$ as many bushels as the above? *Ans.* 42.61 in.+

564. OPTIONAL EXAMPLES.

22.	$\sqrt[3]{405224} = ?$	26.	$\sqrt[3]{\frac{1}{8}} = ?$
23.	$\sqrt[3]{513922401} = ?$	27.	$\sqrt[3]{.64} = ?$
24.	$\sqrt[3]{1404928000} = ?$	28.	$\sqrt[3]{16\frac{5}{11}} = ?$
25.	$\sqrt[3]{176.000258} = ?$	29.	$\sqrt[3]{64\frac{8}{27}} = ?$

☞ For Dictation Exercises in Cube Root, see Key.

NOTE TO THE TEACHER. — The method here illustrated for extracting the second and third roots is applicable to the roots of higher powers.

Thus to extract the fifth root of 7962624, we have the accompanying formula and operation: —

FORMULA. — $tens^5 + 5\ tens^4 \times units + 10\ tens^3 \times units^2 + 10\ tens^2 \times units^3 + 5\ tens \times units^4 + units^5$.

$$\begin{array}{lrl}
 & & \dot{7}96262\dot{4}\ (\ 24 \\
2^5 = & & 32 \\
5 \times 2^4 = & 80\ ) & 476 \text{ Dividend.} \\
 & 80 \times 4 = & 320 \\
 & & 1562 \\
10 \times 2^3 \times 4^2 = & & 1280 \\
 & & 2826 \\
10 \times 2^2 \times 4^3 = & & 2560 \\
 & & 2662 \\
5 \times 2 \times 4^4 = & & 2560 \\
 & & 1024 \\
4^5 = & & 1024
\end{array}$$

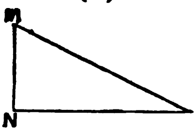
[For fuller explanation, see Key, page .]

MENSURATION.

NOTE. — The definitions of various lines, surfaces and solids, are given on pages 159, 162, 168. Such as are in general use, and not found there, are given in this section.

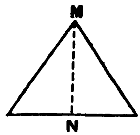
TRIANGLES.

(1.)



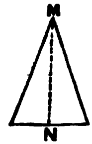
Right-angled triangle.

(2.)



Equilateral triangle.

(3.)



Isosceles triangle.

Define a triangle ; a right-angled triangle. (See Arts. 550, 551.)

565. A triangle which has its sides all equal, as Fig. 2, is an **equilateral triangle.**

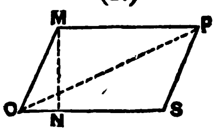
566. A triangle which has two of its sides equal, as Fig. 3, is an **isosceles triangle.**

EXERCISES.

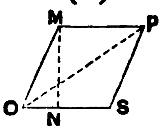
Draw a right-angled triangle; an equilateral triangle; an isosceles triangle; a right-angled isosceles triangle.

QUADRILATERALS.

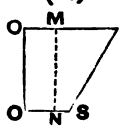
(4.)



(5.)

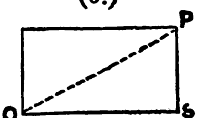


(8.)



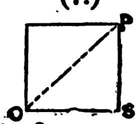
Trapezoid.

(6.)



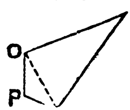
Rectangle.

(7.)



Square.

(9.)



Trapezium.

567. Each of the preceding figures has four sides and four angles; such figures are **quadrilaterals.**

NOTE.—{ A——B / C——D } Lines which are equally distant from each other throughout their whole extent, as A B and C D, are **parallel lines.**

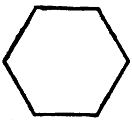
568. Figs. 4, 5, 6, and 7 have their opposite sides parallel; such figures are **parallelograms.**

569. A parallelogram whose angles are all right angles, as Figs. 6 and 7, is a **rectangle.**

570. A rectangle, whose sides are all equal, as Fig. 7, is a **square.**

571. A quadrilateral, only two of whose sides are parallel, as Fig. 8, is a **trapezoid.**

572. The name **polygon** is applied to any figure bounded by straight lines. A polygon of five sides is a **pentagon,** one of six sides is a **hexagon,** one of eight sides is an **octagon,** and one of ten sides is a **decagon.** A polygon whose sides are all equal is an **equilateral polygon**; a polygon whose angles are all equal is an **equiangular polygon.** A polygon that is equilateral and equiangular is a **regular polygon.**



EXERCISES.

Draw a parallelogram; a rectangle; a square; a trapezoid; a pentagon; a regular hexagon; an octagon.

573. The line upon which a figure is supposed to stand, as O S, Fig. 4, is the **base.**

574. In any figure, the shortest distance from the farthest point above the base to the line of the base, as M N, Fig. 4, is the **height.**

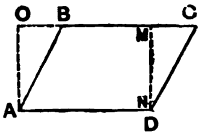
575. A line joining any two angles of a figure, not adjacent, as O P, Fig. 4, is a **diagonal.**

EXERCISES.

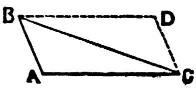
What lines are bases of the preceding figures? What lines indicate heights? What are diagonals?

TO FIND THE AREAS OF RECTILINEAR FIGURES.

576. If in the parallelogram A B C D the part D M C, cut off by the perpendicular M N, is placed upon the other side of the figure at A O B, a rectangle A O M D will be formed, which will have the same base and height as the parallelogram A B C D, and the same area. In the same way it may be shown that the area of any parallelogram equals that of a rectangle of the same base and height. Hence



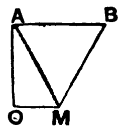
To find the area of a RECTANGLE, a SQUARE, or any PARALLELOGRAM, *Multiply the number of units in the base by the number of like units in the height.* (Art. 310.) The product will equal the number of square units in the area.



577. If the parallelogram A B D C is cut by the diagonal B C, the two triangles thus formed will be found to be exactly alike; therefore the triangle A B C equals one half of the parallelogram A B D C. Hence,

To find the area of a TRIANGLE, *Multiply the number of units in the base by half the number of like units in the height.*

578. The trapezoid A B M O may be cut by the diagonal A M into two triangles whose bases are the parallel sides of the trapezoid and whose height is the distance between them. Hence,



To find the area of a TRAPEZOID, *Multiply one half of the sum of the number of units of the two parallel sides by the number of like units in the distance between them.*

579. To find the area of any POLYGON, *Divide it into triangles and find the sum of their areas.*

580. EXAMPLES.

NOTE. — The pupil will be greatly assisted by drawing figures to illustrate the examples which follow.

1. How many square feet are there in the surface of both sides of a slate that is 12 inches long and 8 inches wide?

2. How many square yards in the surface of a table that is 8 feet long and $3\frac{1}{2}$ feet wide?

3. How many square yards in the surface of a floor that is 10 feet long and 9 feet 2 inches wide?

4. How many square rods in a garden that is 42 feet square?

Ans. $6\frac{58}{121}$ rds.

5. How many acres in a rectangular field that is $20\frac{1}{2}$ chains long and 18 chains 20 links wide? (Art. 311, Note II.)

Ans. 37.31 A.

6. What is the area of a triangle whose base is 7 feet and whose height is 2 feet 8 inches? *Ans.* $9\frac{1}{3}$ sq. ft.

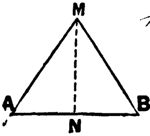
7. What is the area of a right-angled triangle whose base is 3 feet and whose perpendicular is $2\frac{1}{2}$ feet?

8. What is the area of a right-angled triangle whose base is 5 feet and whose hypothenuse is 13 feet? (See Art. 553, Rule II.)

Ans. 30 sq. ft.

9. What is the area of a right-angled triangle whose perpendicular is 3 ft. 8 in. and whose hypothenuse is 4 ft. 7 in.?

Ans. 5 sq. ft. 6 sq. in.



10. What are the height and area of a triangle each of whose sides is 20 feet long?

NOTE. — The perpendicular M N divides the equilateral triangle into two equal right-angled triangles; therefore the line A N equals $\frac{1}{2}$ of A B.

Ans. Height 17.32 ft.+; Area 173.2 sq. ft.+

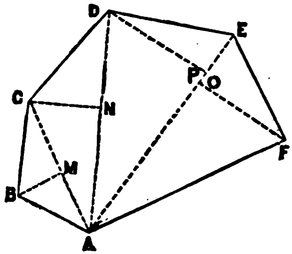
11. What are the height and area of a triangle whose base is 3 ft. in length, and each of the other two sides 2 ft. 6 in.?

Ans. Height 2 ft.; Area 3 sq. ft.

12. How many square inches in a trapezoid whose parallel sides are 5 and 6 inches long respectively, and the distance between them 4 inches? *Ans.* 22 sq. in.

13. What is the area of a board 8 ft. long, one end of which is 2 ft. wide and the other 1 ft. 9 in.?

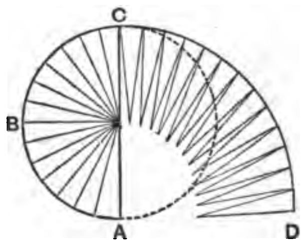
14. I have an irregular four-sided field a diagonal of which measures 5 ch. 3 l., and lines drawn from the opposite corners perpendicular to the diagonal, 2 ch. 8 l. and 4 ch. 5 l. respectively; how many acres are there in the field? *Ans.* 1.54 A.+



15. What is the number of square rods in a field shaped like the figure in the margin, and whose measurements are as follows: A C 6 rd.; A D 9 rd.; A E 10 rd.; B M $2\frac{1}{2}$ rd.; C N 3 rd.; D O 5 rd.; P F $4\frac{1}{2}$ rd.? *Ans.* $68\frac{1}{2}$ sq. rds.

CIRCLES.

Define circle; circumference; radius; diameter; arc. (See Arts. 326 – 329.)



581. A circle may be considered as made up of triangles whose bases form the circumference of the circle and whose vertices are at the centre, the height of the triangles being equal to the radius of the circle; hence,

The number of square units in the area of the circle *is equal to the number of units in the circumference multiplied by half the number of like units in the radius, or by one fourth the number of like units in the diameter.*

582. It has been found that the circumference of every circle equals its diameter multiplied by 3.1416 nearly; hence

when the diameter only of a circle is given, its circumference and area may easily be obtained by the following formulas.*

When the Diameter is given,

583. The **Circumference** = Diameter × 3.1416.

584. The **Area** = (Diameter × 3.1416) × $\frac{\text{Diameter}}{4}$ = Diameter^2 × .7854.

When the Circumference is given,

585. The **Diameter** = $\frac{\text{Circumference.}}{3.1416}$

When the Area is given,

586. The **Diameter** = $\sqrt{\frac{\text{Area}}{.7854}}$.

587. EXAMPLES.

16. What is the circumference of a circle whose diameter is 15 feet? *Ans.* 47.124 ft.+

17. What is the length of binding required for a circular mat the diameter of which is 3 feet?

18. What is the length of the tire of a wheel, if from the centre of the hub to the tire the distance is 2½ feet? *Ans.* 15.708 ft.+

19. How far is it from one side of a circular pond to the opposite side, the distance round the pond being 120 feet? *Ans.* 38.19 ft.+

20. What must be the diameter of a damper to fit a stove-pipe which measures 16 inches around the outside, $\frac{1}{10}$ of an inch being allowed at each end of the diameter for the thickness of the pipe? *Ans.* 4.89 in.+

21. What is the area of a circle whose diameter is 5 ft. 2 in.? *Ans.* 20.96 sq. ft.+

22. What is the area of a circle whose radius is 1 ft. 3 in.? *Ans.* 4.90 sq. ft.+

23. How many square feet are there in a circular grass-plat that is 10 feet in diameter?

* In the formulas given above, by "Circumference," "Diameter," and "Area" are not meant the lengths or the surfaces themselves, but the *number* of units of length or of surface which they contain.

24. What must be the diameter of a circle to contain 20 square feet? *Ans.* 5.05 ft.—

25. The bottom of a basket just covers a circle of 36 square inches; what is the diameter of the circle? *Ans.* 6.77 in.+

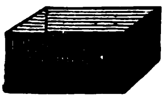
26. How many more square yards in a grass-plat 10 feet square than in a circular plat 10 feet in diameter? *Ans.* 2.38$\frac{4}{9}$ sq. yds.

27. How many more rods of fencing are required to enclose a square field that contains an acre, than to enclose a circular field of the same area? *Ans.* 5.76 rds.

☞ For Dictation Exercises upon these Examples, see Key.

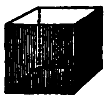
SOLIDS.

Fig. 1.



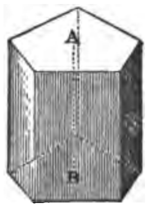
Rectangular Solid.

Fig. 2.



Cube.

Fig. 3.



Prism.

588. Each of the above diagrams represents a solid.

589. A solid bounded by rectangles (Fig. 1) is a **rectangular solid.**

590. A solid bounded by six equal squares (Fig. 2) is a **cube.**

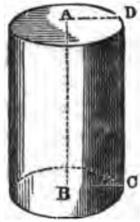
591. A solid (Fig. 3) bounded by parallelograms which terminate in equal and parallel polygons is a **prism.**

NOTE. — The equal and parallel polygons form the **bases** of the prism, and the parallelograms form its **convex surface.** When the bases are regular polygons and the parallelograms are perpendicular to the bases, the prism is a **regular prism.**

Fig. 4.

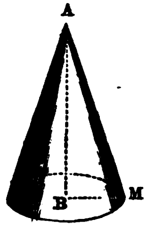
Pyramid.

Fig. 5.



Cylinder.

Fig. 6.



Cone.

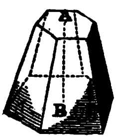
592. A solid (Fig. 4) which has a polygon for a base, and the rest of whose surface is formed of triangles which terminate in a common point called the vertex, is a **pyramid.**

NOTE.—The triangles form the **convex surface** of the pyramid. When the base of a pyramid is a regular polygon, and a line drawn from the vertex to the middle of the base is perpendicular to the base, the pyramid is a **regular pyramid.**

593. A solid (Fig. 5) which may be formed by the revolution of a rectangle, as A B C D, about one of its sides, as A B, is a **cylinder.**

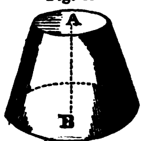
594. A solid (Fig. 6) which may be formed by the revolution of a right-angled triangle, as A B M, about one of its sides, as A B, is a **cone.**

Fig. 7.



Frustum of a Pyramid.

Fig. 8.



Frustum of a Cone.

595. If the upper part of a pyramid or of a cone is cut off by a plane parallel to the base, the part that remains (see Figs. 7 and 8) is a **frustum** of the pyramid or cone.

596. In any of the solids now defined, the shortest distance from the highest point above the base to the plane of the base is the **height.** (See the lines A B in the preceding diagrams.)

NOTE.—In a regular pyramid or in a cone, the shortest distance from the vertex to the perimeter (boundary) of the base is the **slant height**.

In the frustum of a regular pyramid or of a cone, the shortest distance between the perimeters of the two bases is the **slant height**.

597. A solid (Fig. 9) bounded by a surface every part of which is equally distant from a point within called the centre, is a **globe** or **sphere**.

Fig. 9.

Sphere.

NOTE.—A circle which divides a sphere into two equal parts is a **great circle** of the sphere. A circumference, a diameter, or a radius of a great circle of a sphere, are also a **circumference**, a **diameter**, or a **radius** of the sphere itself.

TO FIND THE CONTENTS OF SOLIDS AND THE AREAS OF THEIR CONVEX SURFACES.

NOTE.—To find the contents of a cube or of any rectangular solid, see Art. 318.

598. TO FIND THE CONTENTS OF A PRISM OR OF A CYLINDER. It is evident that a prism or a cylinder 1 inch high contains as many cubic inches as there are square inches in the base, and that if the height is increased to 2, 3, or any number of inches, the solid contents will be increased in the same proportion. Hence,

To find the contents of a prism or of a cylinder, *Multiply the number of square units in the base* by the number of units in the height;* the result will equal the number of cubic units required.

599. TO FIND THE CONVEX SURFACE OF ANY UPRIGHT PRISM OR OF A CYLINDER. It is evident that if a prism or a cylinder is 1 inch high, its convex surface contains as many square inches as there are inches in the perimeter of the base, and that if the height is increased to 2, 3, or any number of inches, the convex surface will be increased in the same proportion. Hence,

* The units in the height must correspond with the units in the base; that is, if the base is in square feet, the height must be in feet; if the base is in square inches, the height must be in inches, etc.

To find the convex surface of an upright prism or of a cylinder, *Multiply the number of units in the perimeter of one of its bases by the number of units in the height.*

600. TO FIND THE CONTENTS OF A PYRAMID OR OF A CONE. It can be proved that a pyramid or a cone equals $\frac{1}{3}$ of a prism or a cylinder of the same base and height. Hence,

To find the contents of a pyramid or of a cone, *Multiply the number of square units in the base by $\frac{1}{3}$ of the number of units in the height.*

601. TO FIND THE CONVEX SURFACE OF A REGULAR PYRAMID OR OF A CONE. The convex surface of a regular pyramid or a cone may be regarded as composed of triangles whose bases form the perimeter of the base of the solid, and whose height is the slant height of the solid. Hence,

To find the convex surface of a regular pyramid or of a cone, *Multiply the number of units in the perimeter of the base by $\frac{1}{2}$ of the number of units in the slant height.*

602. TO FIND THE CONTENTS OF THE FRUSTUM OF A PYRAMID OR OF A CONE. It can be proved that the frustum of a pyramid or a cone equals three pyramids or cones whose bases are the upper and lower bases of the frustum and a mean proportional between the two, and whose height is the height of the frustum. Hence,

To find the contents of a frustum of a pyramid or a cone, *Multiply the sum of the number of units in each of the two bases, plus the square root of their product, by $\frac{1}{3}$ of the number of units in the height.*

603. TO FIND THE CONVEX SURFACE OF THE FRUSTUM OF A REGULAR PYRAMID OR OF A CONE. The convex surface of each solid may be regarded as made up of trapezoids whose parallel sides form the perimeters of the bases, and whose height is the slant height of the frustum. Hence,

To find the convex surface of a frustum of a regular pyramid or of a cone, *Multiply the number of units in the sum of the perimeters of the two bases by $\frac{1}{2}$ of the number of units in the slant height.*

604. TO FIND THE SURFACE OF A SPHERE. It can be proved that the surface of a sphere equals *the number of units in the circumference of the sphere multiplied by the number of units in the diameter*, or equals *the area of four great circles of the sphere.*

605. TO FIND THE CONTENTS OF A SPHERE. A sphere may be regarded as composed of pyramids whose bases form the surface of the sphere and whose height is the radius. Hence,

To find the contents of a sphere, *Multiply the number of units in the convex surface by the number of units in $\frac{1}{3}$ of the radius, or in $\frac{1}{6}$ of the diameter.*

606. EXAMPLES.

1. How many cubic feet of air may be contained in a room 20 ft. long, 16 ft. 6 in. wide, and 12 ft. high? *Ans.* 3,960 cu. ft.

2. How many cubic feet are there in a prism 2 feet high, whose base contains $3\frac{1}{2}$ sq. ft.? *Ans.* 7 cu. ft.

3. How many cubic feet are there in a pyramid whose slant height is $6\frac{1}{2}$ feet and whose base is 5 feet square?

NOTE.—First find the perpendicular height. *Ans.* 50 cu. ft.

4. How many square feet in the convex surface of an octagonal shaft of granite, each face of which is 9 feet long and 18 inches wide? *Ans.* 108 sq. ft.

5. How many gallons of water will fill a circular tub which is $1\frac{1}{2}$ ft. deep and $2\frac{1}{2}$ ft. across the top and bottom? *Ans.* 55.08 gal.

6. How many square yards in the surface of a four-sided pyramidal roof, the length of each side being 22 ft. and the slant height 16 ft.? *Ans.* $78\frac{2}{9}$ sq. yds.

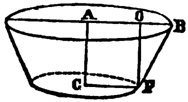
7. How many square yards in the surface of a four-sided pyramidal roof, the length of each side being 36 feet and the highest point $7\frac{1}{2}$ feet above the eaves? *Ans.* 156 sq. yds.

8. How many square yards of pasteboard in 100 circular paper-collar boxes, each of which is 2 inches high and 4 inches across the bottom, the rim of the lid being $\frac{1}{2}$ of an inch deep, no allowance being made for the thickness of the pasteboard? *Ans.* $4.36\frac{1}{3}$ sq. yds.

9. How many quarts of berries can be put into a box the bottom of which is 14 inches square on the inside, and the top 18 inches square, the depth of the box being 8 inches? *Ans.* 30.6 qt.+

10. How many tons of hay does a conical haystack contain which is 32 feet around the base and 18 feet high, allowing 384 cubic feet to a ton? *Ans.* 1.273 T.

11. How many yards of cloth 22 inches wide, will be required to make a conical tent 12 feet high and 10 feet across, measured on the ground? *Ans.* 37.128 yds.



12. How many quarts of water will fill a circular dish whose inside dimensions are 20 inches across the top, 15 inches across the bottom, $6\frac{1}{2}$ inches slant height? *Ans.* 25.16 qts.—

13. How many square inches are there in the surface of a foot-ball that is 8 inches in diameter? *Ans.* 201.06 sq. in.+

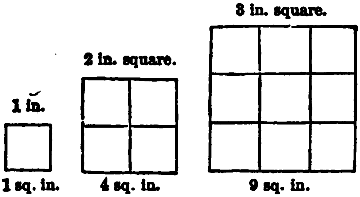
14. How many square inches of leather will cover a base-ball, whose circumference is 12 inches? *Ans.* 45.8 sq. in.+

15. How many cubic feet are there in a globe, the circumference of which is 10 feet? *Ans.* 16.88 cu. ft.+

☞ For Dictation Exercises on these examples, see "Manual and Key," page 159.

RELATIONS OF SIMILAR SURFACES AND SOLIDS.

SIMILAR SURFACES.



607. All the accompanying figures correspond in form though they differ in size.

Figures which correspond in form are **similar figures.**

608. From the above diagrams it will be seen that a figure 1 in. square contains 1 sq. inch; a figure 2 in. square contains 4 sq. inches; a figure 3 in. square contains 9 sq. inches.

In a similar manner it may be shown that

The areas of all similar figures are to each other as the squares of their corresponding dimensions.

EXAMPLES.

1. If the base of a triangle is 15 feet, and its area 28 sq. feet, what is the area of a similar triangle whose base is 45 feet?

OPERATION. — From the foregoing illustrations, we have the proportion

$15^2 : 45^2 = 28 \text{ sq. ft.} : \left(\frac{28 \times \overset{3}{\cancel{45}} \times \overset{3}{\cancel{45}}}{\cancel{15} \times \cancel{15}} \text{ sq. ft.}\right)$ *Ans.* 252 sq. ft.

2. If a room 15 feet wide can be carpeted with 20 yds. of carpeting, what is the width of a room of the same shape which can be carpeted with $51\frac{1}{5}$ yds. of carpeting of the same width?

OPERATION. — $20 : 51\frac{1}{5} = 15^2 : \frac{\overset{3}{\cancel{15}} \times \overset{3}{\cancel{15}} \times \overset{64}{\cancel{256}}}{\underset{4}{\cancel{20}} \times \cancel{5}} =$ the square of the number of feet of width. $\sqrt{576} = 24$. *Ans.* 24 ft.

3. If a sheet of lead 1 ft. square costs \$ 1.00, what will a sheet of the same kind of lead 4 ft. square cost? *Ans.* \$ 16.

4. If a triangle whose base is 3 rods has an area of 36 sq. rods, how many square rods of area has a similar triangle whose base is 5 rods? *Ans.* 100 sq. rds.

5. If it costs \$ 750 to pave a circular court whose diameter is 200 feet, what will it cost to pave a similar court whose diameter is 60 feet? *Ans.* \$ 67.50.

6. If a pipe 5 centimeters in diameter will drain a cistern in 2 hours, in what time will a pipe 3 centimeters in diameter drain the same cistern, no allowance being made for friction? *Ans.* $5\frac{5}{9}$ hrs.

7. Two similar lots of land are to be mowed; the first, which is 800 rods wide, can be mowed in 12 hours, the second can be mowed in 15 hours; what is the width of the second? *Ans.* 894.42 rds.+

8. If a rectangular lot of land whose width is 80 feet can be seeded with $\frac{1}{2}$ bushel of grain, what is the width of a similar lot which can be seeded with 4 bushels of grain? *Ans.* 226.27 ft.+

9. A and B have circular gardens; the diameter of A's is 25 rods; what is the diameter of B's, if its area equals $\frac{1}{5}$ the area of A's? *Ans.* 11.18 rds.+

10. What must be the diameter of a pipe to convey away the water received from 100 pipes, each 7 inches in diameter? *Ans.* 70 in.

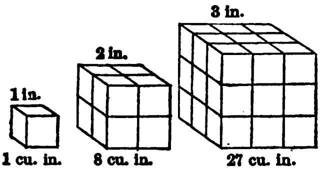
11. The gilding of a globe whose diameter was 4 inches cost \$ 1.50; what is the diameter of a globe of which the gilding cost \$ 9?

Ans. 9.7979.+

12. 20 feet of the upper part of a conical church-spire, whose slant height was 74 feet, was painted for \$ 20; what will it cost to paint the remainder?

Ans. \$ 253.80.

SIMILAR SOLIDS.



609. All of the accompanying diagrams represent solids which correspond in form, though they differ in size.

Solids which correspond in form are **similar solids.**

610. From the above diagrams it will be seen that a cube whose edge is 1 inch contains 1 cu. inch; a cube whose edge is 2 inches contains 8 cu. inches; a cube whose edge is 3 inches contains 27 cu. inches.

In a similar manner it may be shown that

The volumes of all similar solids are to each other as the cubes of their corresponding dimensions.

EXAMPLES.

1. If the contents of a pyramid whose base is 10 in. sq. are 5 cu. ft., what are the contents of a similar pyramid whose base is 20 in. sq.?

OPERATION. — From the foregoing illustrations, we have the proportion

$$10^3 : 20^3 = 5 \text{ cu. ft.} : \left(\frac{5 \times \overset{2}{\cancel{20}} \times \overset{2}{\cancel{20}} \times \overset{2}{\cancel{20}}}{\cancel{10} \times \cancel{10} \times \cancel{10}} \text{ cu. ft.}\right) = 40 \text{ cu. ft.}$$

Ans. 40 cu. ft.

2. If a cubical vessel whose edge is 1 ft. will contain $62\frac{1}{2}$ lbs. of water, what is the edge of a cubical vessel that will contain 50 lbs.?

OPERATION. — $62\frac{1}{2} : 50 = 1^3 : \dfrac{1^3 \times \overset{2}{\cancel{50}} \times 2}{\underset{5}{\cancel{125}}} = \frac{4}{5}$, the cube of the number of feet in the edge. $\sqrt[3]{\frac{4}{5}} = \sqrt[3]{.8} = .9283+$ *Ans.* .9283 ft.+

3. If the trunk of a tree whose girth is 30 inches contains 1 cord ft. of wood, what will a similar trunk contain whose girth is 60 inches?

Ans. 8 cord ft.

4. If the edge of a cube of ice which weighs 56.25 lbs. is 12 inches, what is the edge of a cube which weighs 100 lbs.?

Ans. 14.53 in.+

5. If a cubic vessel whose edge is 1 meter contains 1000000 grams of water, what weight of water will a cubic vessel contain whose edge is 1 centimeter? *Ans.* 1 gram.

6. I have a bin in my granary which is 3 feet deep, and which contains 800 bushels of grain; what must be the depth of a similar bin to contain 1500 bushels? *Ans.* 3.699 ft.+

MISCELLANEOUS MEASURES.

611. MEASUREMENT OF LUMBER.

If a board is 1 inch or less in thickness, its contents are estimated by *the number of square feet in its surface.*

If a board is more than 1 inch in thickness the contents are found *by multiplying the number of square feet in its surface by the number of inches in thickness.* This measure is called **Board Measure.**

The contents of timber, as joist, beams, etc., are estimated *by the foot of board measure* or *by the cubic foot.*

EXAMPLES.

1. What are the contents of 2 boards each of which is 12 feet long, $2\frac{1}{2}$ feet wide at one end, 2 feet wide at the other end, and 1 inch thick?

NOTE. $(2\frac{1}{2}$ ft. $+ 2$ ft.$) \div 2 = 2\frac{1}{4}$ ft., the average width. $2\frac{1}{4} \times 12 = 27.$

Ans. 27 feet.

2. What is the cost of a piece of board 7 ft. long, 2 ft. wide, and $\frac{3}{4}$ of an inch thick, at 3 cents per foot?

3. How many feet, board measure, in a piece of timber 8 ft. long, 8 in. wide, and 4 in. thick?

4. How many feet, board measure, in a joist 12 ft. long, 3 in. wide, and 4 in. thick?

612. Shingling and other plain work, as flooring and partitioning, are generally estimated by the square of 100 feet. 1000 shingles are allowed for a square.

613. Painting is measured by the square yard.

614. Plastering is measured by the square foot, square yard, or square of 100 ft.

615. Glazing is measured by the square foot, including the sash.

616. Paving is measured by the square foot, or square yard.

617. Bricklaying is generally estimated by the thousand bricks; sometimes it is estimated by the square yard, square rod, or square (of 100 ft.), allowing 1½ bricks, or 12 in. in thickness.

A great variety of methods for measuring prevail. Some workmen make no allowance for doors and windows, others make allowance of half the space occupied by doors and windows, and others still estimate the exact amount of material and labor employed. Measurements are taken on the outside of walls, no allowance being made for corners. In estimating the number of bricks used, an allowance of one sixth of the solid contents is made for mortar.

618. TO FIND THE CAPACITY OF A CASK OR BARREL.

NOTE I. — The following rule for finding the capacity of casks, though given here without demonstration, for want of space, may be of service.

Add the squares of the number of inches in the head diameter, in the bung diameter, and twice the middle diameter; multiply this sum by .0005667 *of the number of inches in the length, and the result will be the contents in gallons.*

NOTE II. — The middle diameter is the diameter of the section midway between the bung and the head. It may be determined, when the staves are of a uniform thickness, by dividing the circumference of the section by 3.1416, and taking out of the quotient twice the thickness of the staves.

EXAMPLE.

1. Find the capacity, in gallons, of a cask whose length is 35 inches, the head diameter 25 inches, the bung diameter 32 inches, and the middle diameter 29 inches. *Ans.* 99.43 gal.+

619. TOPICAL REVIEW IN RATIO, PROPORTION, INVOLUTION, ETC.

The pupil may present the following topics to his class, using common illustrations, giving definitions and deriving rules from illustrative examples which he will solve before the class: —

1. Ratio (Arts. 507 – 511); how found (Arts. 512, 513). Compound Ratio (Art. 514).

2. Proportion (Arts. 515, 516). To find a missing term (Arts. 517, 518). Deductions (Art. 519). Applications to Examples (Art. 520).

3. Compound Proportion with application to examples (Art. 521).

4. Partnership (Arts. 522 – 527).

5. Involution (Arts. 528 – 533). 6. Evolution (Arts. 534 – 537).

7. Square Root; how to find number of terms of the root (Arts. 538 – 540). $(\text{tens} + \text{units})^2$ (Arts. 541, 542). To extract a root of 2 terms (Art. 543). To extract roots of more than 2 terms, with rule (Arts. 544, 545). Roots of fractional numbers (Art. 546, with notes).

8. Right-angled triangles (Arts. 550 – 552).

9. Cube Root; how to find number of terms of the root (Arts. 554 – 556). $(\text{tens} + \text{units})^3$ (Art. 557). To extract a root of 2 terms (Art. 559). To extract roots of more than 2 terms, with rule (Arts. 560, 561). Roots of fractional numbers (Art. 562, with notes).

10. Mensuration; definitions (Arts. 565 – 575). 11. Areas of polygons (Arts. 576 – 579). 12. Areas of circles (Arts. 581 – 586).

13. Solids, definitions (Arts. 588 – 597).

14. Contents of solids and their convex surfaces (Arts. 598 – 605).

15. Similar surfaces (Arts. 607, 608). 16. Similar solids (Arts. 609, 610).

17. Measurement of lumber, etc. (Arts. 611 – 617).

620. GENERAL REVIEW, No. 8.

1. A and B bought a plantation in Mississippi, for which A paid \$ 6500 and B paid \$ 8000; they afterwards sold the plantation at a loss of \$ 2650.60; what was the loss of each?

Ans. A's \$ 1,188.20; B's \$ 1,462.40. done

2. At \$ 20 per M., what cost 290 boards, each 7 ft. 2 in. long, 5 in wide, and $\frac{3}{4}$ in. thick? *Ans.* \$ 17.32.

3. At $10 per hund., what will be the cost of evergreens placed 18 in. apart, to border a rectangular park 20 rods long and 12 rods wide? *Ans.* $70.40.

4. Find the square root of 1.02. done *Ans.* 1.009+

5. Find the cube root of 372. done *Ans.* 7.188+

6. At $.90 per sq. yd., what is the cost of a concrete walk $7\frac{1}{2}$ ft. wide upon two sides of a corner lot 93 ft. upon one street and 79 ft. upon the other? *Ans.* $$134.62\frac{1}{2}$.

7. The first term of a proportion is 22 and the ratio $1\frac{4}{7}$; what is the second term? *Ans.* 14.

8. What is the length of the minute-hand of a clock which describes a circle whose circumference is 2 ft. in length?
Ans. .318 ft.+ or 3.8 in.+

9. What are the solid contents of a cone the diameter of whose base is 12 ft. and whose slant height is 15 ft.? *Ans.* 518.277 cu. ft.+

10. How many acres are there in a farm two of whose sides are parallel, 34 ch. and 22 ch. in length respectively, the distance between them being 30 ch.? *Ans.* 84 A.

11. If a cannon-ball whose diameter is 4 in. weighs 8 lbs., what is the weight of a cannon-ball whose diameter is 12 in.? *Ans.* 216 lb.

12. What must be the width of a rectangular tin pipe whose depth is 2 in. that its capacity shall equal that of a circular pipe whose diameter is 6 in.? *Ans.* 14.137 in.+

☞ Perform the above examples, substituting 3 for 2 in each example. For other dictations on this Review, see "Manual," page 161.

621. MISCELLANEOUS EXAMPLES.

1. In how many days will a steamer go from New York to Liverpool, a distance of 3500 miles, if she goes at the rate of 12 miles per hour? *Ans.* 12 d. $3\frac{2}{3}$ h.

2. Make and receipt a bill for the following items:—

17468 bricks at $18.75 per M.
4250 shingles at $$3.62\frac{1}{2}$ per M.
1200 laths at $$.33\frac{1}{3}$ per C. *Ans.* $346.93.

3. The interest of a certain sum for 1 y. 6 m. is $67.20; what is the interest of the same sum for 10 m.?

4. What cost a quarter-section of government land at $12.40 per acre? *Ans.* $1,984. done

5. A watch which loses 1 m. 5 sec. a day is found after a journey of 3 days to be 1 h. 7 m. too fast; how many degrees of E. or W. longitude must have been travelled? *Ans.* 17° 33′ 45″ W.

6. What is the equated time for paying $ 200 due Sept. 15, $ 400 due Oct. 3, and $ 280 due Oct. 18? *Ans.* Oct. 4.

7. What was the cost of a piece of cloth containing $28\frac{1}{4}$ yds. on which a tailor lost $2\frac{1}{2}$% by selling it at $12\frac{1}{2}$ cents per yd. less than cost? *Ans.* $ 141.25.

8. When $ 40.40 is paid for 5.05 tons of coal, 2000 lbs. to the ton, which cost, including freight, etc., $ 5.60 per ton of 2240 lbs., what % of profit is made? *Ans.* 60%.

9. It is estimated that the coal-beds near Pittsburg contain 54 billions of tons of workable coal; at $ 2 per ton, how many debts equal to the national debt, $ 2,480,000,000 (July, 1869), would this coal pay? *Ans.* 43: Rem. $ 1,360,000,000.

10. If the annual product of the gold-mines of California is $ 100,000,000, in how many years would the gold product of California pay for the above-mentioned coal? *Ans.* 1,080 yrs.

11. At $ .30 per hund. for laths, what will the laths cost to cover the walls and ceiling of a room 24 ft. long, 18 ft. wide, and 10 ft. high, deducting $\frac{1}{8}$ for openings, and allowing 100 laths to lay 5 sq. yds.? *Ans.* $ 6.78.

12. A woman bought a sewing-machine, paying $\frac{2}{15}$ of the price down, and promising to pay $\frac{1}{13}$ of the remainder each month afterwards; when she had made 5 monthly payments she still owed $ 40, what was the cost of the machine? *Ans.* $ 75.

13. How many cubic feet of stone will be required to lay a cellar wall 2 ft. thick, to enclose a cellar 15 ft. long, 12 ft. wide, and 8 ft. deep? *Ans.* 992 cu. ft.

14. What is the cost of removing the earth for the above-mentioned cellar and wall at 45 cts. per cu. yd.? *Ans.* $ 40.53.

15. At a concert, $ 275 was received for 1700 tickets; there were $\frac{7}{10}$ as many tickets for adults as for children, and the price of the children's ticket was $\frac{3}{5}$ the price of the adults'; what was the price of each? *Ans.* 15 cts.; 25 cts.

16. If one acre of land produces 500 bushels of onions, which sell at an average of \$ 1.12$\frac{1}{2}$ a bushel, how many acres must be cultivated when the price of onions is doubled and the yield $\frac{2}{3}$ as great, that the value of the crop may be \$ 600? *Ans.* $\frac{4}{5}$ A.

17. Which is the better investment, stock at 12% discount which pays 5%, or stock at 3% premium which pays 7%?

Ans. The latter, which yields $6\frac{82}{103}$%, while the former yields $5\frac{15}{22}$%.

18. A note for \$ 5000, dated Jan. 19, 1869, on 3 mo., was discounted at the First National Bank, Providence, at 10% on the day of its date; what were the proceeds? *Ans.* \$ 4,870.83.

19. A railroad corporation declares a 6% dividend, payable in stock at \$ 100 per share; to how many additional shares will a stockholder be entitled who previously held 50 shares?

20. A drover sold a yoke of oxen at \$ 225 each; on one ox he gained 20%, and on one ox he lost 20%; what did he gain or lose on the yoke? *Ans.* Lost \$ 18.75.

21. What will be gained by hiring \$ 1000 for 1 year, in New York, at the legal rate, transmitting it by draft to St. Louis, purchased at $\frac{1}{4}$% premium, and loaning it at the highest rate allowed at the latter place? *Ans.* \$ 27.50.

22. If 1 silver rouble of St. Petersburg is equal to 38$\frac{1}{4}$ pence sterling, and £ 1 sterling to \$ 4.84 U. S. money, how many roubles of St. Petersburg may be bought for \$ 185.13 U. S. money? *Ans.* 240.

23. My correspondent at Boston sold on my account 40 bales of cotton, averaging 420 lbs. per bale, at 20 cents per pound. After deducting charges, \$ 140.60, and his commission of $\frac{1}{2}$% upon the sale, with the net proceeds he purchased a draft for remittance at $\frac{1}{4}$% premium; what was the face of the draft? *Ans.* \$ 3,194.61.

24. How many square feet of glass in the roof, ends, and front of a greenhouse, the length of the house being 40 ft., the width 24 ft., the height of the back wall being 14 ft., and of the front 4 ft.?

Ans. 1,632 sq. ft.

25. One fourth of a lot of hats, purchased at \$ 9 per doz., were spoiled in transporting; at what price per hat must the remainder be sold that 10% may be cleared on the purchase?

26. If the pyramid of Cheops in Egypt, which is said to contain 82,111,000 cu. ft. of masonry, could be arranged in a solid cubical pile, what would be the length of one of its edges? *Ans.* 434.6 ft.+

27. What is the length of one side of a square table-cover whose corners hang 2 ft. from the edge of a round table which is 4 ft. across? *Ans.* 5.65 ft.+

28. The corners of a checker-board just reach the edge of a round table upon which it lies; the board being 2 ft. square, how many square feet are there in the surface of the table? *Ans.* 6.28 sq. ft.+

29. There are three cubes whose diameters are respectively 10, 12, and 15 inches; what is the diameter of a cube of the same material whose weight is equal to that of the three? *Ans.* 18.27 in.+

30. Wishing to use $ 800 for 6 months, Lee borrowed $ 400 of A for the first 2 months, and $ 400 of B for the next 4 months; he also borrowed $ 300 of C for 3 months and $ 500 of D for 3 months, promising half of his gains for the use of the money. He gained $ 1800; what should each person receive?

A, $ 150; B, $ 300; C, $ 168.75; D, $ 281.25.

31. A agreed to contribute for the building of a chapel $ 10 on a certain Sabbath, $ 20 on the next, $ 30 on the next, and so on for 6 mo. B said as he was not so wealthy as A, he would contribute 5 cts. on the first Sabbath, and make his contribution on each succeeding Sabbath for 6 mo. double that of the preceding. If each kept his promise, what was the contribution of each on the last Sabbath? What was the amount of the contribution of each?

Ans. { A's, $ 260; $ 3,510.
B's, $ 1,677,721.60; $ 3,355,443.15.

32. X and Y commenced business on the 1st of Jan. X contributed $ 4000 to the capital and Y $ 6000. By contract gains or losses were to be shared equally, interest at 6% on each investment was to be allowed, and interest on all sums withdrawn by either party was to be deducted. X drew out $ 200, July 3d, $ 300, Sept. 10, and $ 300, Dec. 1. Y drew out $ 500, July 3d, $ 500, Oct. 1st, and $ 200, Dec. 6. At the end of the year their resources were $ 18020, and their liabilities $ 7340; what was the net capital due to each?

Ans. X's, $ 4,485.11; Y's, $ 6,194.89.

APPENDIX.

CONTRACTIONS IN MULTIPLICATION AND DIVISION.

NOTE TO THE TEACHER. — Some processes for abbreviating the operations of multiplication and division have been illustrated in Arts. 79, 85, 104, and 106. Some additional processes are indicated below. The appropriate illustrations will suggest themselves to the teacher.

622. TO MULTIPLY BY 9, 99, 999, etc.

9 being one less than 10, 99 one less than 100, 999 one less than 1000, etc.,

To multiply by any number whose digits are all 9's, *Annex as many zeros to the expression for the multiplicand as there are 9's in the expression for the multiplier, and take out of the product thus expressed a number equal to the multiplicand.*

EXAMPLES.

1. $43 \times 99 = ?$ $(4300 - 43 = 4257)$ *Ans.* 4,257.
2. $584 \times 999 = ?$ *Ans.* 583,416.
3. $283 \times 9999 = ?$
4. $54 \times 999999 = ?$
5. $70089 \times 99999 = ?$
6. $69 \times 998 = ?$ $(69000 - 69 \times 2)$ *Ans.* 68,862.
7. $549 \times 9990 = ?$ $(5490000 - 549 \times 10)$ *Ans.* 5,484,510.

623. TO MULTIPLY BY A COMPOSITE NUMBER.

Separate the multiplier into convenient factors, multiply the multiplicand by one of the factors, and that product by another factor, and so on till all the factors are employed; the last product will be the answer.

EXAMPLES.

1. $52 \times 24 = ?$ $(52 \times 3 \times 8 = 1248)$ *Ans.* 1,248.
2. $4307 \times 36 = ?$
3. $894 \times 54 = ?$
4. $4079 \times 121 = ?$
5. $8931 \times 150 = ?$

624. TO MULTIPLY BY THE ALIQUOT PARTS OF 10, 100, ETC.

Multiply by 10, 100, 1000, *etc., as the case may require, and then find the required part; thus,*

(1.) To multiply by 5, *multiply by* 10 *and divide the product by* 2.
(2.) To multiply by 25, *multiply by* 100 *and divide the product by* 4.
(3.) To multiply by 125, *multiply by* 1000 *and divide the product by* 8.
(4.) To multiply by $33\frac{1}{3}$, *multiply by* 100 *and divide the product by* 3.
(5.) To multiply by $16\frac{2}{3}$, *multiply by* 100 *and divide the product by* 6.
(6.) To multiply by $12\frac{1}{2}$, *multiply by* 100 *and divide the product by* 8.

EXAMPLES.

1. $368 \times 5 = ?$
2. $7009 \times 12\frac{1}{2} = ?$
3. $9075 \times 3\frac{1}{3} = ?$
4. $87450 \times 16\frac{2}{3} = ?$
5. $50793 \times 125 = ?$
6. $92687 \times 33\frac{1}{3} = ?$

625. TO DIVIDE BY THE ALIQUOT PARTS OF 10, 100, ETC.

(1.) To divide by 5, *divide by* 10 *and multiply the quotient by* 2.
(2.) To divide by 25, *divide by* 100 *and multiply the quotient by* 4.
(3.) To divide by 125, *divide by* 1000 *and multiply the quotient by* 8.
(4.) To divide by $33\frac{1}{3}$, *divide by* 100 *and multiply the quotient by* 3.
(5.) To divide by $16\frac{2}{3}$, *divide by* 100 *and multiply the quotient by* 6.
(6.) To divide by $166\frac{2}{3}$, *divide by* 1000 *and multiply the quotient by* 6.

EXAMPLES.

1. $868 \div 25 = ?$
2. $3456 \div 12\frac{1}{2} = ?$
3. $80764 \div 50 = ?$
4. $9876 \div 166\frac{2}{3} = ?$

☞ For further contractions, see "Manual," page 162.

ANNUAL INTEREST.

$800 *Portsmouth, Nov. 18, 1865.*

On demand, I promise to pay L. D. Spaulding, or order, Eight hundred dollars, with interest annually. Value received.

Edward Linton.

In New Hampshire and in Vermont, on notes written as the above, "with interest annually," each year's interest that is not paid when due draws interest for the time it remains unpaid. The total of interest accruing upon such a note is called **Annual Interest.**

626. ILL. EX. If no interest is paid upon the foregoing note previously, what is the amount of the note July 24, 1869?

OPERATION.

	Principal on interest from Nov. 18, 1865, ...			$ 800.00
	Interest on $ 800 to July 24, 1869 (3 y. 8 m. 6 d.), . .			176.80
	A year's interest, $ 48, is due and on interest		Interest.	
(1.)	from Nov. 18, 1866, to July 24, 1869,	2 y. 8 m. 6 d.,	$ 7.728	
(2.)	from Nov. 18, 1867, to July 24, 1869,	1 y. 8 m. 6 d.,	4.848	
(3.)	from Nov. 18, 1868, to July 24, 1869,	8 m. 6 d.,	1.968	
	which equals the interest of $ 48 for	5 y. 0 m. 18 d.		14.54
	Amount due July 24, 1869			$ 991.34

From the above may be derived the following

RULE.—TO COMPUTE ANNUAL INTEREST: *Compute interest on the principal for the entire time it is on interest. Compute also interest on each year's interest for the time it remains unpaid.*

The sum of the interests thus found will be the annual interest. The principal with this interest will be the amount at annual interest.

NOTE.—To abbreviate the process (instead of computing interest on *each* year's interest separately), *compute interest on one year's interest for a period of time equal to the sum of all the periods of time for which a year's interest remains unpaid.*

EXAMPLES.

1. What is the amount at annual interest of $ 420 from Feb. 1, 1864, to June 1, 1869, at 6%? *Ans.* $ 572.04.

2. What amount is due Oct. 15, 1869, on a note for $ 750, dated June 20, 1866, with interest annually, at 6%? *Ans.* $ 910.06.

3. What is the amount of $ 468.50 for 4 y. 10 m. 8 d., at 5% annual interest? *Ans.* $ 593.28.

627. ANNUAL INTEREST WITH PARTIAL PAYMENTS.

In case partial payments are made upon notes or other obligations drawing annual interest, the following is the

VERMONT RULE.

1. *Compute annual interest upon the principal to the end of the first year in which any payments are made; also compute interest upon the payment or payments from the time they are made to the end of the year.*

2. *Apply the amount of such payment or payments first to cancel any interests that may have accrued upon the yearly interests, then to cancel the yearly interests themselves, and then towards the payment of the principal.*

3. *Proceed in the same way with succeeding payments, computing, however, no interest beyond the time of settlement.*

THE NEW HAMPSHIRE RULE

is the same as the foregoing, with the following provision: —

If at the time of any payment no interest is due except what is accruing during the year, and the payment or payments are less than the interest due at the end of the year, deduct such payment or payments at the end of the year, without interest added.

ILL. EXAMPLE.

Find the amount due on the following note Jan. 1, 1869, interest at 6%, by the Vermont and by the New Hampshire Rule: —

$ 1000. ....*March* 9, 1865.

On demand, I promise to pay the bearer one thousand dollars for value received, with interest annually, at 6%. C. A. MEACHAM.

INDORSEMENTS. — Nov. 19, 1865, $ 204; March 3, 1867, $ 50; June 15, 1868, $ 600; Nov. 1, 1868, $ 85.

Ans. Vt. $ 243.25; N. H. $ 243.31.

OPERATION.		By Vt. Rule.	By N. H. Rule.
Principal, March 9, 1865,		$ 1000.00	$ 1000.00
Interest to March 9, 1866 (1 year),		60.00	60.00
		1060.00	1060.00
1st payment, Nov. 19, 1865,	204.00		
Interest to March 9, 1866 (3 m. 18 d.),	3.672		
		207.67	207.67
New Principal, March 9, 1866,		852.33	852.33
Interest to March 9, 1867 (1 year),		51.14	51.14
		903.47	903.47
2d payment, March 3, 1867,	50.00		50.00
Interest to March 9, 1867 (6 days),	.05		
		50.05	
New Principal, March 9, 1867,		853.420	853.470
Interest to March 9, 1868 (1 year),		51.205	51.208
Interest to Jan. 1, 1869 (9 m. 23 d.),		41.675	41.677
Interest on 1 year's interest ($ 51.205), to Jan. 1, 1869 (9 m. 23 d.),		2.500	Int. on $ 51.208 } 2.500
		948.800	948.855
3d payment, June 15, 1868,	600.00		
Interest to Jan. 1, 1869 (6 m. 17 d.),	19.70		
4th payment, Nov. 1, 1868,	85.00		
Interest to Jan. 1, 1869 (2 m.),	.85		
		705.55	705.55
Balance due,		$ 243.25	$ 243.305

Ans Vt., 243.25; N. H., $ 243.31.

EXAMPLE.

A note dated at Lancaster, N. H., Mar. 1, 1865, was given for $4000 with annual interest at 6%. On this note were the following

INDORSEMENTS. — Received Oct. 1, 1867, $2000. Nov. 1, 1868, $100.

What balance was due Jan. 1, 1869, by Vermont Rule? by New Hampshire Rule? *Ans.* Vt., $2,747.86; N. H., $2,748.86.

628. The following is the

CONNECTICUT RULE FOR PARTIAL PAYMENTS.

1. *When a year's interest or more has accrued at the time of a payment, and always in case of the last payment, follow the* UNITED STATES RULE. (Art. 478.)

2. *When* LESS *than a year's interest has accrued at the time of a payment, except it be the last payment, find the difference between the amount of the principal for an* ENTIRE *year, and the amount of the payment for the balance of a year after it is made; this difference will form the new principal.*

3. *If the interest which has arisen at the time of a payment exceeds the payment, compute interest upon the principal only.*

A note, dated March 9, 1866, is given for $852.26 on interest at 6%, with the following

INDORSEMENTS: — March 3, 1867, $50; June 15, 1868, $600; Nov. 1, 1868, $85.

What was the balance due Jan. 1, 1869, by the Connecticut Rule?

OPERATION.

Principal, March 9, 1866,		$852.26
Interest to March 9, 1867 (1 year),		51.135
		903.395
1st payment (being less than interest),		50.00
New Principal, March 9, 1867,		853.395
Interest to June 15, 1868 (1 y. 3 m. 6 d.),		64.858
		918.253
2d payment,		600.00
New Principal, June 15, 1868,		318.253
Interest to Jan. 1, 1869 (6 m. 17 d.),		10.449
		328.702
3d payment, Nov. 1, 1868,	$85.00	
Interest to Jan. 1, 1869 (2 m.),	.85	
		85.850
Balance due Jan. 1, 1869,		$242.852

Ans. $242.85.

629. Miscellaneous Examples.

[Principally selected from Waltons' Written Arithmetic, but so modified as to give results wholly different from those of the original examples.]

1. A man gave to his eldest son $ 3575, to his youngest son $ 4680, and to his daughter $ 2495 less than to the youngest son; his whole property was worth $ 20500; what sum remained?

2. The difference between two numbers is 95478. The larger number is 148765; what is the smaller?

3. Out of what number must three 846's be taken to leave 15684?

4. I have four corn-bins, containing severally 63 bushels, 75 bushels, 37 bushels, and 29 bushels. There are 4 pecks in a bushel. How many pecks do they all hold?

5. Paid $ 35328 for 736 acres of land; find the price per acre.

6. If 446 is one factor, and 28544 the product, what is the other factor?

7. A dividend is 76986, and the divisor 4277; what is the quotient?

8. Smith & Co. consume 74 tons of coal in a year; how much more must they pay for their coal in 1868, when coal was $ 14 a ton, than in 1869, when it is $ 9 a ton?

9. Bought 3 dozen pigeons at $ .85 per dozen, 2 dozen at $ 1.10 per dozen, and 1 dozen for $ .90. What should I pay?

10. Bought 2 pieces of flannel, each containing 62 yards, for $ 39.68, and sold them for 48 cents per yard. What did I gain?

11. If 50 barrels of apples were bought for $ 200 and sold for $ 350, what would be gained by selling 45 barrels at the same rate?

12. If a certain piece of work can be performed by 350 men in 14 weeks, how many more must be employed to perform it in a week?

13. A garrison of 10000 men have provisions to last them 6 weeks; if 2500 men be killed in a sally, how long will the provisions last the remainder?

14. If 9872 is the multiplicand, and half that number the multiplier, what is the product?

15. If 9665592 is the product, and 1208199 the multiplicand, what is the multiplier?

16. If 700150 is the divisor, and 3685 the quotient, what is the dividend?

17. What is the width of the widest carpeting that will exactly fit either of two halls, 45 feet and 33 feet wide respectively?

18. What is the width of the narrowest street, across which stepping-stones either 4, 6, or 8 feet long will exactly reach?

19. If a bar of iron 8 feet long weighs 36 pounds, what will a bar of the same size around and 140 feet long weigh?

20. If the work of 8 men is equal to the work of 9 boys, how many men's work will equal the work of 63 boys?

21. If 12 men consume a barrel of flour in 6 weeks, how long would it last 9 men?

22. If the interest of $ 650 for 12 months is $ 52, what is the interest of three times that sum for eight months?

23. If $ 15 will purchase 12 yards of cloth, how many yards will $ 40 purchase?

24. At $\frac{5}{6}$ of a dollar a dozen, how many eggs can be bought for 10 dollars?

25. Having lost $\frac{1}{11}$ of my money in trade, I now have $ 2476.10; what had I at first?

26. A body of 4800 troops has $\frac{1}{7}$ as many cavalry as infantry; what is the number of each?

27. In counting his fowls, a farmer finds that he has 396 in all, which is $\frac{1}{5}$ less than he had the previous year; how many had he then?

28. A can perform a journey on foot in $7\frac{1}{2}$ days; what part of it can he perform in $1\frac{1}{2}$ days?

29. A and B hired a pasture together. A pastured 12 cows, and B 18 cows in it; what part of the price should each pay?

30. How much will be left of a piece of cloth containing 9 yards, after cutting out of it 2 vests and a coat, allowing $\frac{3}{4}$ of a yard for a vest and $4\frac{1}{3}$ yards for a coat?

31. What do I receive per pound by selling $15\frac{5}{6}$ pounds of coffee for $ $6\frac{1}{3}$?

32. Bought $\frac{5}{7}$ of an acre of land for $ 40.75; what would 1 acre cost at the same rate?

33. What costs 5 pieces of calico, $37\frac{1}{2}$ yards in a piece, at $19\frac{1}{4}$ cents per yard?

34. Sold my house and farm of $37\frac{3}{8}$ acres for \$6150; allowing \$3500 for the house, what did I receive per acre for the land?

35. How long will a barrel of flour last a family of 7 persons, if it lasts 3 persons $4\frac{1}{2}$ months?

36. What number is that which diminished by $\frac{7}{10}$ of 1 will leave a remainder of $\frac{1}{60}$?

37. What number is that to which if you add $8\frac{7}{8}$, the sum will be $124\frac{5}{6}$?

38. How long will 200 pounds of meat last 7 persons at the rate of $2\frac{5}{8}$ pounds a day for each person?

39. If a man can build $9\frac{3}{8}$ rods of a wall in a day, how much can he build in $6\frac{1}{2}$ days?

40. How many bushels of wheat can a man purchase for \$$2724\frac{1}{2}$, at $81\frac{3}{4}$ cents per bushel?

41. If I buy 325 bushels of corn at $41\frac{3}{8}$ cents per bushel, and sell it at $52\frac{1}{2}$ cents per bushel, what do I gain?

42. Owning $\frac{2}{5}$ of a paper-mill, I sold $\frac{3}{4}$ of my share for \$2750; what is the value of the whole mill at the same rate?

43. When hay was \$25 per ton, I gave $\frac{3}{4}$ of a ton for $1\frac{3}{7}$ tons of coal; what was the coal worth per ton?

44. If $\frac{3}{4}$ of a yard of cloth will pay for 6 hats worth \$$9\frac{1}{2}$ per dozen, what is the price of the cloth per yard?

45. If a man walks $8\frac{1}{2}$ miles in $2\frac{1}{8}$ hours, how far will he walk in $4\frac{3}{4}$ hours?

46. If $\frac{3}{4}$ of $\frac{7}{8}$ of a ship cost \$42000, what is $\frac{2}{5}$ of her worth?

47. $\frac{1}{4}$ of my money is in gold, $\frac{1}{6}$ of the remainder in silver, and the balance, \$420, is in bank-notes; how much money have I in all?

48. A, B, and C shared \$102 among them, so that A had \$17 more than B, and B had \$8 more than C; what had each?

49. Bought a horse and saddle for \$95, giving $\frac{3}{7}$ as much for the saddle as for the horse; what was the cost of each?

50. A certain piece of work can be performed by A in 9 days, by B in 12 days, and by C in 15 days; in what time can all do it, working together?

51. In what time can A and B do it together?

52. In what time can A and C do it together?

53. In what time can B and C do the above-named work together?

54. If A and B can slate a roof in $5\frac{1}{4}$ days, and A, B, and C can do it in $3\frac{1}{3}$ days, in what time can C do it alone?

55. What will it cost to fence both sides of a road, 26 r. 2 yd. long, at $.75 per yd.?

56. What are 2 A. 135 r. 3 y. 8 ft. of land worth at 20 cents a foot?

57. A garden containing $\frac{3}{4}$ of an acre measures on one side 190 feet; required the length of the other side.

58. What will be the cost of oil-cloth to cover a floor 24 feet by $16\frac{1}{2}$ feet, at 75 cents per square yard?

59. Sold 3 bu. 3 pk. 5 qt. of peaches for $8.75; what did I receive per quart?

60. How many hours in the 1st century?

61. How many days from March 1st, 1859, to January 1st, 1864?

62. What is the time in years, days, etc., from November 19, 1860, 18 min. of 4, P. M., to April 9, 1863, 10 min. past 2, P. M.?

63. How much land have I in 4 pastures, the 1st containing 7 A. 83 r. 13 yd., the 2d, 15 A. 146 r., the 3d, 22 A. 52 r. 18 yd., and the 4th, 5 A. 9 r. 2 yd.?

64. If a car runs 18 m. 149 r. $2\frac{1}{2}$ yd. in $\frac{1}{2}$ of an hour, how far will it run in 9 h.?

65. How many bins, each containing 63 bu. 1 pk. will be required to hold 885 bu. 2 pk. of potatoes?

66. At $9 a cord, what cost a pile of wood 33 ft. long, 8 ft. 10 in. high, and 4 ft. wide?

67. What will be the cost of fencing a lot of land 20 rods by 260 rods, at $37\frac{1}{2}$ cts. per foot?

68. How many yards of carpeting $1\frac{1}{4}$ yd. wide will cover a floor 16 feet square?

69. If a cotton-mill can make 1200 yds. of cloth per hour, how many yards could be made by working 8 hours a day from July 7th to January 4th, allowing for 26 Sabbaths?

70. Divide 30 miles by 7, carrying out the quotient to the lowest denomination.

71. Of a pile of wood 48 ft. long, 4 ft. high, and 4 ft. wide, there was sold at one time 3 cd. 5 cd. ft.; at another 2 cd. 32 cu. ft.; what was the remainder worth at $8 per cord?

72. A floor 30 ft. by 24 ft. is to be covered with carpeting $\frac{5}{8}$ of a yard wide. How many yards will be required?

73. How many barrels of 31 gallons each will be contained in a water-tank 7 ft. square and 11 ft. deep?

74. When it is 1 o'clock, A. M. at A, which is 44° 15′ 2″ W. long., what is the time at B, which is 8° 4′ 40″ E. long?

75. Add $\frac{3}{7}$ of the month of February, 1860, to $\frac{1}{5}$ of the days from February 15, 1861, to May 6, 1861?

76. How many paving-stones 12 in. by 8 in. will be required to pave a street 27 rods long by 50 ft. wide?

77. When snow is uniformly 9 inches deep, how many cubic feet are there on one acre of land?

78. How many square feet in a walk around a garden inside and next to the fence, the garden being $27\frac{1}{2}$ rods long, $20\frac{1}{4}$ rods wide, the walk being 8 feet wide?

79. What cost 12 bu. 2 pks. of plums at $.08 a pint?

80. What cost 2 qts. $1\frac{1}{2}$ pts. of oil at $ 2.12 per gallon?

81. How many cu. ft. of space in a cellar measuring on the inside of the wall 5 yd. 1 ft. in length, 5 yd. in width, and 10 ft. in depth?

82. What is the difference of time in two places whose longitudes differ 9° 8′ 4″?

83. When the difference of time is 5 h. 4 m. 7 s., what is the difference of longitude between two places?

84. The difference between two numbers is 6.797, the less being 9874.08; what is the greater?

85. The difference between two numbers is 298.75, the larger being 1909; required the smaller.

86. Divide 18.2 by 10; multiply that result by 100; divide that by 1000; multiply that by 10, and that by 10, and find the sum of the results?

87. What is the cost of whitewashing the ceiling of a room 18.75 yd. long, 12.82 yd. wide, at $.015 per sq. yd.?

88. Divide 600 by .012, multiply the quotient by .05, and by that product divide .005.

89. Change to decimals, and add, $\frac{3}{4}$, $\frac{855}{15625}$, $3\frac{1}{400}$.

90. Change 60 cu. ft. to the decimal of a cord.

91. Change 87 rd. 12 ft. to the decimal of a mile.

92. What is the amount of 3.75 tons, .085 tons, 11.7$\frac{5}{8}$ tons, and 775 lbs.?

93. If 2537.5 feet of boards cost $ 240.555, what is the price per M.?

94. How many cords in a load of wood, 6.5 ft. long, 4.8 ft. wide, and 4.2$\frac{1}{3}$ ft. high?

95. What is the cost of fencing 180 rd. 3 yd. 2 ft. of road, both sides, at $ 2.25 per rd.?

96. Multiply 675 by $\frac{1}{10}$, the product by 100; divide the last product by 1000, this quotient by $\frac{1}{8}$; and add the four results.

97. Out of 1 lb. take .0678 lb.

98. Multiply 30.5$\frac{1}{3}$ by .056$\frac{1}{4}$.

99. What is the cost of 3 pk. 7 qt. of peas, at $ 4.85 per bushel?

100. Change 10 h. 12 m. to the fractional part of a day.

101. What is the 6th power of .09?

102. Change 2.1 pt. to the decimal of a peck.

103. What is 25% of 125% of 75% of 384 inches?

104. The owner of a field of wheat allows 8$\frac{1}{3}$% of the wheat for harvesting; what will be the owner's share if 88 bushels are harvested?

105. A and B had each $ 2800 bequeathed to them. A gained 25% on his bequest, and B lost 12$\frac{1}{2}$% of his bequest. How much had each then?

106. Having lost 33$\frac{1}{3}$% of my money, I have $ 84 remaining; what had I at first?

107. An attorney receives $ 1.26 for collecting a bill which is $\frac{3}{10}$% of the bill; what was the amount of the bill?

108. Drew out 25% of my deposit in a bank; of this I have spent $ 500, which is 8% of what I drew out; what have I remaining in the bank?

109. What per cent of one score is 1 doz.?

110. What per cent of £ 2 is 1 s.?

111. 9 gal. leaked out of a cask containing 120 gal. of oil; what per cent was lost?

112. A field which yielded 90 bu. of rye last year, yields 1 this year; what is the gain per cent?

113. If wood, which should be cut 4 ft. in length, falls short 4 in., what per cent should be deducted from the price?

114. What must I ask apiece for lamps that cost $ 4 a doz., that I may make 50%?

115. Sold a carriage for $ 240, which was 20% less than it cost; required the cost.

116. Lost $ 20 by selling goods at 25% below cost; what was the cost?

117. What was my property worth 10 years ago, if it has since increased 100%, and it is now worth $ 6000?

118. What must be the amount of my sales for a year, that I may clear $ 800 at a profit of 25%?

119. Bought paper at $ 2.75 per ream, and sold it at 20 cents per quire; what per cent did I gain?

120. What % is lost by selling a lot of goods for $\frac{3}{4}$ of their cost?

121. What % is gained by selling a lot of goods for thrice their cost?

122. Bought a cow for $ 87.50, which was 25% more than her real worth; what was her worth?

123. In settling with a person Jan. 1, 1859, I found I owed him $ 487.20; for this sum I gave my note on interest at 7%; what should I pay to discharge this note Oct. 20, 1859?

124. Suppose a note for $ 1958.42, dated Aug. 9, 1851, to be on interest at 6% till Feb. 15, 1852, when a payment of $ 1732.59 is made; what sum will remain due?

125. What is the amount of $ 250 for 1 y. 3 m. 18 d., at 8% per annum, interest compounding semiannually?

126. What is the true discount of $ 661.37½ for 3 m. 15 d. at 7%?

127. A bill for $ 240 is dated June 1, 1860, due in 8 m. 15 d.; what money will discharge it at date, by true discount at 10%?

128. What is the value, by true discount at 5%, of a bill for $ 139.50 Dec. 11, 1863, which is dated Sept. 9, 1863, and given for 1 year?

129. Find the bank discount on a 60 days' note for $ 500, dated May 10, and discounted June 9 at 6%. 2.75

130. A trader buys 450 pairs of shoes at \$ .75 a pair cash, and immediately sells them at \$ .90 on a note payable in 4 months without interest; suppose he gets his note discounted at a bank for the 4 months at 6%, what will he have made? 39.20.

131. For what must a note, dated Sept. 1, 1860, on 4 months, be given to yield at its date \$ 600, when interest is 7%? *Ans.* \$ 614.95.

132. What cash must be paid to discharge the above note at its date by true present worth ~~(without grace)~~?

133. What would be the avails of it at a bank Dec. 5, 1860? 411.34

134. What would be its cash value March 17, 1861? 428.24

135. What would be the true discount on it Nov. 5, 1860?* 4.74

136. What would be the bank discount on it Nov. 5, 1860? 7.17.

137. Change 25%, $16\frac{2}{3}$%, $37\frac{1}{2}$%, 95%, and $83\frac{1}{3}$% to their smallest terms, and give their sum in a common fractional number.

138. What is the simple interest of \$ 300 from May 5, 1860, to Feb. 2, 1862, at $2\frac{1}{2}$% a month? \$157.

139. What is the amount of \$ 271.36 at compound interest, for 2 y. 6 m. at 8%?

140. What is the amount at simple interest of £ 6 4 s. 6d. for 2 y. at 5%? (Change 4 s. 6 d. to decimal of a £.)

141. The interest of \$ 400 for a certain time at 6% was \$ 90; what was the time?

142. Given a note for \$ 2400, dated Sept. 5, 1862, on which were paid \$ 50 Jan. 29, 1863, \$ 500 July 1, 1864. The note being on interest at 6% from its date, what was due Sept. 5, 1864?

143. When gold is at a premium of 25%, what must be paid for \$ 375 of gold?

144. What are the net proceeds from the sale of 1260 barrels flour, at \$ 9 50 per bbl., charges for freight and storage being 40 c. per bbl., commission for selling being 2%, and for guaranteeing sales $1\frac{1}{4}$%?

145. If I buy 12 shares of stock, originally worth \$ 100, at 18% above par, and sell it at 7% below par, what do I lose?

146. How many shares of stock at \$ 100 each can a broker purchase with a remittance of \$ 525, allowing himself a brokerage of 5%? 5

147. Insured $\frac{2}{3}$ of a store valued at \$ 15000 at $\frac{7}{8}$%, and paid \$ 1 for the policy. What amount was paid?

* Without grace; see example 132.

148. What is the equated time for paying \$ 50 due in 5 m. from May 14, 1863, \$ 35 due in 4 m., and \$ 25 due in 3 m. from the same date?

149. A owes B \$ 2000 Oct. 5; if he should pay \$ 1500 of it Sept. 8, at what time should the balance be paid?

150. What is the specific duty, at 25 cents per gallon, on 25 barrels spirits turpentine, containing 32 gallons each, 5% being deducted for leakage?

151. My city tax is $1\frac{1}{2}$%, my State tax .15%, my poll tax \$ 2; the sum of my taxes is \$ 117.50; on what sum am I taxed?

152. A house which cost \$ 6000 rents for \$ 600 a year; if \$ 80 be deducted for taxes, etc., what per cent does the investment pay?

153. I received \$ 73.50 for the use of \$ 150 a certain time at 7%. Required the time.

154. Lent a certain sum for 1 y. 6 m. at 9%; the interest being \$ 9.45, what was the sum?

155. What principal will amount to \$ 63.25 in 1 y. 3 m. at 6%?

156. What must be the face of a note, which, discounted at a bank for 60 days and grace at 6%, would yield \$ 500?

157. In what time will the interest of a sum of money at 4% equal the principal?

158. What would be due May 1, 1865, on a note for \$ 1000, dated March 26, 1862, at 8% interest, on which \$ 200 was paid at the end of each year from the date of the note?

159. What is the cost of insuring \$ 3500 at \$ 17.50 on \$ 1000?

160. Paid \$ 28.77 for insuring my schooner at a premium of $\frac{1}{2}$%; what was the sum covered?

161. What is the difference between the true and bank discount of a 90 days' note for \$ 900, where the legal rate is 7%?

162. A debtor owes \$ 200, $\frac{1}{2}$ due in 2 months, $\frac{1}{4}$ in 4 months, and the remainder in 5 months; what is the equated time for paying the whole?

163. If I lose $33\frac{1}{3}$% by selling goods at 18 cents per yard, for what should they have been sold to gain 20%?

164. For what must hay be sold per ton to gain $12\frac{1}{2}$%, if by selling at \$ 16 the gain is $33\frac{1}{3}$%?

165. What would you receive from a bank, June 20, '68, for a note of $ 820, dated April 12, '68, payable 6 months after date, discount 10% ?

166. A takes a note on 2 months' credit for $ 100 in payment for a watch; on getting the note discounted at a bank at 6%, he finds that he has lost 16⅔% on the first cost of the watch. Required the cost.

167. If, by selling goods at 60 cents per lb., 25% is gained, what per cent would have been gained by selling them at 75 cents per lb. ?

168. Sold 4 ploughs at $ 24 each; on 3 of them I made 20%, and on 1 I lost 20%; what did I gain or lose on the whole?

169. If I buy coal at $ 4.12 per ton on 6 months' credit, for what must I sell it immediately to gain 40% ? (By true discount at 6%.)

170. If for the above coal I pay $ 3.90 cash, for what must I sell it on 4 months without grace at 6%, to gain 40% ?

171. If 25% of what I receive for an article is gain, what is the gain per cent?

172. What amount of current money must a broker give me for $ 20 in gold, when the premium on gold is 28% ?

173. If 2 men build 17 rods of wall in a week, how many rods will 100 men build in twice the time?

174. If 65 pairs of boots can be made from 85 lbs. of calfskin, how many pairs can be made from 150 lbs. ?

175. If $ 500 purchase 400 hats, how many hats can be purchased for $ 87½ ?

176. What time would be required for 5 men to mow an acre of land, if 4 men can mow it in 1½ days of 10 hours in length?

177. If my friend lends me $ 7000 for 15 days, for what time should I lend him $ 7500 to requite the favor?

178. A deer, 150 rods before a hound, runs 35 rods a minute; the hound follows at the rate of 42 rods a minute; in what time will the deer be overtaken?

179. How much cloth that is ¾ yd. wide will cover 14 tables 6 ft. long and 3 ft. wide?

180. How long must a piece of land be to contain 3 acres, if it is 8 rods wide?

181. If 120 rods of wall were laid by 63 men in 33 days of 14 hours each, how many rods would be laid by 77 men, working 12 hours a day for 8½ days?

182. If the wages of 75 men for 84 days were $ 68.75, for how many days could 45 men be employed for $ 41.25 ?

183. If the freight on 450 lbs. of merchandise is 30 cents for 26 miles, how many miles can 3 tons be carried for $ 14 ?

184. If 25 men, in $9\frac{1}{2}$ days of 10 hours each, build 200 rods of wall, how many rods would be built in 1 day of 8 hours by 12 men ?

185. If 5 men, working 12 hours a day for 4 days, cut 22 loads of wood, each 8 ft. long, 4 ft. wide, and 4 ft. 6 in. high, in how many hours would 16 men cut $49\frac{1}{2}$ loads 8 ft. long, 4 ft. wide, and 5 ft. 6 in. high ?

186. If 3 men dig a cellar 33.75 ft. long, 18 ft. wide, and 9.6 ft. deep, in 4.5 days of 11.25 hours each, in how many days of 11.7 hours will 12 men dig a cellar 15.2 yds. long, 7.8 yds. wide, and 10.8 ft. deep, the latter cellar being twice as hard to dig as the former ?

187. Two traders, A and B, shipped coal from Philadelphia to Boston. A had on board 350 tons and B 800 tons. It became necessary to throw overboard 250 tons. What was the loss of each ?

188. Banks & Ward traded in company, and gained $ 675. It was agreed that B. should have $ 8 of the gain as often as W. had $ 7. What was the share of each ?

189. Divide $ 7500 among three persons so that their shares shall be in the proportion of 3, 4, and 5.

190. A and B engaged in business, and gained $ 2008.25. A put in $ 4500 for 6 months, and B $ 5690 for 7 months. What was the gain of each ? A. 811.35 B 1196.90

191. A, B, and C formed a partnership. A furnished $\frac{2}{5}$ of the capital for the first 6 months, B $\frac{1}{3}$ of the capital for 10 months, and C the balance. At the end of 10 months their gain was $ 1560 ; what was the share of each ?

192. Hooker & Brown were in business together for 3 years, and gained $ 5750. Hooker put in $ 2000 for the first year, and $ 1500 for the other two ; Brown put in $ 1500 for the first two years, and $ 2500 for the last year. What was the gain of each ? H. — 738.10
B 3[illegible]

193. A and B received $ 857.50 for grading a road. A furnished 5 hands for 16 days, and 6 for 15 days ; B furnished 10 hands for 9 days, and 9 for 20 days. What was the share of each contractor ? A. 331.31 B. 526.19

194. A company of persons spent $ 3.61; each person spending as many cents as there were persons, how many cents did each spend? 19, 19

195. What is the length of one side of a square farm containing 102 acres, 80 rods of land? 128.06 rds +

196. There is a circular lot which contains $3\frac{1}{2}$ acres; what is the length of one side of a square lot whose area is the same?

197. What is the cost of fencing a square lot which contains 1 acre, at $ 4 per rod? $ 202.37

198. The side of a square is 8 ft. 6 in.; what is the side of a square having 15 times the area?

199. My orchard contains 1200 trees; the number of trees in width is to the number in length as 4 to 3; what is the number each way? 30 ft 40

200. What is the side of a square that will contain as many square feet as a rectangle whose sides are 225 and 81 feet respectively? 135 ft

201. The base of a right-angled triangle being 10 feet, the perpendicular 24 feet, what is the hypothenuse? 26 ft.

202. The hypothenuse of a right-angled triangle being 45.5 feet, the base 42 feet, what is the perpendicular?

203. What must be the length of a ladder to reach to the top of a chimney 52 feet high, the foot of the ladder being 20 feet from the chimney? 55.71 ft

204. How far from the foot of a flagstaff 45 feet high must a ladder 43 ft. 4 in. long be placed, that it may reach to within 5 feet of the top?

205. The diagonal of the floor of a square room is 120 feet; what is its area?

206. What is the length of one side of a cubical block of granite which contains $47\frac{325}{512}$ solid feet?

207. What will be the edge of a cubical pile of wood, composed of 1000 loads, each 8 feet long, 4 feet wide, and $4\frac{1}{2}$ feet high? 52.41 ft

208. What will be the length of a cubical pile of wood that will contain 100 cords? 23.39 ft.

209. What will be the length of a cube which will contain $\frac{1}{8}$ as much as another whose edge is 25 feet?

210. What is the depth of a cubical cistern which will contain 5 times as much as one whose depth is 5 feet?

320

160

0

160) 400 (2

320

800

1 5-30
6 1 2

1 5-3 2
6 1 3 6
6 0 2
6 0
1
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